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STEEL STRUCTURE



BY JASPAL SIR

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General Design Consideration.

- Design of steel structure consists of design of steel members and their connections so that applied load can be resisted and transferred safely & economically.
- Design of steel structure is based on following theories.
 - A) Working stress design method / Elastic method.
 - B) Ultimate load method / Plastic method.
 - C) Limit state method.

Note: → The basic difference b/w 3 design method is the manner in which safety is considered in designing

A) WORKING STRESS DESIGN METHOD. (WSM)

- In this method, designing is based on elastic theory.
- Here structures are analysed elastically for worst combination of working loads & the members are proportioned such that allowable or permissible stresses are never exceeded.
- Here permissible stress, is defined as ratio of yield stress to FOS (factor of safety). and this FOS is found on the basis of experience.

$$\begin{aligned} \text{i.e. Working stress} &\leq \text{permissible stress.} \\ &\leq \frac{\text{yield stress}}{\text{FOS.}} \end{aligned}$$

- This theory possess certain inherent drawbacks such as:

a) as in this case, linear elastic behaviour is assumed, reserved strength of material (strength beyond elastic limit) is not utilised, resulting

in uneconomical design.

- b) The assumption that working stress would be kept less than permissible stress is not possible due to.
- (i) stress concentration (Bolt, weld)
 - (ii) residual stress
 - (iii) other secondary effects. (Temperature change $\Rightarrow$ thermal stress)
(or sinking of support).
- c) It assumes, all the loads to act simultaneously with same degree of uncertainty.

B) Ultimate load method / Plastic method.

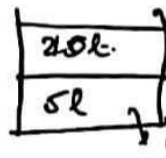
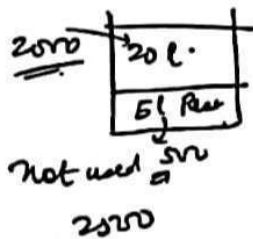
- In this method, the designing is based on ultimate strength.
- This method of design is based on failure condition rather than working load conditions.
- Here failure implies collapse or excessive large deformation.
- when sufficient plastic hinges are formed in the structure at maximum stressed point, a mechanism is developed, that causes failure of the structure.

Note: $\rightarrow$ When a system of load is applied to an elastic body, it deforms & shows resistance against it, this body is termed as STRUCTURE, but if no resistance is set up against deformation, then it is termed as MECHANISM.

- In this method, service loads are multiplied by the load factor. & cross-section of the members are selected on the basis of collapse strength.

i.e. $\boxed{\text{load factor} \times \text{working load} \leq \text{ultimate load or collapse load}}$

- Since, in this method, reserved strength of material (steel) is used, it results in smaller sized section w.r.t. those designed using working stress design method.
- However it does not consider serviceability conditions (deflection, vibrations, fatigue)
- Here FOS, is ^{not} considered on material strength.
- Also, all the loads are considered to be acting simultaneously with same load factor in this case.



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$$15L + 5L + 20L = 2000L$$

state after which our struct cannot perform its function.

C) Limit State Method.

- All the drawback of WSM, and ultimate load method are overcome in this case.
- In this method, partial safety factors for loading & material strength is applied (which is based on acceptable probability of failure)
- These partial FOS accounts for possible overload, and under strength of material.

$$\text{Factored load} = \text{characteristic load} \times \gamma_f$$

$$\text{Factored strength} = \frac{\text{characteristic strength}}{\gamma_m}$$

γ_f = partial safety factor for load

γ_m = partial safety factor for material strength

Note: → All the characteristic values are the values that are not expected to be exceeded with more than 5% probability during the life of the structure

Lesson 2, 14 Oct.

- Partial FOS on load (γ_f) accounts for the following.
 - a) Possibility of load exceeding characteristic load.
 - b) Possibility of inaccurate assessment of load.
 - c) Uncertainty in the assessment of effect of load.
 - d) Uncertainty in the assessment of limit state being considered.
- Partial FOS on material strength (γ_m) accounts for the following.
 - a) Possibility of strength falling below characteristic strength.
 - b) Reduction in the member size.
 - c) Reduction in the strength due to fabrication and tolerance.
 - d) Uncertainties in the calculation of strength of the member.

Note: → Here limit states are the states beyond which the structure no longer satisfies the performance requirements.

- These limit are classified as follows :
 - A) Limit state of collapse / strength.
 - B) Limit state of serviceability.

- Limit state of collapse / strength includes

- a) loss of equilibrium of structure as a whole or any of its part or components.
- b) loss of stability of the structure (including the effect of sway & overturning) on any of its part or structure or as a whole.
- c) Failure by excessive deformation, rupture of structure or any of its parts.
- d) Fracture due to fatigue.
- e) Brittle failure

Note: For brittle failure to occur, a combination of tensile stress, low temperature, thick material & rapid change of stress

⇒ Limit state of serviceability includes

- a) It includes deformation and deflection which may adversely affect the appearance or use of structure.
- b) vibration in the structure or any of its component causing discomfort to the occupants and limits the functioning of structure.
- c) Corrosion and durability.
- d) Fire resistance.
- e) Ponding of structure (accumulation of water)
- f) Repairable damage due to fatigue.

In LSM, structures are designed for limit state of collapse and are checked for limit state of serviceability.

Note: $\rightarrow$ In IS 800 : 2007, design criteria are largely based on LSM; however it still retains the ^{WSM} design by, but it accounts for reserved strength also, beside elastic strength.

Partial safety factor for material strength (γ_m)

Description	Partial safety factors for material (γ_m)	
a) Resistance governed by yielding	1.1	
b) Resistance of member to buckling	1.1	
c) Resistance governed by ultimate stress	1.25	
d) Resistance of connections	shop fabrication	field fabrication
(i) Bolted connection, friction type (γ_{mf})	1.25	1.25
(ii) Bolted connection, bearing type (γ_{mb})	1.25	1.25
(iii) Riveted connection (γ_{mr})	1.25	1.25
(iv) Welded connection (γ_{mw})	1.25	1.5

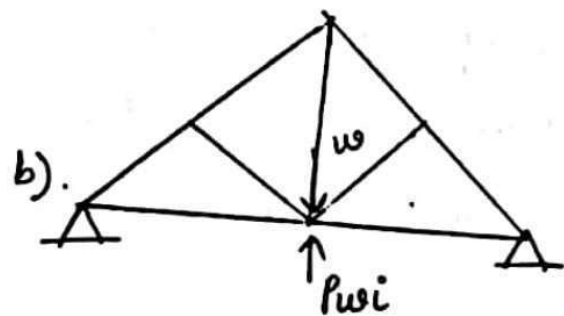
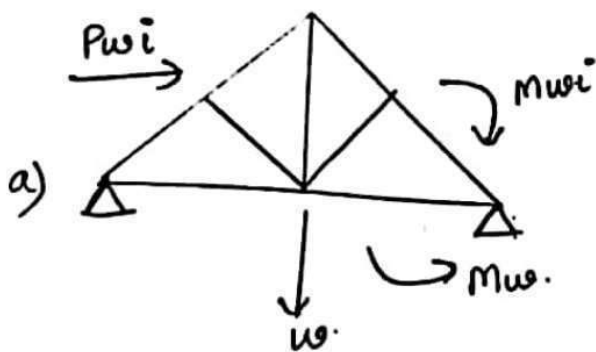
Partial safety factor for load (γ_f)

Combination	Limit state of collapse.				
	DL	LL		EL/WL	AL
		leading LL	accompanying LL		
a) DL + LL + CL	1.5	1.5	1.05	—	—
b) DL + LL + CL + WL/EL	1.2	1.2	1.05 + EL	0.6 + EL	—
	1.2 + WL	1.2 + WL	0.53 + WL	1.2 + WL	—
c) DL + WL/EL	1.5	—	—	1.5	—
	(0.9)*	—	—	—	—
d) DL + ER	1.2	1.2	—	—	—
	(0.9)*	—	—	—	—
e) DL + LL + AL	1	0.35	0.35	—	1.0

Combination	Limit state of serviceability.				
	DL	LL		EL/WL	AL
		loading LL	accompanying LL		
a) DL + LL + CL	1	1	1	—	—
b) DL + LL + CL + WL/EL	1	0.8	0.8	0.8	—
c) DL + WL/EL	1	—	—	1	—
d) DL + ER	—	—	—	—	—
e) DL + LL + AL	—	—	—	—	—

Note 1: γ_f for DL = 0.9* is to be considered when it contributes to the stability against overturning or causes stress reduction.

For eg. $\rightarrow$ a) overturning of roof truss caused by wind
 $\rightarrow$ b) wind uplift in roof truss



Here DL = dead load

LL = live load

CL = crane load

AL = accidental load (such as those due to blast)

SL = snow load. (Type of LL)

EL = earthquake load

WL = wind load.

ER = erection load.

ER = erection load

such as due to storage or positioning of construction, erection of equipment, provision of temporary bracings etc.

Note 2: → When action of different live load is simultaneous the leading live loads are one which causes higher load effects in the member / section and all the remaining live loads are classified as accompanying live loads.

Note 3: → Wind load & earthquake load are not considered to act simultaneously.

2. The load on a floor beam of a commercial building are as follows.

Roof load

Dead load = 12 kN/m^2 .

live load = 9 kN/m^2 .

Roof finish = 4 kN/m^2 .

Determine the design load for

a) limit state of strength

b) limit state of serviceability.

$$DL + LL \neq 12 + 9 \neq 21.$$

a) for limit state of strength

$$(DL + LL) \times 1.5$$

$$DL = 12 + 4 = 16$$

$$LL = 9.$$

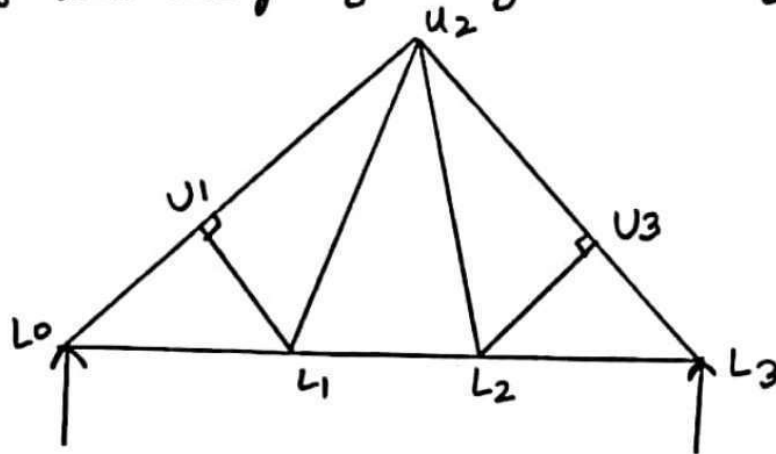
$$\begin{aligned} \text{factored} &= 1.5(16 + 9) \\ &= 37.5 \text{ kN/m}^2 \end{aligned}$$

b). $(DL + LL) \times 1$

$$\Rightarrow 16 + 9$$

$$= 25 \text{ kN/m}^2$$

Q The primary force in the member of principal rafter in a roof truss of an industrial building is given below; due to DL, LL, & WL. Determine the design force for the rafter.



(+) → Tension
(-) → Compression

Member	DL (KN)	LL (KN)	WL (KN)
L0U1	-19	-19	65
U1U2	-16	-16	65
U2U3	-16	-16	65
U3L3	-19	-19	65

load combination.

- $1.5 (DL + LL)$
- $1.2 (DL + LL + WL)$
- $0.9 DL + 1.5 WL$

Member	$1.5 (DL + LL)$	$1.2 (DL + LL + WL)$	$0.9 DL + 1.5 WL$
L0U1	-57	32.4	80.4
U1U2	-48	39.6	83.1
U2U3	-48	39.6	83.1
U3L3	-57	32.4	80.4

Hence, rafter will be designed for 83.1 KN (Tension) load & will be checked by 57 KN (comp) load.

Note 4: for 2nd load combination γ_f in case of WL being taken.

DL = 1.2, Leading LL = 1.2, accompanying LL = 0.53.

WL = 1.2

for limit state of collapse.

Lesson 4 Oct 16

Loads

- For the purpose of designing of any element or member or structure, following loads & their effects must be considered

- Dead load (IS: 875 Part-I)
- Live load (IS: 875 Part-II & IV)
- Wind load (IS: 875 Part III)
- Earthquake / Seismic load (IS: 1893).
- Erection load (IS 875 Part-II)
- accidental load.
- secondary effects due to contraction & expansion resulting from temp. changes, differential settlement, eccentric connections.

(C) WIND LOAD. (as per IS 875 Part III)

- The wind pressure at any height of a structure depends on the following.

- The velocity and density of the air.
- The height above the ground level
- The shape and aspect ratio of the structure.
- The topography of the surroundings ground surface.
- The angle of wind attack.

- It is analysed as follows :-

(i) Design wind speed. (V_z)

$$V_z = V_b K_1 K_2 K_3$$

V_z = design wind speed at a height 'z' m in m/s.

V_b = basic wind speed. as given in code.

K_1 = probability factor or risk coeff.

K_2 = Terrain, height and structure coeff.

K_3 = Topographic factor.

(ii) Design wind pressure (P_z)

$$P_z = 0.6(V_z)^2$$

P_z = design wind press. at "z" m in N/m^2 .

(iii) Design wind load (F)

$$F = C_f A_e P_z$$

C_f = force coeff. of the structure.

A_e = effective frontal area.

P_z = design wind pressure.

Mechanical Properties of steel.

a) Modulus of Elasticity (E) = 2×10^5 MPa.

b) Shear modulus (G) = 0.769×10^5 MPa.

c) Poisson's ratio (μ).

(i) Elastic range = 0.3

(ii) Plastic range = 0.5

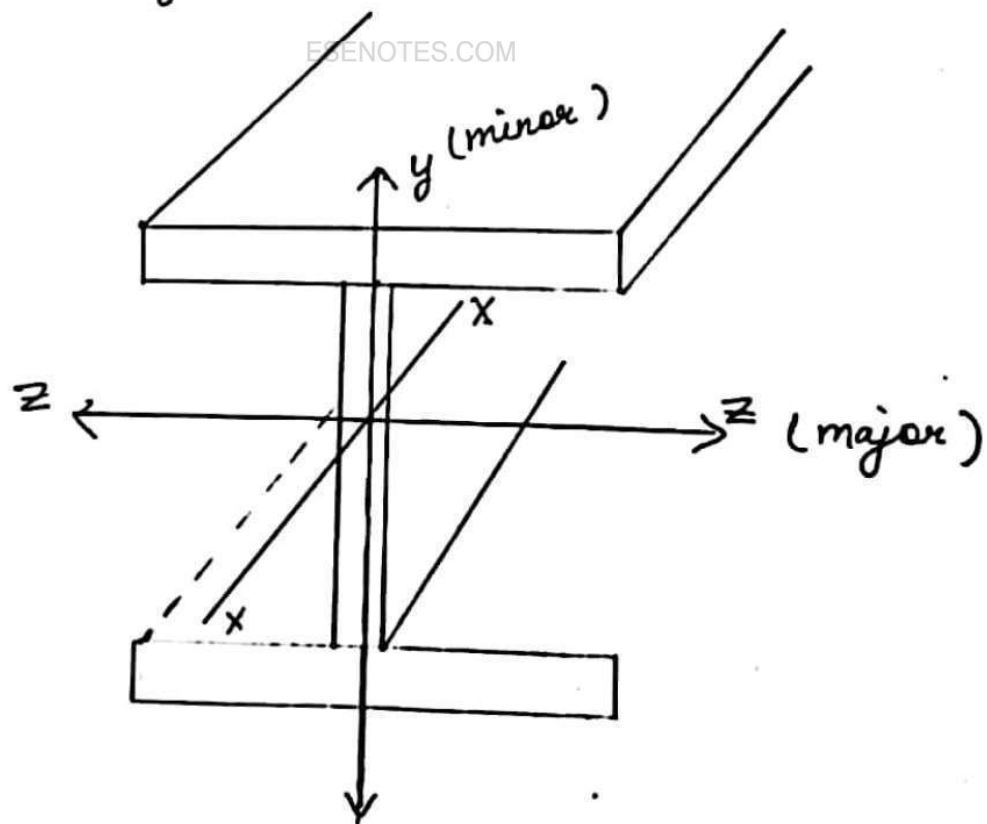
d) Density (ρ) = 7850 kg/m^3

e) Ultimate & yield strength as per code.

→ Pg-14

Grade designation	Yield strength (MPa)			Ultimate str. (MPa)
	Thickness (mm).			
Engineering steel	< 20	(20-40)	> 40	
yield str. (MPa) ultimate str. E250 (Fe 410) A E250 (Fe 410) B E250 (Fe 410) C	250	240	230	410
E300 (Fe 440)	300	290	280	440
E350 (Fe 440)	350	330	320	490
E410 (Fe 540)	410	390	380	540
E450 (Fe 540)	450	430	420	570
E450 (Fe 590)	450	430	420	590

Convention for member Axis.



$x-x$ axis $\Rightarrow$ along the length of member

$y-y$ axis $\Rightarrow$ axis of cross-section

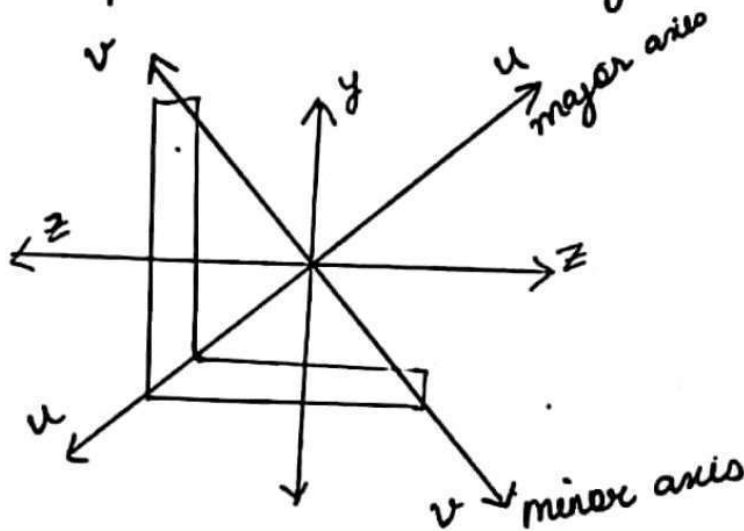
a) an axis $\perp$ to flange.

b) an axis $\perp$ to the smaller leg in angle section

$z-z$ axis $\Rightarrow$ axis of cross-section.

a) an axis parallel to flange.

b) an axis parallel to smaller leg in angle section.



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$u-u \rightarrow$ major axis

$v-v \rightarrow$ minor axis.

CONNECTIONS.

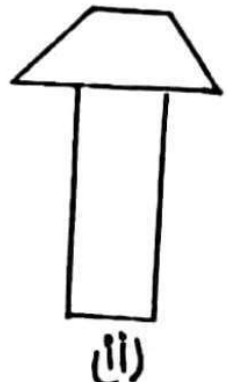
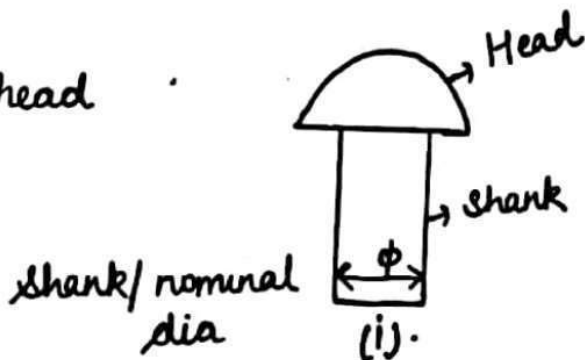
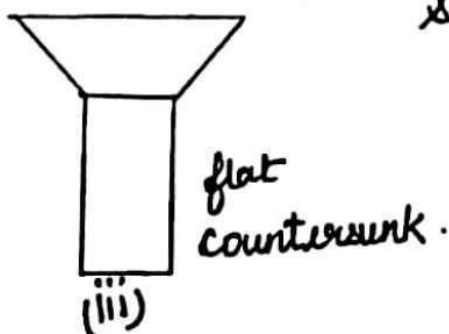
- In steel structure, various types of elements are connected using different types of connections as follows.

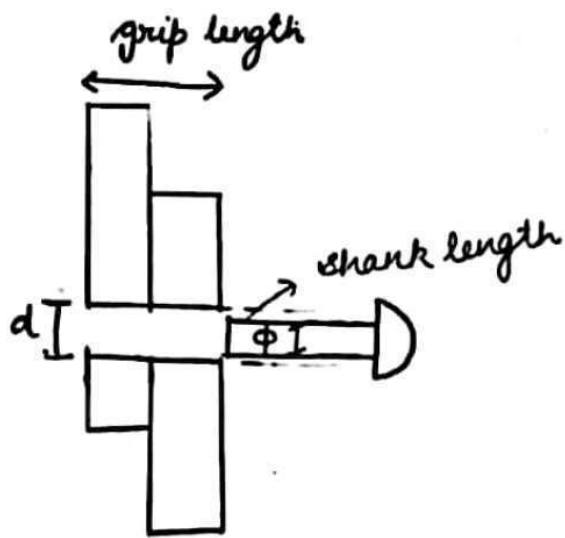
- a) Riveted connection
- b) Bolted connection
- c) Welded connection

(A) RIVETED Connection.

- In the riveted connection, rivets are inserted in the hole made to join the two members together and hammering is done to make the head on the other side.
- A rivet is made up of round ductile steel bar piece either mild or high tensile, called as shank, with head on one side (end).
- The shank is made of length to extend through the hole parts to be connected & with sufficient extra length for second head to be made on the other side.
- Head of the rivet may be of following type.

- (i) Snap head
- (ii) Pan head
- (iii) countersunk head
- (iv)





- Shank length $>$ grip length
- When the grip of rivet becomes more (long), the rivet is subjected to bending along with bearing and shearing.
- grip length should not be more than $4 \times \text{diameter} \cdot (d)$

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- Riveting can be done as (a) cold riveting (b) Hot Riveting

a) In cold riveting process, riveting is done at normal temperature requiring higher clamping force & equipment for making the head on the other side, which makes it uneconomical.

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ⓑ) Although the strength of cold riveting is more, its use is limited due to its cost

ⓐ) Cold riveting is not preferred adopted for $\text{dia} > 10\text{mm}$

b) In hot riveting, rivets are heated to $550-1000^\circ\text{C}$ & hammering is done on the other side to make the head

- During hot riveting when temp is increased, dia & length of rivet increases, but after cooling both dia & length shrinks leading to the development of residual tension in rivet & ^{some} compression in plates to be connected.
- This compression causes friction to slide b/w the plate to be connected & is called as clamping Action

- Hammering of the rivets can be done manually or mechanically accordingly they are termed as.

a) Power driven rivets b) Hand driven rivets.

- Power driven rivets have better quality control and hence have a higher permissible stress.

- Riveting can be done in the factory or in the field and accordingly they are classified as

a) Shop riveting b) field riveting.

⇒ The nominal dia of rivet is said to be shank, dia under cold condition & gross dia of rivet is taken as dia of hole.

⇒ The strength of the rivet is based on its gross dia under the assumption that rivet fills the hole completely.

⇒ For ease in connection, dia of hole is taken to be larger than dia of rivet; as follows. (as per IS code)

$$\begin{aligned} \text{Gross dia} &= \text{nominal dia} + 1.5 \text{ mm, for nominal dia} \leq 25 \text{ mm} \\ &= \text{nominal dia} + 2 \text{ mm, for nominal dia} > 25 \text{ mm} \end{aligned}$$

- Due to several drawbacks riveting is obsolete now

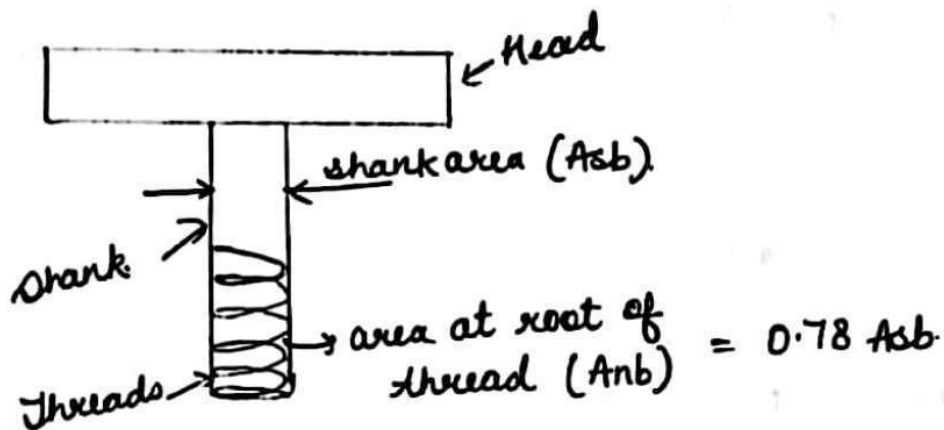
- Design of riveted connection is same as that of Bolted connection, but with following differences ⇒

a) The dia of rivet to be used in the calculation is dia of hole whereas for bolted connection, it is the nominal dia.

b) The design stresses are different for both connections

B) BOLTED CONNECTION

- The process of inserting the bolt in the hole created in the plates to be connected is termed as BOLTING.
- Bolt is a metal pin with head at one end and a shank threaded at the other end to receive the nut.



- Various type of Bolts available for connection are as follows.

- a) Black bolt / Ordinary bolt / Unfinished Bolt.
- b) Turned bolt / Close tolerance bolt.
- c) high strength bolt.

a) Black Bolt.

- It is the least expensive amongst all & is used for light structure subjected to static load or for secondary member such as Bracing, purlin etc.
- They are not recommended for connections subjected to impact loading, vibrations & fatigue or stress reversal.
- These bolts are available from 5mm - 36mm in dia. & are designated as M5 to M36.

(Here M stands for metric).

- Bolt of property class. 4.6 means

- a) Ultimate strength of bolt (f_{ub}) = $4 \times 100 = 400 \text{ N/mm}^2$
- b) Yield " " " (f_{yb}) = $0.6 \times 400 = 240 \text{ N/mm}^2$

b) Turned Bolt / Close Tolerance Bolt.

- It has small tolerance & are used in no slip condition
- They are mainly preferred for machines and under dynamic loading condition.

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c) High strength bolt

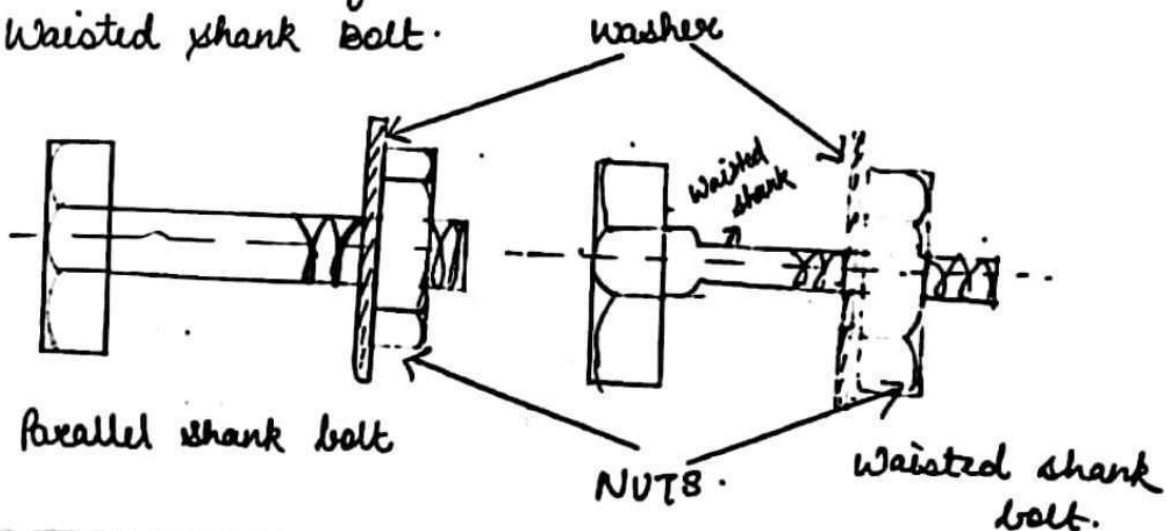
- These bolts are made from high strength steel.
- They can be tightened to give high shank tension. and are also termed as "HIGH STRENGTH FRICTION GRIP" (HSFG) Bolts. and joint using these bolts are termed as "NON-SLIP connection / slip critical connection friction type connection".

- These bolts are of 2 types.

a) Parallel shank.

b) Waisted Shank.

Note $\rightarrow$ A bolt whose shank dia is less than 0.9 times the root dia of the thread is termed as Waisted shank Bolt.



- These bolts (waisted bolts) are preferred when they are subjected to cyclic bending. as waisted shank, results in less fatigue as shank area is reduced.

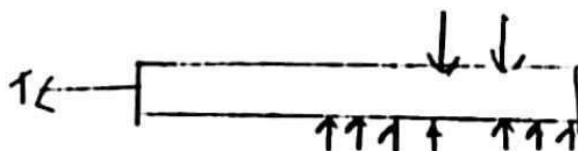
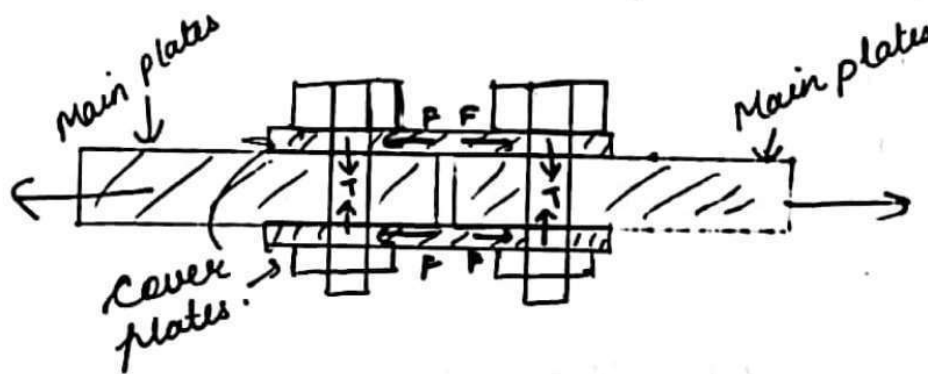
- Steel washers of hard steel / carburized steels are provided in such cases as it helps in.

a) Evenly distribution of clamping pressure on the bolted member.

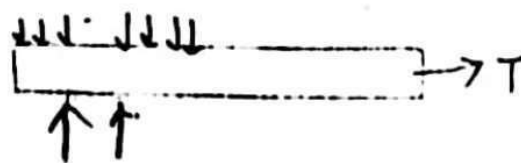
b) It helps in preventing the threaded portion of the bolt from bearing on the connecting parts.

⇒ Since these bolts are tightened upto proof tension the connected parts are clamped tightly together between the bolts and nut which permits the load to be transferred primarily by friction & not by shear.

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Frictional force.



- High strength bolts are available in size ranging from 16-36 mm & are designated as M16, M20, M24 etc.
- These bolts are identified by the 'property class' as 8.8S and 10.9S where 'S' stands for high strength.
- The material cost of these bolts is 50% more but overall cost is less.
- Less manpower is required in this case.
- Noise nuisance is less.
- It provides rigid joint; hence no slip & due elements takes place.
- They are suitable to be used for structure subjected to cyclic loading.

Classification of Bolted Connections.

- These connections can be classified on the basis of following.

A) On the basis of resultant force transferred.

(i) concentric connection

(ii) eccentric "

(iii) moment resisting connection.

(i) If the load passes through C.G. of section, it is termed as concentric connection.

eg → axially loaded tension & compression members.

(ii) If the load is away from the C.G. of the connection, it is termed as eccentric connection.
eg → Bracket connection.

(iii) When the joints are subjected to moment, it is termed as moment resisting connection.
eg $\Rightarrow$ beam to column connection.

B) On the basis of type of force.

- (i) Shear connection
- (ii) Tension connection
- (iii) Combined shear & tension connection.

(i) When the load transfer from the member to the connection is through shear, it is termed as shear connection. eg. Lap Joint., BUTT JOINT.

(ii) When the load is transferred by tension on connection it is termed as TENSION CONNECTION.
eg $\rightarrow$ Hanger connection.

(iii) If the load is transferred by both shear & tension it is termed as combined connection.
eg $\rightarrow$ Bracings.

C) On the basis of force mechanism.

- (i) Shear & bearing connection.
- (ii) Friction connection.

(i) If the load transfer from one connected part to another may be by shearing & bearing of the bolts, it is termed as bearing type connection. / Slip type connection.

(ii) If the force is transferred by friction b/w the plates due to tensioning of bolts, it is termed as friction type / slip critical connection.

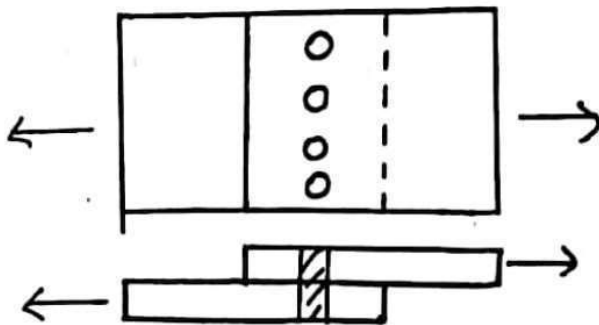
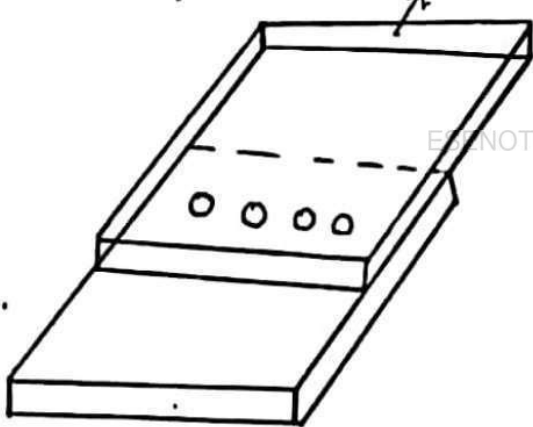
Lesson 8 Oct 21

Type of Belted Joints

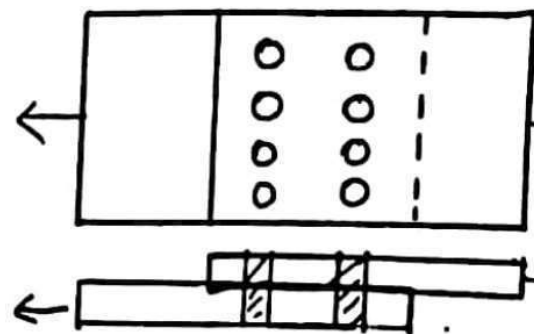
- Belted joints which are subjected to axial forces are as follows:-

A) LAP JOINT

- The two members to be connected are overlapped and connected together in this joint with the help of belts
- If single layer of belts is being provided for connection, it is termed as Single Bolted Lap Joint.
- If two layers of belts are being provided for connection, it is termed as Double Bolted Lap Joint.

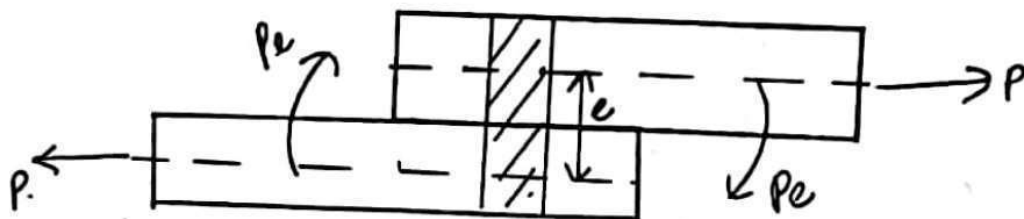


Single Bolted Lap Joint



Double Bolted Lap Joint.

- The load in lap joint has eccentricity, as centre of gravity of load in 1 member and the C.G. of load in second member are not in the same line.
- Hence, due to this couple is formed, which that causes undesirable bending. in the connection & bolt may fail in tension.
- To minimise the effect of bending, at least two bolts in a line must be provided.
- Also, distribution of stresses is not uniform across the contact area, due to this eccentricity.

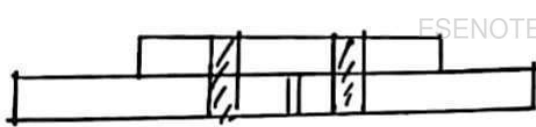
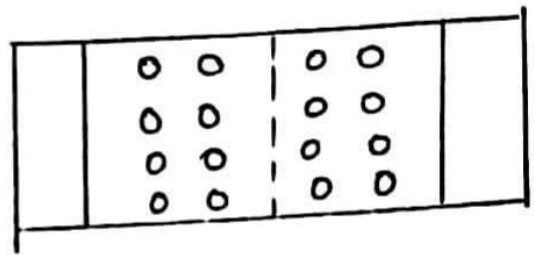
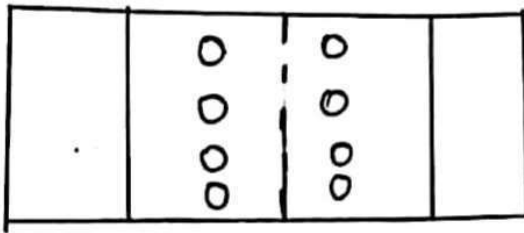
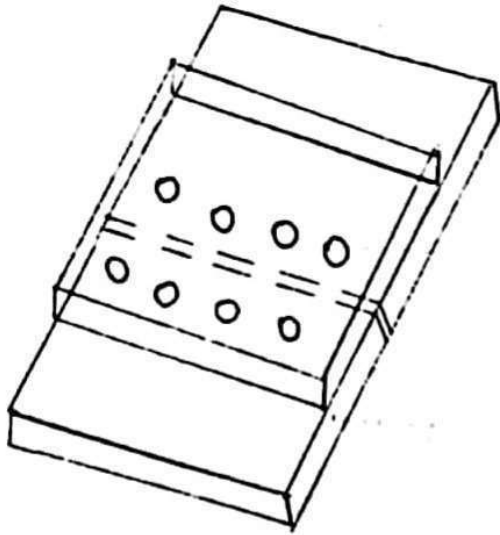


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B) BUTT JOINT

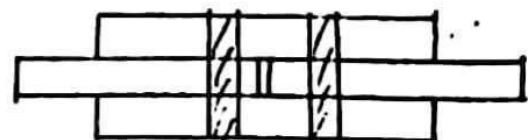
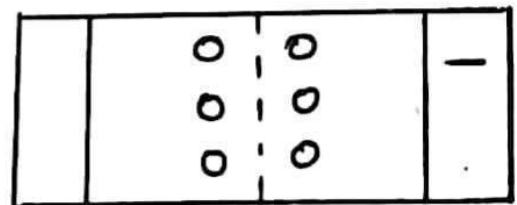
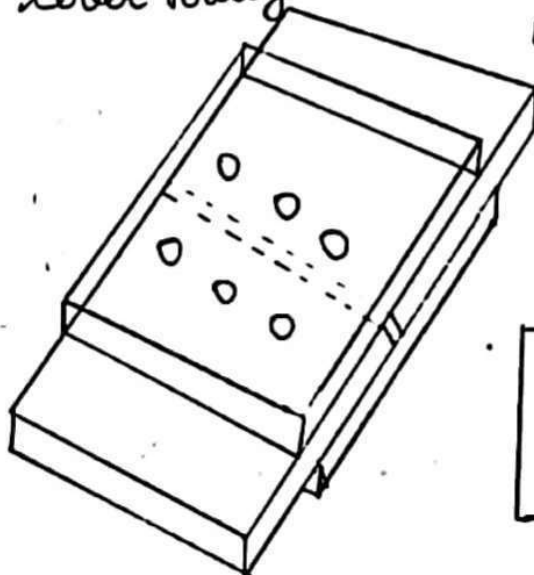
- In this case 2 members to be connected are placed end to end and additional plates on either one side or both sides (cover plates) are placed.
- According to which, these are termed as "Single Cover Butt joint" & "Double cover butt joint".
- These joints are preferred over Lap joints as
 - (i) In case of double cover butt joint, eccentricity of forces is eliminated, hence no bending takes place.
 - (ii) In case of double cover butt joint, the total shear force to be transmitted by the members is split into two parts. & force acts on each half, hence shear capacity of bolt in this

Case is doubled than in lap joint.

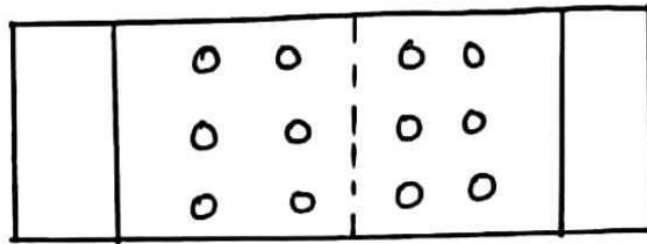


Single cover Butt joint.

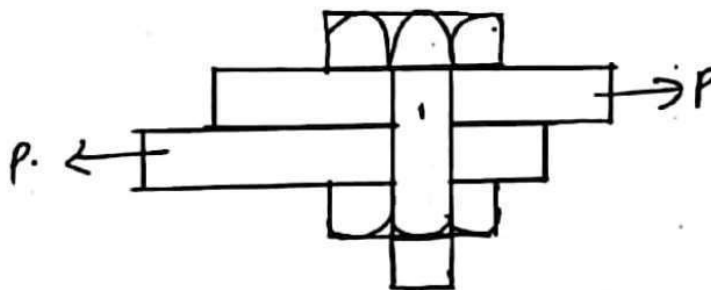
Single cover double bolted Butt joint.



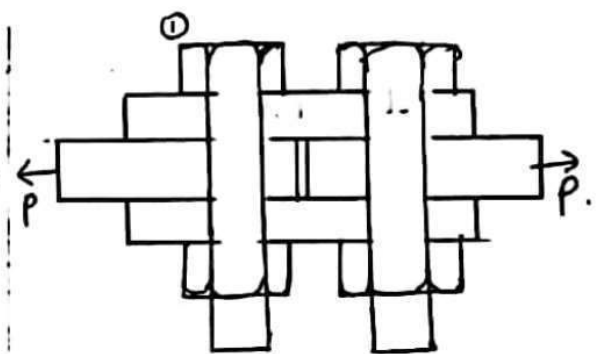
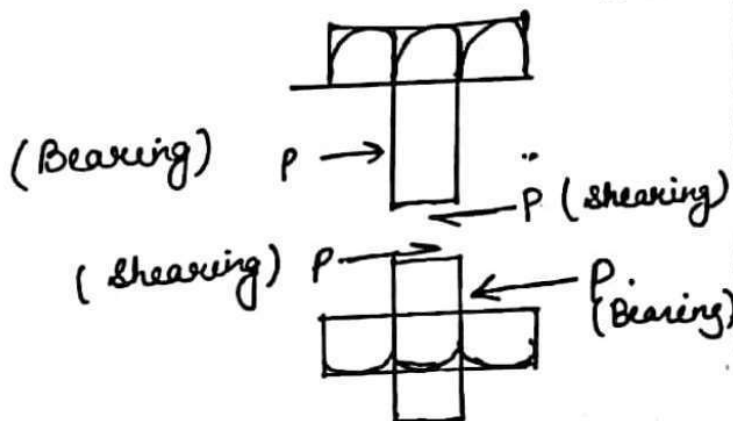
Double cover single bolted butt joint



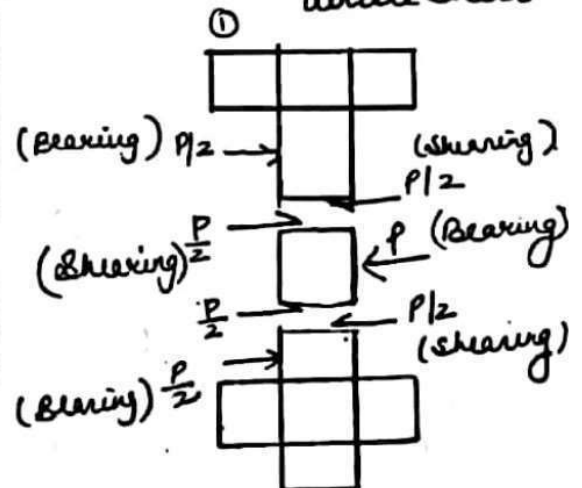
Double cover
double bolted
butt joint.



LAP JOINTS: Bolts are in single shear.

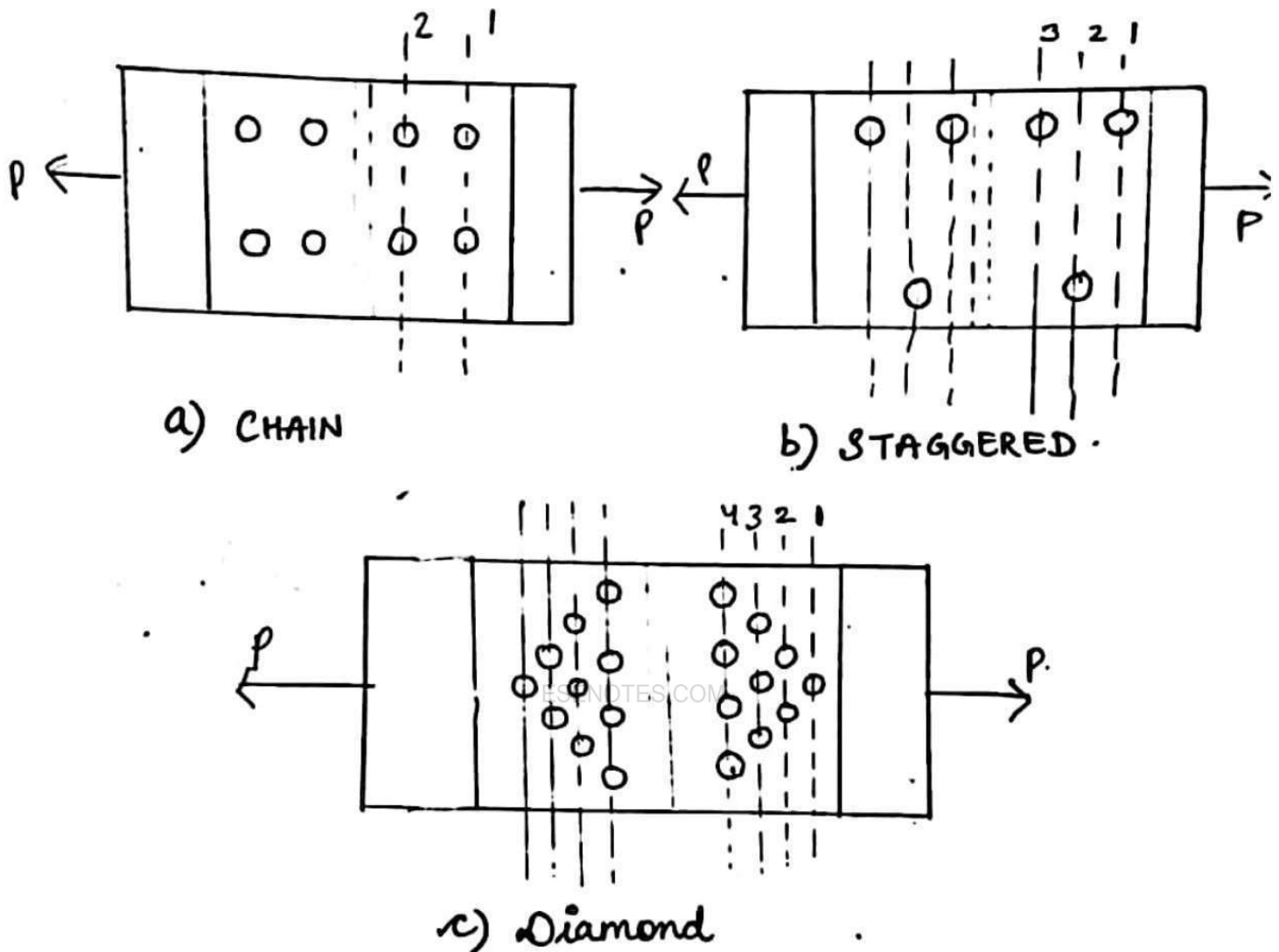


BUTTJOINT: Bolts are in double shear



Lesson 9. Oct 22

Patterns of Bolted Joints.



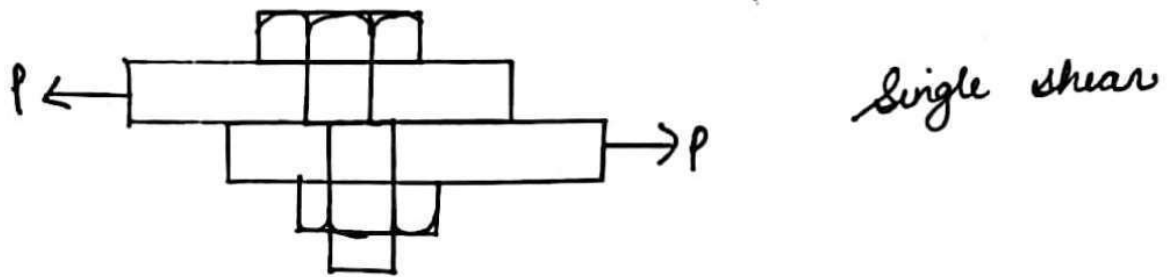
- The bolts may be placed in variety of patterns, depending upon the space available for connection & the shape of the members to be connected.
- The most common type of Bolt pattern are
 - A) CHAIN BOLTING.
 - B) STAGGERED CHAIN BOLTING
 - C) DIAMOND BOLTING.
- Staggered chain bolting is preferred over chain bolting as it results in a greater net area of the member.

- The diamond pattern is the most efficient one amongst all.

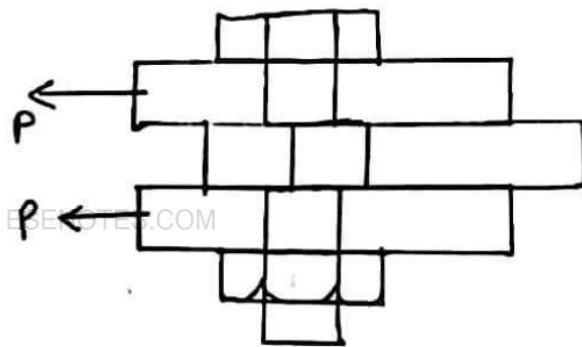
Modes of Failure of Bolted Joint.

- The bolted joint may fail in any of the following ways.

(a) Shear failure of Bolt



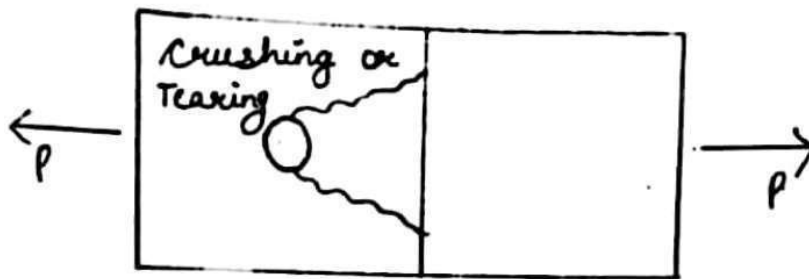
Double shear.



- Shear stresses are generated when the plates slip due to applied forces.
- If the maximum design shear force exceeds the shear strength of the bolt, shear failure of the bolt takes place (shear) at shear plane.
- The bolt may fail in single shear or in double shear.

b) Shear failure of Plate.

- If the design shear stress is less than shear strength of bolt but more than shear strength of plate, it leads to tearing or crushing of plate due to shear.



C) Bearing failure of Bolts.

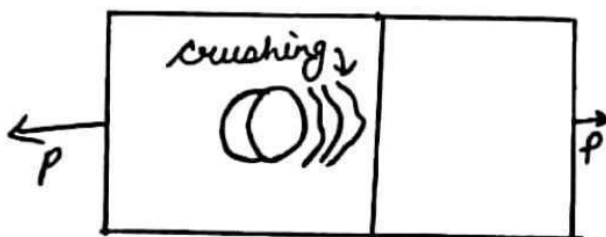


- The bolts may get crushed around half circumference when the connected plates slip due to applied force.
- If plate is strong in bearing and the heaviest stressed plate

may press the bolt shank.

- It occurs when plates are made of high strength steel & bolts are of very low grade steel.

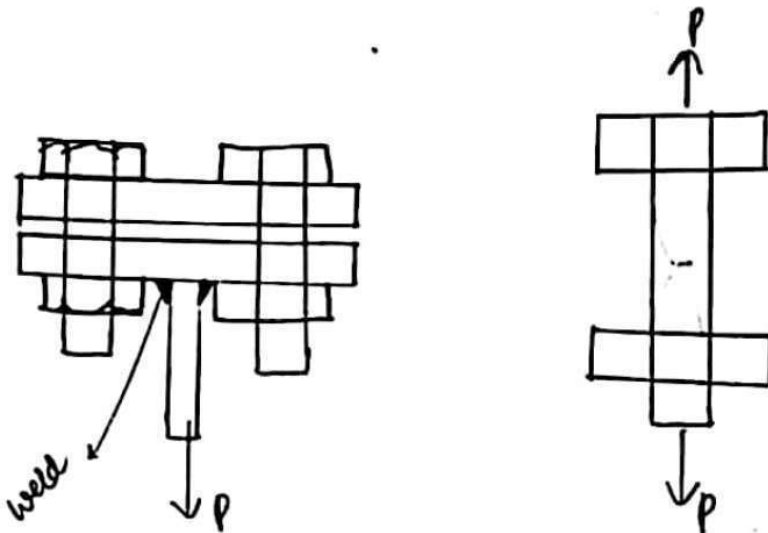
D) Bearing failure of plate:



- When a connection with ordinary bolts is subjected to shear force, slip takes place & bolts come in contact with plate

- If bearing strength of plate is less, it may get crushed, due to bearing.

E) Tension failure of Bolt.



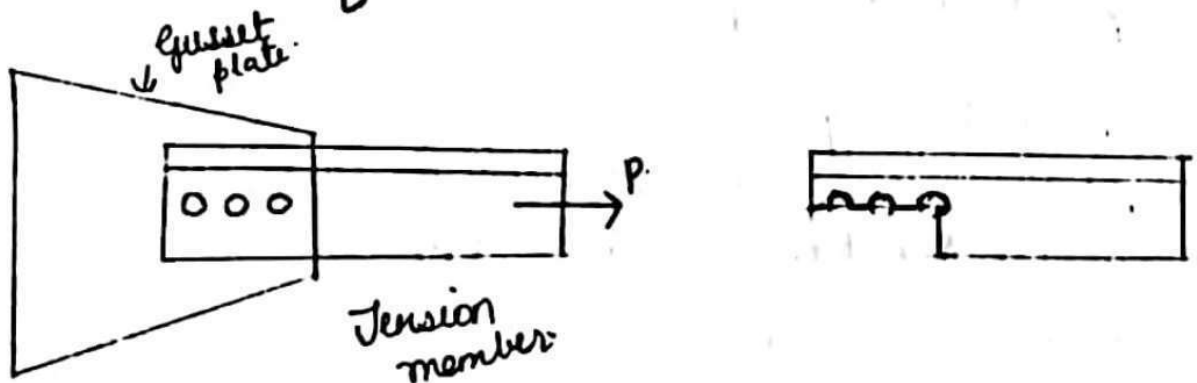
- When subjected to tension, bolts may fail at the stressed area.

F) Tension / Tearing failure of plate.



- The tension failure of a plate in rupture occurs when the bolts are stronger than the connected plates.

G) Block Shear failure



- The failure of connection in block shear occurs when a block of material within the bolted area breaks away from the remainder area.

- This failure occurs when shear on one plane & tension on another (perpendicular) plane occurs.

Lesson 10 Oct 23

Specification for Bolted Joints.

(A) Diameter of Bolts

- In general, a connection with few large-diameter bolts costs less than one of the same capacity with many small diameter bolts
- As lesser are the bolts, lesser are the no. of holes to be formed & less is the installation work.
- The larger diameter bolts are favourable where the connection is governed by shear
- Under normal situation, the dia of bolt hole is made larger than the shank dia in order to facilitate erection.
- Dia of Bolt hole (d_o) in relation to dia. of bolt (d) is

$\xleftarrow{\text{12-14 mm} \atop +1 \text{ mm}} \quad \xleftarrow{\text{16-24 mm} \atop +2 \text{ mm}} \quad \xleftarrow{\text{>24 mm} \atop +3 \text{ mm}} \rightarrow$

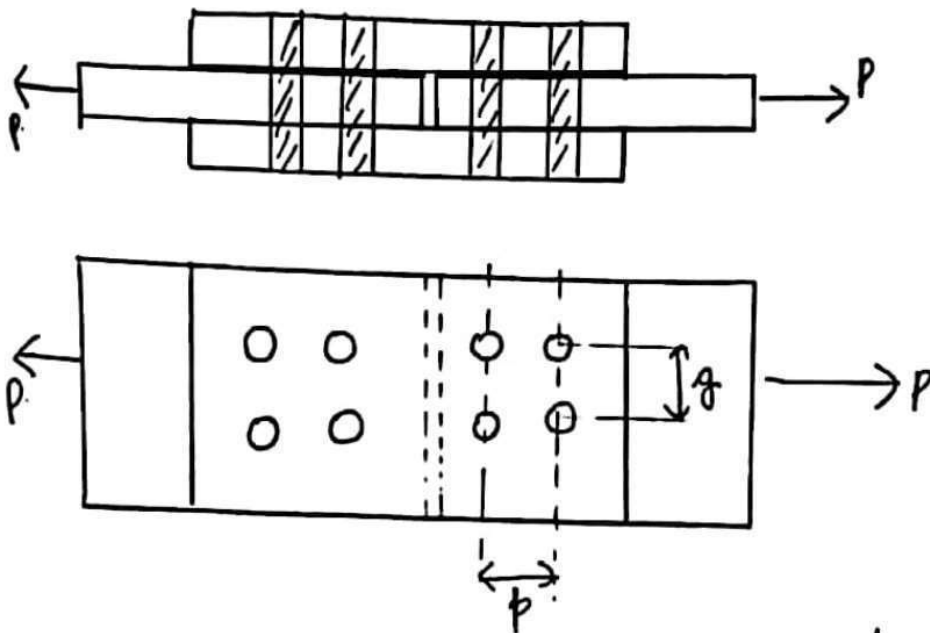
Nominal dia. of Bolt (d) (mm)	12	14	16	20	24	30	36
Dia of hole (d_o) (mm)	13	15	18	22	26	33	39

Note → Oversized holes & slotted holes are used in slip critical or slip resistant connection

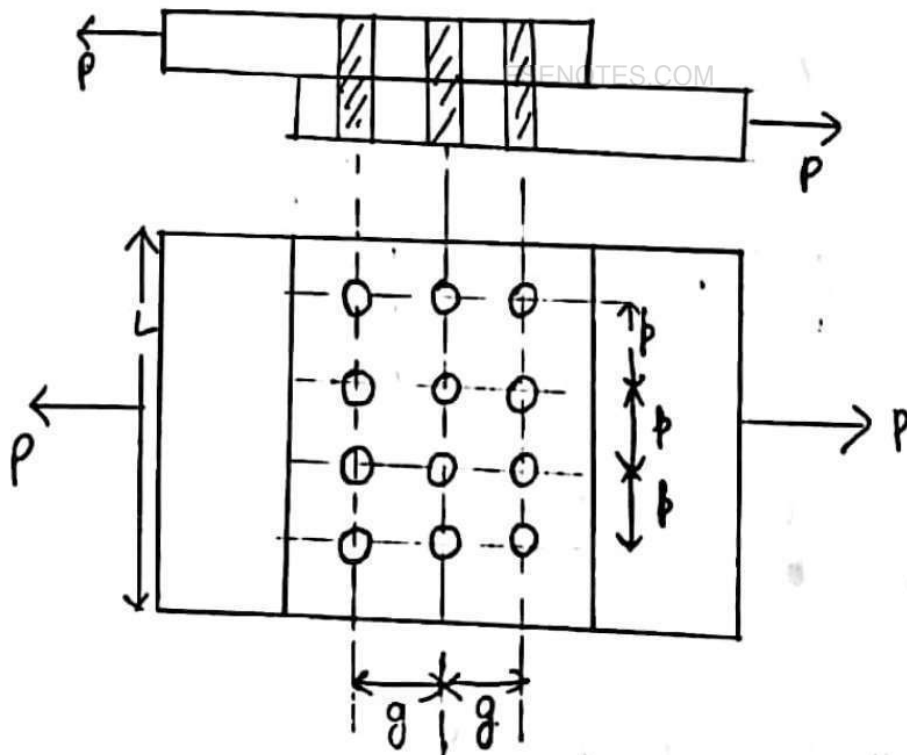
(B) Pitch & Gauge.

- Pitch is the distance between two consecutive bolts in the direction of force. (p)
- Gauge is the distance b/w two consecutive bolts in the direction perpendicular to the direction of force. (g)





Note: For wide plates, pitch may also be defined as c/c distance of bolt measured along the length of the member or connection.



(i) Minimum pitch & Gauge.

- $\boxed{\text{Min pitch \& gauge} = 2.5d}$

where d = nominal dia of bolt
or gauge

- Minimum pitch ^{or gauge} is ensured

(a) to prevent bearing failure of members between the two bolts.

(b) to permit efficient installation of bolts i.e. to prevent overlapping of the washers & to provide adequate resistance to tear out of bolts.

(ii) Maximum Pitch & Gauge.

Q For 1) Pitch.

a) for tension member, $p \nless (16t, 200\text{mm})_{\text{min}}$

t = thickness of thinner member.

b) for compression member, $p \nless (12t \text{ or } 200\text{mm})_{\text{min}}$

c) For rows near the edge, $p \nless (4t + 100\text{mm})_{\text{min}}$ or 200mm

d) for staggered pitch, where the gauge does not exceed 75mm , the pitch in part (a), (b), (c) can be increased by 50% .

Note: when bolts are staggered the pitch will be referred as staggered pitch.

2) Gauge.

a) maximum gauge (g) $\nless (32t, 300\text{mm})_{\text{min}}$.

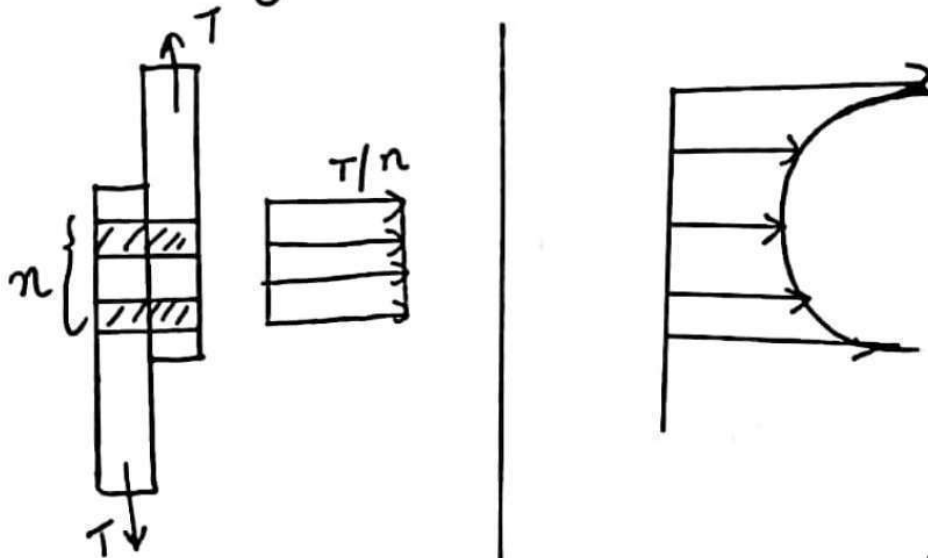
t = thickness of thinner member.

- Max. pitch or gauge is ensured for following purposes:

a) To reduce the length of the connection & gusset plate so as to reduce cost.

b) In case of compression member, to avoid the problem of local buckling.

c)



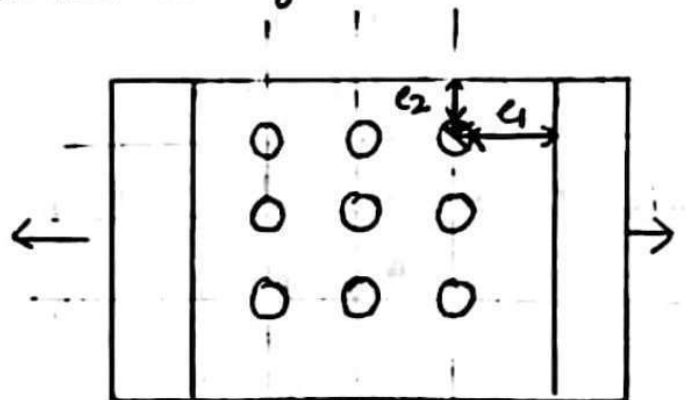
- To have uniform stress in the bolts, the joint should be compact.
- It is assumed that the load on the joint is distributed equally amongst all the bolts.
- But in long joints (> 15 times dia. of bolt) the outer bolts are stressed more and its failure may be progressive in nature which is termed as unbuttoning.

C) EDGE AND END DISTANCE

- Distance from the centre of bolt to the adjacent edge of the member at right angles to the direction of stress or force is called 'edge distance' while in the dirⁿ of stress or force is called as 'end distance'.

e_1 : end distance

e_2 : edge distance.



Lecture 11 Oct 24

Minimum end or edge distance = $1.7d_o$, for sheared or hand flame cut edges.

= $1.5d_o$, for machine cut and planed edges.

d_o = dia of bolt hole

Nominal dia of Bolt (mm). (d)	Dia of bolt hole (mm) (d_o)	e_1 or e_2	
		To sheared or hand flamed cut edges	To rolled and machine cut edges
12	13	20	19
14	15	26	23
16	18	30	27
18	20	34	30
20	22	37	33
22	24	40	36
24	26	44	39
30	33	56	50
over 33	$d + 3$	$56 + 1.7(d + 3)$	$1.5(d + 3)$

— The min. edge distance is ^{provided to} ~~to be~~ ^{to be} ~~avoid~~ ^{avoid} :-

a) The failure of plate in tension.

b) To avoid the bulging & cracking of the steel plate opposite to the hole.

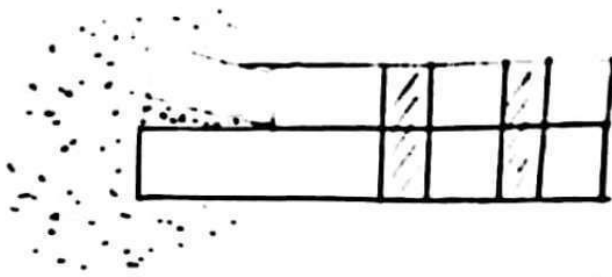
b) Maximum end or edge distance = $12t \leq$

t = thickness of thinner member.

$$E = \sqrt{\frac{250}{b_f}}$$

- When the bolts are too far away from the edges of the plate being connected, the

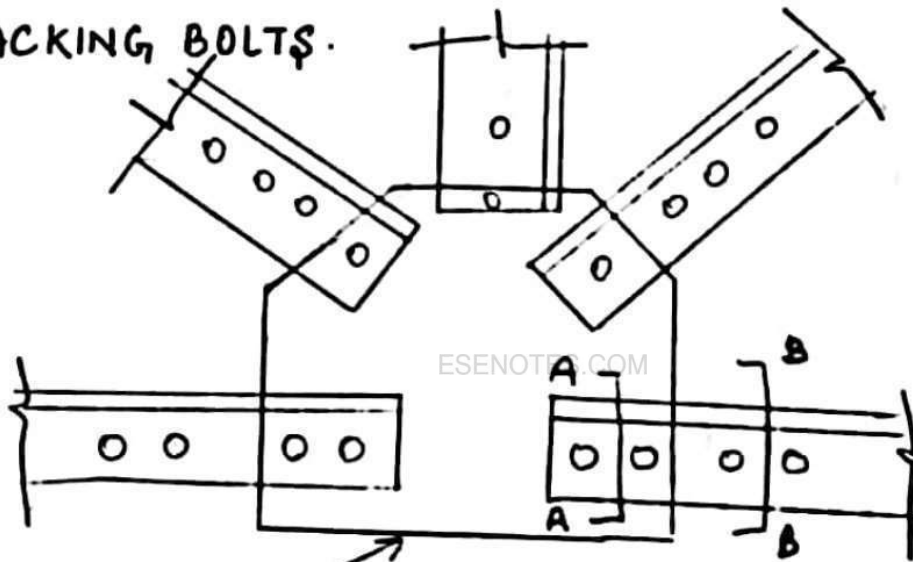
edges may sometime, separate, thus permit the entrance of the moisture which leads to corrosion & increased separation of plates.



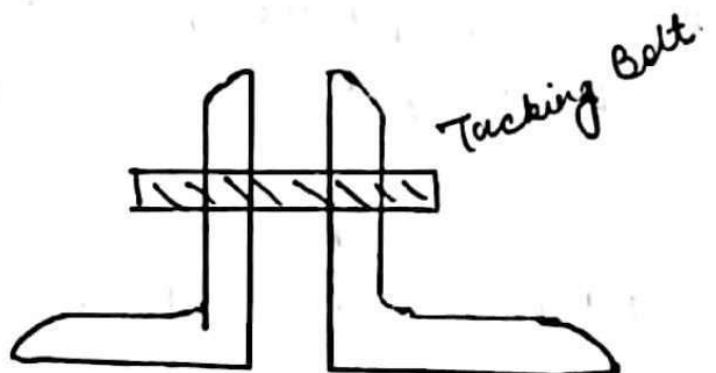
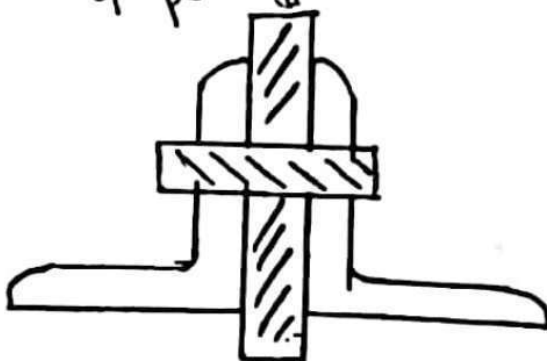
- When the members are exposed to corrosive environment.

max. edge distance $> 40\text{mm} + 4t$

D) TACKING BOLTS.



Gusset plate



- The purpose of tacking or stitching bolts is to ensure that the two component members act together & also to equalise the stress in two component members.
- In compression member, it also prevents the local buckling.
- Max pitch = $(32t \text{ or } 300 \text{ mm}) \text{ min.}$

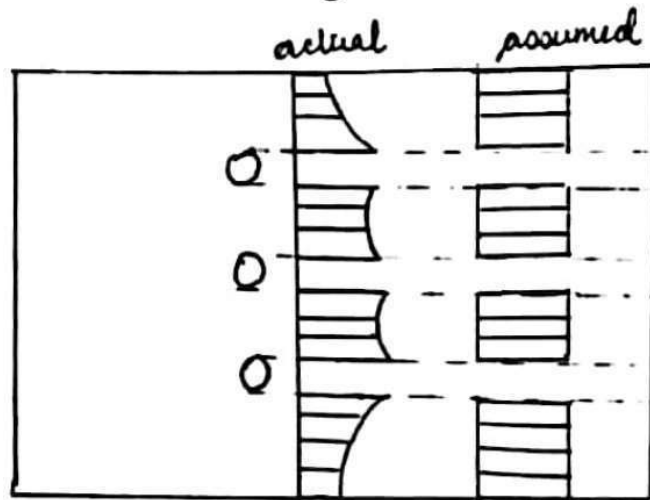
t = thickness of thinner member.
(when the plates are not exposed to weather).

- Max pitch = $(16t \text{ or } 200 \text{ mm}) \text{ min.}$
 { when the plates are exposed to weather }
- For two members placed back to back, the max. pitch of tacking bolts $\nless 1000 \text{ mm}$ in tension member and $\nless 600 \text{ mm}$ in compression member.

Assumptions in the analysis of Simple Bolted Joint. { Bearing type connections }.

- Friction b/w the plate is neglected & load is resisted by bolts in bearing & shearing.
- In case of bolt if thread occur in the plane of shear, the effective area resisting shear is taken as area at the root of thread.
and if thread do not occur in the plane of shear, effective area is the cross-sectional area of shank.
- The applied load is equally resisted by all the bolts.

- (iv) Distribution of stress on the portion of plate b/w the bolt holes is uniform i.e. stress concentration around the holes is neglected.



{ This assumption is taken for simplicity in computation but at the time of failure, this assumption would actually be valid? }

- (v) The bending stresses in the bolt is neglected.

- (vi) Bearing stress is assumed to be uniform over the nominal contact area b/w the plate & the bolt.

Lec 12 Oct 26

Strength of Bolt in Bearing Type Connection

$$\text{Design strength of bolt } (V_{db}) = \min \begin{cases} \text{(i) design shear strength of bolt } (V_{dsb}) \\ \text{(ii) design bearing strength of bolt } (V_{dpb}) \end{cases}$$

$$\text{(i) design shear strength of bolt } (V_{dsb}) = \frac{\text{Nominal shear strength of bolt } (V_{nsb})}{\text{Partial safety factor for material of bolt } (\gamma_{mb})}$$

$$(ii) \text{ Design bearing strength of bolt } (V_{dpb}) = \frac{\text{nominal bearing strength of bolt } (V_{npb})}{\text{partial safety factor for material of bolt } (\gamma_{mb})}$$

$$\gamma_{mb} = 1.25.$$

Hence, for safety of bolt,

$$\text{factored shear load in bolt } (V_{sb}) \leq \text{design strength of bolt } (V_{db})$$

Note $\Rightarrow$ Design strength of the bolt (V_{db}) is also termed as "BOLT VALUE".

(i) Shear strength of Bolt.

$$V_{dsb} = \frac{V_{nsb}}{\gamma_{mb}}$$

$$V_{nsb} = \frac{f_{ub}}{\sqrt{3}} (n_n A_{nb} + n_s A_{sb}) \phi_f \phi_g \phi_{pk}$$

f_{ub} = ultimate tensile strength of Bolt.

n_n = no. of shear plane with thread intercepting the shear plane.

A_{nb} = net tensile area of bolt or area at root of thread

$$A_{nb} = 0.78 A_{sb}$$

n_s = no. of shear plane without thread intercepting the shear plane.

A_{sb} = nominal shank area of bolt.

$\phi_f, \phi_g, \phi_{pk}$: Reduction factors taking into account the effect of long joints, large grip, packing plate.

For eg. : for single shear case.

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} (1 \times A_{nb})$$

$$n_n = 1, n_s = 0$$

Note :- Unless specified V_{dsb} will be computed corresponding to shear plane intercepting the thread.

For eg. : for double shear case.

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} (1 \times A_{nb} + 1 \times A_{sb})$$

$$n_n = 1, n_s = 1$$

- In above calculation, it is assumed that one of the shear plane is intercepting thread and other is intercepting shank.
- nominal shear strength of bolt is also compromised on the account of

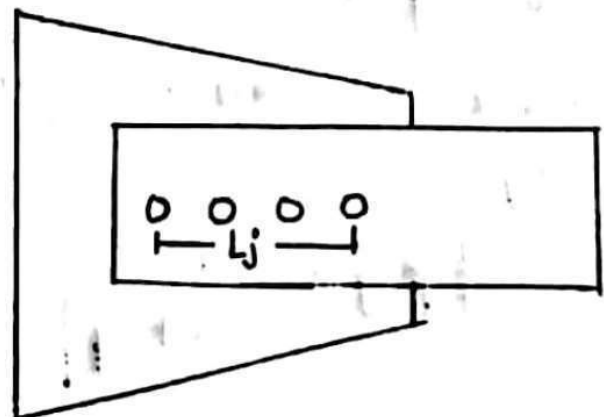
A) For long joints

- A joint is termed as long joint when the c/c distance b/w the extreme bolts (L_j) in the direction of load transfer or length of joint is greater than 15 times the nominal dia of shank (d)

$$L_j > 15d.$$

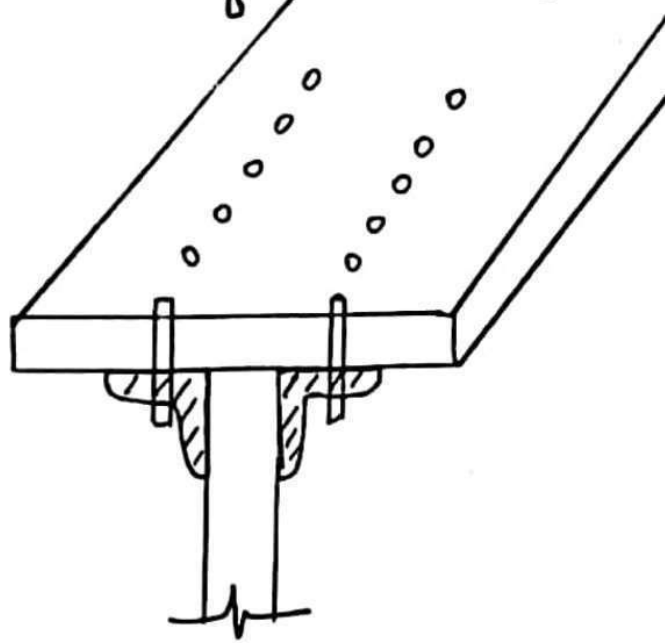
- Due to this long joint stress distribution is non uniform that results in reduced shear capacity, hence to avoid excessive stress in the extreme bolts of the joint, this reduction factor is applied (β_{Lj})

$$\beta_{Lj} = 1.075 - \frac{L_j}{200d}$$



Note: β_{ij} is taken 1.0 for joints having uniform shear over the length of joint.

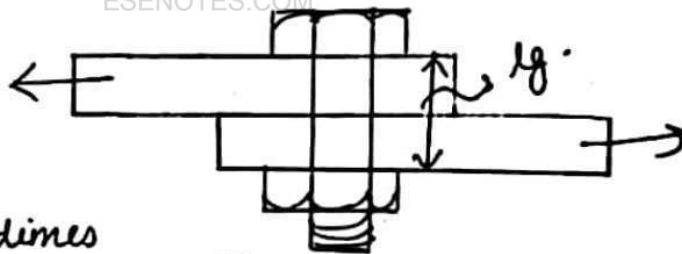
for eg: Connection of web & flange of a plate



b) For long large grip

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- A grip termed as large grip if the length of grip (l_g) is greater than 5 times the nominal dia. of bolt (d).



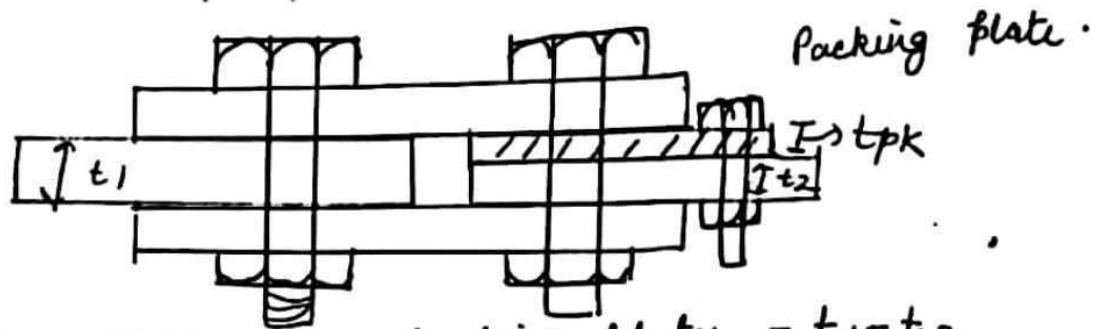
- When grip is large, bolt is subjected to bending action, due to shear force acting on the shank, resulting in reduced _{shear} capacity of bolt.

- Hence this correction factor is applied (β_{lg}), given by.

$$\beta_{lg} = \frac{8d}{3d + l_g}$$

Here $5d \leq l_g \leq 8d$
also $\beta_{lg} \neq \beta_{ij}$

c) Packing plates



$t_{PK} = \text{Thickness of packing plates} = t_1 - t_2$

When packing plates are thick, the shank of the bolt is subjected to bending which reduces its shear capacity.

- Hence, if $t_{PK} > 6\text{mm}$, reduction factor due to packing plate is applied.

$$\beta_{PK} = 1 - 0.0125 t_{PK}$$

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(ii) Bearing strength of Bolt.

$$V_{dpb} = \frac{V_{npb}}{\gamma_{mb}}$$

Bolt will only fail in this when bearing str. of plate is more..

$$V_{npb} = 2.5 K_b \cdot d \cdot t \cdot f_u$$

d = nominal dia. of bolt.

t = summation of thickness of plate elements experiencing bearing in same direction.

f_u = ultimate strength of plate

f_{ub} = ultimate strength of bolt.

$$K_b = \begin{cases} \frac{e}{3d_o} \\ \frac{p}{3d_o} - 0.25 \\ \frac{f_{ub}}{f_u} \\ 1 \end{cases} \quad \begin{array}{l} \phi = \text{pitch} \\ e = \text{end distance} \end{array}$$

min.

- The constant '2.5' in the above equation corresponds to ~~what~~ hole elongation of about 6mm.
- Bearing failure in the bolt is possible only if the bolt used are of very low grade and plate used is of very high grade. [which is generally not provided].
- Hence, the bearing strength of a bolted connection is a function of strength of connected part. (plate) & the arrangement of bolt; rather than the grade of bolt.
- A possible failure due to excessive bearing is shear tearing out at the end of the connection.

Note: The bearing strength is also a function of bolt hole.

- The bearing strength calculated above is for standard bolt hole.
- For oversized & short slotted bolt holes, it is 0.7 times and for long slotted bolt holes it is 0.5 times the bearing strength of standard bolt.

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Tensile strength of Bolt.

$$T_{db} = \frac{T_{nb}}{\gamma_{mb}}$$

$$T_{nb} = 0.9 f_{ub} A_{nb}$$

also

$$T_{nb} < f_{yb} A_{sb} \frac{\gamma_{mb}}{\gamma_{mo}}$$

$$\gamma_{mb} = 1.25$$

$$\gamma_{mo} = 1.1$$

(partial safety factor for material resistance governed by yielding)

- This criteria is checked so that excessive yielding does not results in non functional joint before ~~rupture~~ rupture

- Factor '0.9' accounts for safety at ultimate load.

Design strength of plate.

- The plate may fail by

a) Shearing of plate $\rightarrow$ It can be ~~also~~ avoided by providing sufficient end & edge distance.

b) Yielding of the plate $\rightarrow$ for gross section yielding

$T_{dg} = \text{design tensile str. of plate in gross yielding}$

$$T_{dg} = \frac{f_y A_g}{\gamma_{mo}}$$

$$\gamma_{mo} = 1.1$$

(partial safety factor for material resistance: governed by yielding)

A_g = gross area of solid plate

f_y = yield strength of plate

c) Rupture of plate for net section rupture.

$$T_{dn} = \frac{0.9 f_u A_n}{\gamma_{m1}}$$

T_{dn} = design tensile strength in net section rupture

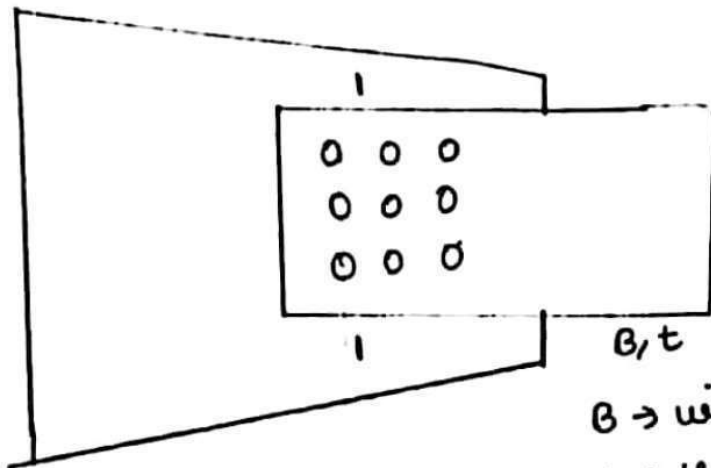
$$\gamma_{m1} = 1.25$$

γ_{m1} = partial safety factor for resistance governed by ultimate stress.

→ '0.9' is being introduced for safety at ultimate load.
(As there is no reserve strength beyond ultimate str.)

A_n = effective net area of the section.

for eg. a) Chain Bolting.



$$A_n = (B - 3d_o)t$$

In general

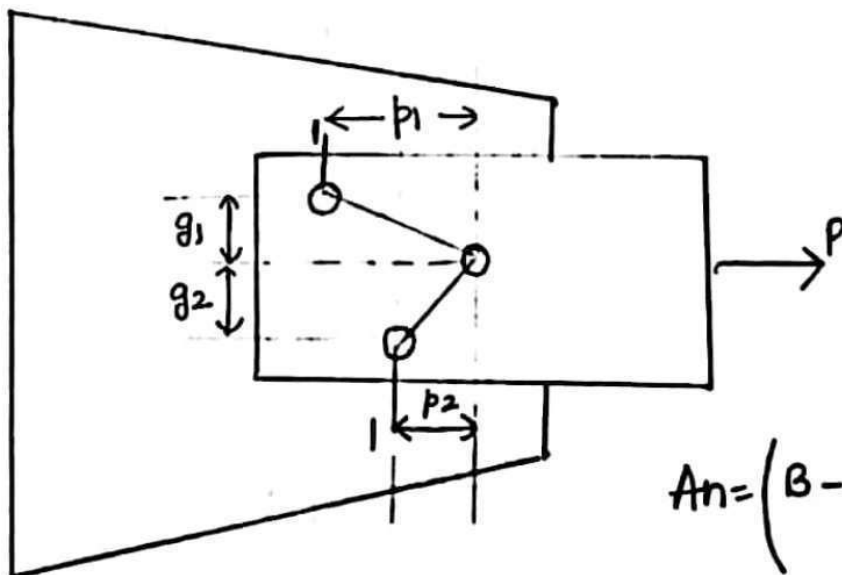
$$A_n = (B - nd_o)t$$

B → width of plate

t → thickness of plate.

n → no. of bolt holes along the

b) for staggered Bolting.



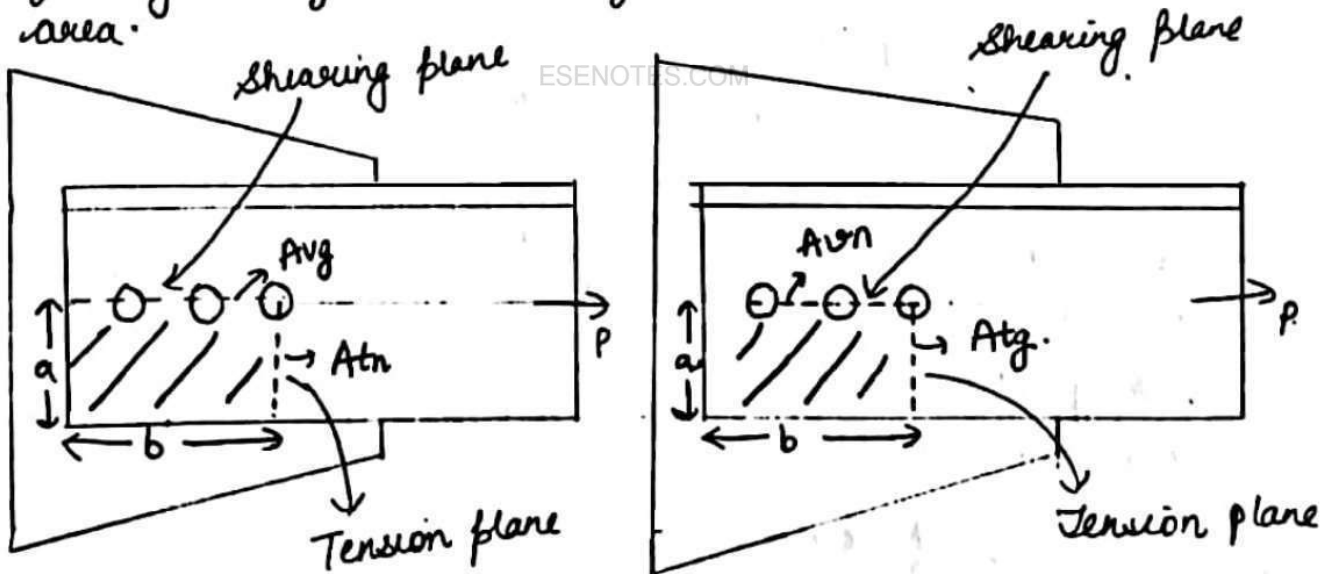
$$A_n = \left(B - nd_o + m \sum \frac{p_i^2}{4g_i} \right) t$$

m = no. of inclined lines.

$$A_{n1-1} \Rightarrow \left(B - 3d_o + \frac{p_1^2}{4g_1^2} + \frac{p_2^2}{4g_2^2} \right) t$$

D) Block Shear Failure

- When a block of material within the bolted area breaks away from the remaining section, the failure is termed as block shear failure.
- The block shear failure along a path involving tension on one plane & shear on a 1st plane.
- For the block shear failure to occur, one of the surfaces must fracture while the other must yield.
- The stronger plane generally fractures while weaker plane yields.
- Yielding always occurs on gross area & fracture on net area.



* Shear yielding & Tension rupture

* Shear rupture & tension yielding.

Q If $a = 150 \text{ mm}$, $b = 350 \text{ mm}$ $d_o = 25 \text{ mm}$, $t = 10$.

$$A_{vg} = bt = 350 \times 10 = 3500 \text{ mm}^2$$

$$A_{tn} = \left(b - 2d_o - \frac{d_o}{2} \right) t = \left(350 - 2 \times 25 - \frac{25}{2} \right) \times 10 = 2875 \text{ mm}^2$$

$$A_{tn} = \left(a - \frac{d_o}{2} \right) t = \left(150 - \frac{25}{2} \right) \times 10 = 1375 \text{ mm}^2$$

$$A_{tg} = at = 150 \times 10 = 1500 \text{ mm}^2$$

- The block shear strength (T_{dB}) is given by

$$T_{dB} = \min \left\{ \begin{array}{l} \frac{f_y A_{vg}}{\sqrt{3} \gamma_{m0}} + \frac{0.9 f_u A_{tn}}{\sqrt{3} \gamma_{m1}} \quad (\text{shear yielding} + \text{tension rupturing}) \\ \frac{0.9 f_u A_{tn}}{\sqrt{3} \gamma_{m1}} + \frac{f_y A_{tg}}{\gamma_{m0}} \quad (\text{shear rupture} + \text{tension yielding}) \end{array} \right.$$

- Block shear failure occurs in joint made with high strength bolts, where fewer bolts are reqd. for making the connection.

- Also, when sufficient end distance is not provided.

Efficiency of Joint: (η)

- It is defined as ratio of strength of bolted connection to the strength of solid plate.

$$\eta = \frac{\text{strength of bolted connection}}{\text{strength of solid plate}} \times 100$$

- strength of the bolted connection = $\min. \left\{ n V_{db}, T_{dg}, T_{dn}, T_{dB} \right\}$

- strength of solid plate is always governed by yielding.
i.e. strength of solid plate (T_{dg}) = $\frac{f_y A_g}{\gamma_{mo}}$.

V_{db} : Design strength of Bolt.

T_{dg} : design strength of plate in yielding.

T_{dn} : design strength of plate in rupture

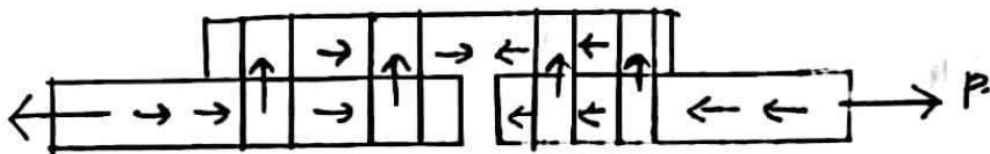
T_{dB} : Design strength of plate in Block Shear.

Lecture 15 Oct 29

Note % By choosing the different ~~array~~ arrangements of bolts, we can ensure economy in design.

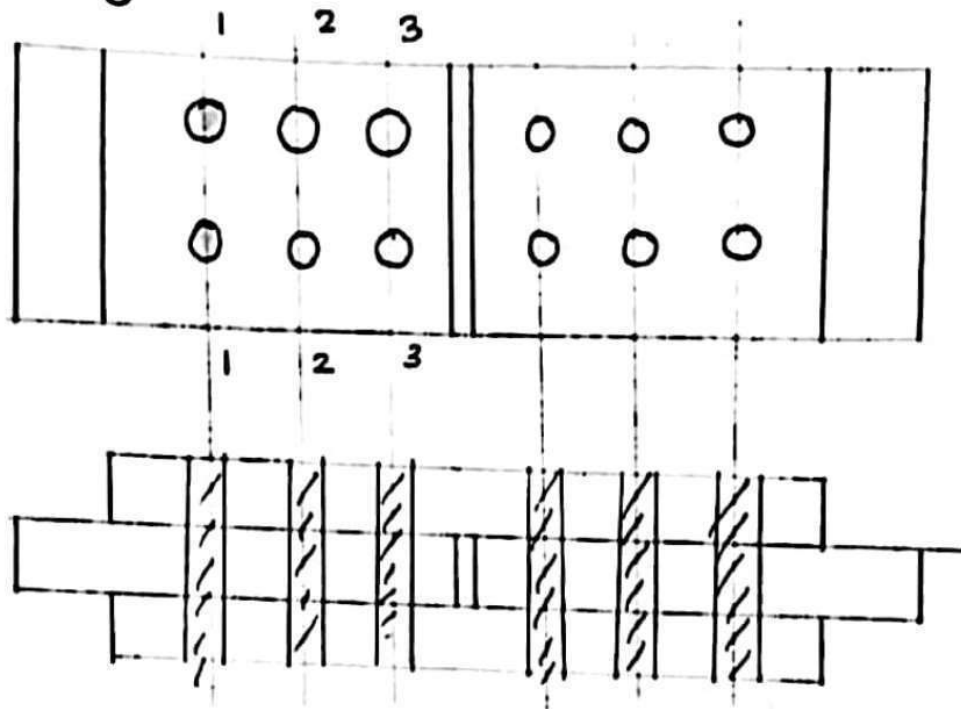
- In tension members, our main aim should be minimum reduction in the net area of the plate at the section at which it is carrying the max. force.

- For eg.



- To understand this point, consider chain & diamond bolting.

a) Chain Bolting



- Tearing strength of Bolt.

(i) at section 1-1.

$$T_{dn 1-1} = \frac{0.9 f_u}{1.25} (B - 2d_o) t$$

(ii) at section 2-2.

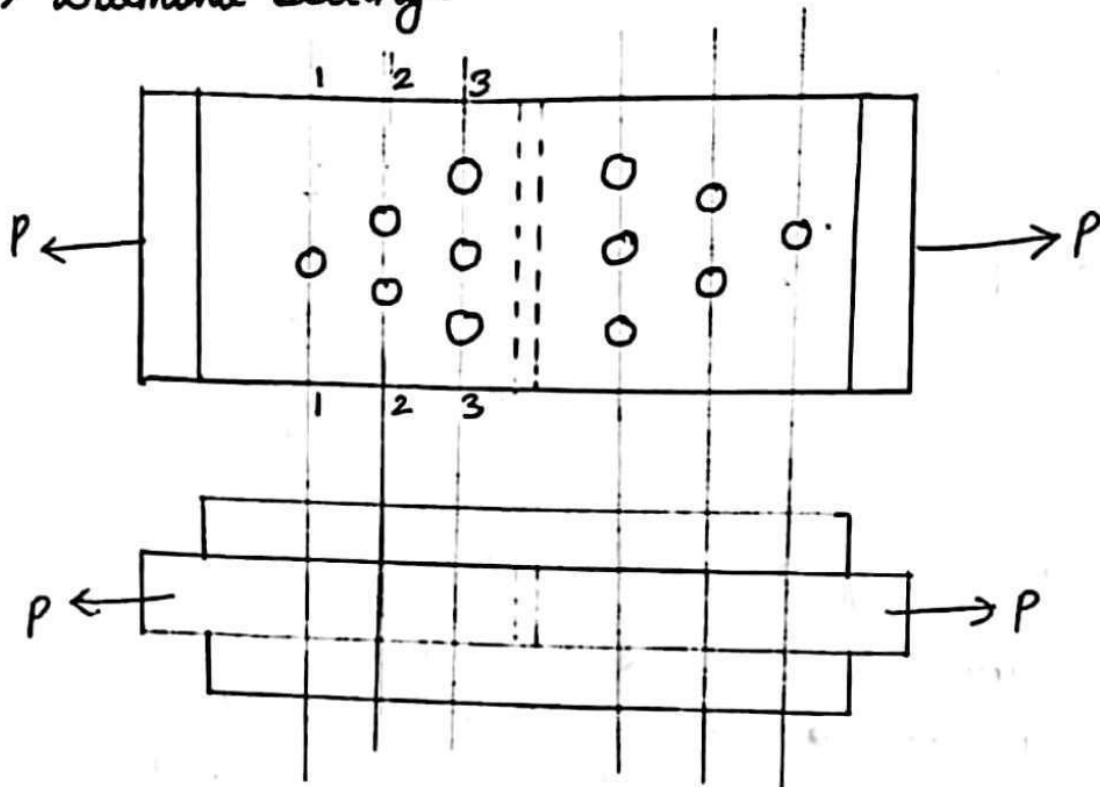
$$T_{dn 2-2} = \frac{0.9 f_u}{1.25} (B - 2d_o) t + 2 V_{db}$$

(iii) at section 3-3.

$$T_{dn 3-3} = \frac{0.9 f_u}{1.25} (B - 2d_o) t + 4 V_{db}$$

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b) Diamond Bolting.



- Tearing strength of bolt

(i) at section 1-1.

$$T_{dn \ 1-1} = \frac{0.9 f_u}{1.25} (B - d_o) t$$

(ii) at section 2-2.

$$T_{dn \ 2-2} = \frac{0.9 f_u}{1.25} (B - 2d_o) t + V_{db}$$

(iii) at section 3-3.

$$T_{dn \ 3-3} = \frac{0.9 f_u}{1.25} (B - 3d_o) t + 3V_{db}$$

⇒ Diamond pattern is most efficient & economical since net area of plate at sec 1-1 is max. in this case (i.e. reduced by 1 bolt hole only), where for load to be resisted is max.

⇒ We do not bother about cover plate, because safety requirements for cover plate can be met by

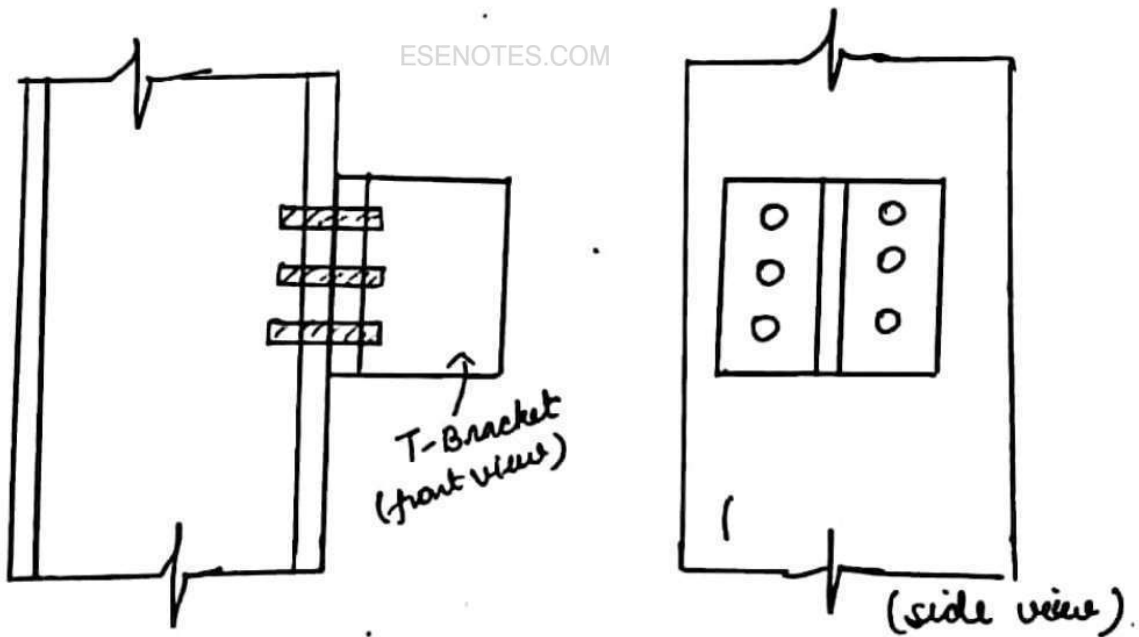
increasing the thickness.

- Moreover cover plate does not extend to full length of member; hence its increased thickness will not cause much increase in cost.
- Thickness of cover plate $\neq \frac{5}{8}t$ in double cover butt joint.
 $\neq \frac{9}{8}t$ in single cover butt joint.

t = thickness of the plate (main)

Bolts Subjected to Combined Shear & Tension.

- When Bolts are subjected to both Tension & shear, then their combined effect may be analysed by using "Interaction formula".



$$\left(\frac{V_{sb}}{V_{db}} \right)^2 + \left(\frac{T_b}{T_{db}} \right)^2 \leq 1$$

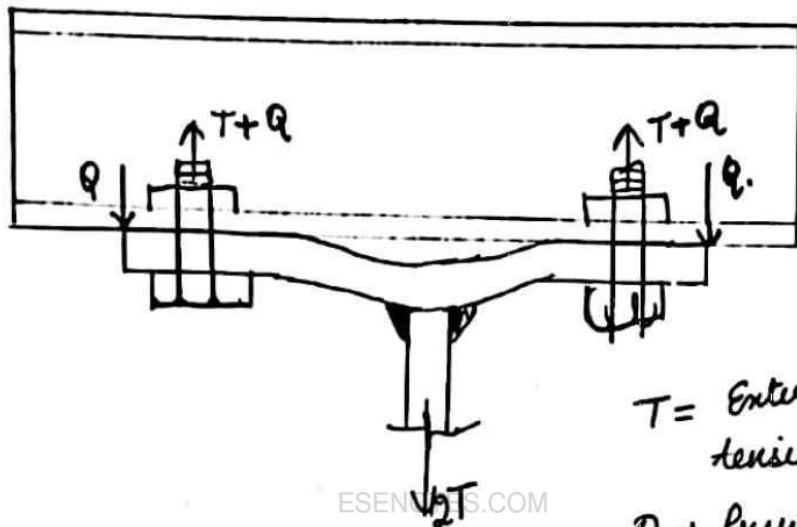
$V_{db} = \text{Design str. of bolt} = \min (V_{dsb}, V_{dpsb})$.

$T_{db} = \text{Tensile str. of bolt.}$

$V_{sb} = \text{factored shear force in the bolt.}$

$T_b = \text{factored tension force in the bolt.}$

Notes PRYING FORCE.



$T = \text{External factored tensile force}$
 $Q \rightarrow \text{Prying force.}$

- Due to this flexibility of connection, when the bolts are subjected to direct tension, they develop an additional force in them termed as prying force.

Q Calculate the strength of M20 bolt of grade 4.6 for the following case. The main plate to be joined are 12mm thick & steel is of grade Fe 410

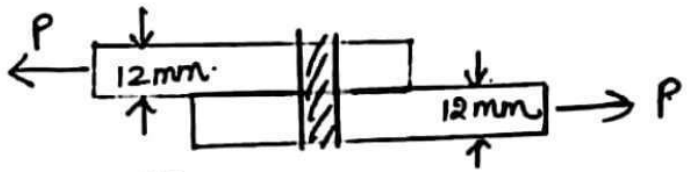
- Lap Joint
- Single cover Butt Joint, cover plate is 10mm thick
- Double " " " ; cover plate is 8mm thick.

$$f_{ub} = 400 \text{ N/mm}^2 \quad f_{yb} = 0.6 \times 400 = 240 \text{ N/mm}^2$$

$$d = 20 \quad d_o = 20 + 2 = 22 \quad f_u = 410 \text{ N/mm}^2$$

a) Lap Joint

(i) design shear strength of bolt



$$V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} [n_n A_{nb} + n_s A_{sb}]$$

Here bolt is in single shear & assuming the plane of shear intercept the thread.

$$n_n = 1, n_s = 0$$

$$A_{nb} = 0.78 A_{sb} = 0.78 \times \frac{\pi}{4} d^2$$

$$= 0.78 \times \frac{\pi}{4} \times 20^2$$

$$= 245.04 \text{ mm}^2$$

$$V_{dsb} = \frac{400}{\sqrt{3} \times 1.25} [1 \times 245.04 + 0]$$

$$= 45.27 \text{ kN}$$

(ii) design bearing strength of the bolt.

$$V_{dpb} = 2.5 k_b d \leq \frac{f_u}{1.25} = 2.5 \times 0.5 \times 20 \times 12 \times \frac{410}{1.25} \Rightarrow 98.4 \text{ kN}$$

$$k_b = \min \left\{ \frac{e}{3d_o}, \frac{p}{3d_o} - 0.25, \frac{f_{ub}}{f_u}, 1.0 \right\}$$

$$e = e_{\min} = 1.5 d_o$$

$$= 1.5 \times 22$$

$$= 33 \text{ mm}$$

$$\left\{ \frac{33}{3 \times 22}, \frac{50}{3 \times 22} - 0.25, \frac{400}{410}, 1.0 \right\}$$

$$k_b \Rightarrow \{0.5, 0.501, 0.97, 1.0\} \min \Rightarrow k_b = 0.5$$

$$p = p_{\min} = 2.5 d = 2.5 \times 20$$

$$\Rightarrow 50 \text{ mm}$$

$$V_{db} = \min \begin{cases} V_{dsb} \\ V_{dpb} \end{cases} \Rightarrow 45.27 \text{ kN}$$

b) Single cover butt joint

(i) Design shear str. of Bolt.

Since bolt in this case also are in single shear so design shear str. of bolt is same as previous.



$$V_{dsb} = 45.27 \text{ kN}.$$

(ii) Design bearing str. of Bolt.

$$V_{dpb} = 2.5 K_b d t \frac{f_u}{1.25}$$

$$= 2.5 \quad K_b = \min \left\{ \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1.0 \right\}$$

$$= 0.5.$$

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t = min. of aggregate thickness of cover plate and main plate.

$$t = \min \{ t_1, t_2 \}$$

$$= \min \{ 12, 10 \} \Rightarrow t = 10 \text{ mm}.$$

$$V_{dpb} = 2.5 \times 0.5 \times 20 \times 10 \times \frac{410}{1.25}$$

$$= 82 \text{ kN}.$$

Design strength of the bolt (V_{db}) = $\min \{ V_{dsb}, V_{dpb} \}$
 $\Rightarrow 45.27 \text{ kN}.$

Working Stress Method.

- The provision of limit state design also apply to design of bolts by working stress design method, except for actual and permissible stress calculation.

a) Shear strength in Bolt.

$$\text{Actual shear stress in bolt } (f_{sb}) = \frac{V_{sb}}{A_{sb}}$$

V_{sb} = shear force in bolt due to working load.

A_{sb} = nominal shank area of bolt.

- Allowable or permissible shear stress in bolt (f_{asb})

$$f_{asb} = 0.6 \frac{V_{nsb}}{A_{sb}}$$

V_{nsb} = nominal shear strength of bolt as calculated in LSM.

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b) Bearing Stress in Bolt.

- Actual bearing stress in the bolt (f_{pb}) = $\frac{V_{sb}}{A_{pb}}$

V_{sb} = shear/Bearing force in bolt due to working load.

A_{pb} = Bearing area of the bolt on plate.

- Allowable bearing stress in bolt (f_{apb}) = $0.6 \frac{V_{npb}}{A_{pb}}$

V_{npb} = Nominal bearing strength of Bolt as computed in LSM.

c) Tensile stress in a Bolt.

$$\text{actual tensile stress in the bolt } (f_{tb}) = \frac{T_s}{A_{sb}}$$

T_s = Tension force in bolt due to working load.

A_{sb} = Nominal shank area of bolt.

$$\text{Allowable tensile stress in bolt } (f_{atb}) = 0.6 \frac{T_{nb}}{A_{sb}}$$

T_{nb} = nominal tensile strength of a bolt. as computed in LSM.

d) Bolts subjected to Combined Shear & Tension

$$\left(\frac{f_{sb}}{f_{asb}} \right)^2 + \left(\frac{f_{tb}}{f_{atb}} \right)^2 \leq 1.0 \quad \left\{ \begin{array}{l} \text{Interaction} \\ \text{formula.} \end{array} \right.$$

e) Strength of plate. ESENOTES.COM

$$\text{actual tensile stress } (f_t) = \frac{T_s}{A_g}$$

$T_s \rightarrow$ tensile force in plate.

- allowable tensile stress in plate is min. of following

(i) If governed by yielding

$$f_{at} = 0.6 f_y.$$

(ii) If governed by rupture

$$f_{at} = 0.69 \frac{T_{dn}}{A_g}$$

(iii) If governed by block shear.

$$f_{at} = 0.69 \frac{T_{dB}}{A_g}$$

T_s = actual tension force under working load.

A_g = gross area of plate

T_{dn} = design str. of plate in tension (rupture) as computed in LSM.

T_{db} = design block strength as computed in LSM.

Ex

Note: The dia. of bolt chosen to fit suit the thickness of plate bearing being connected is given by.

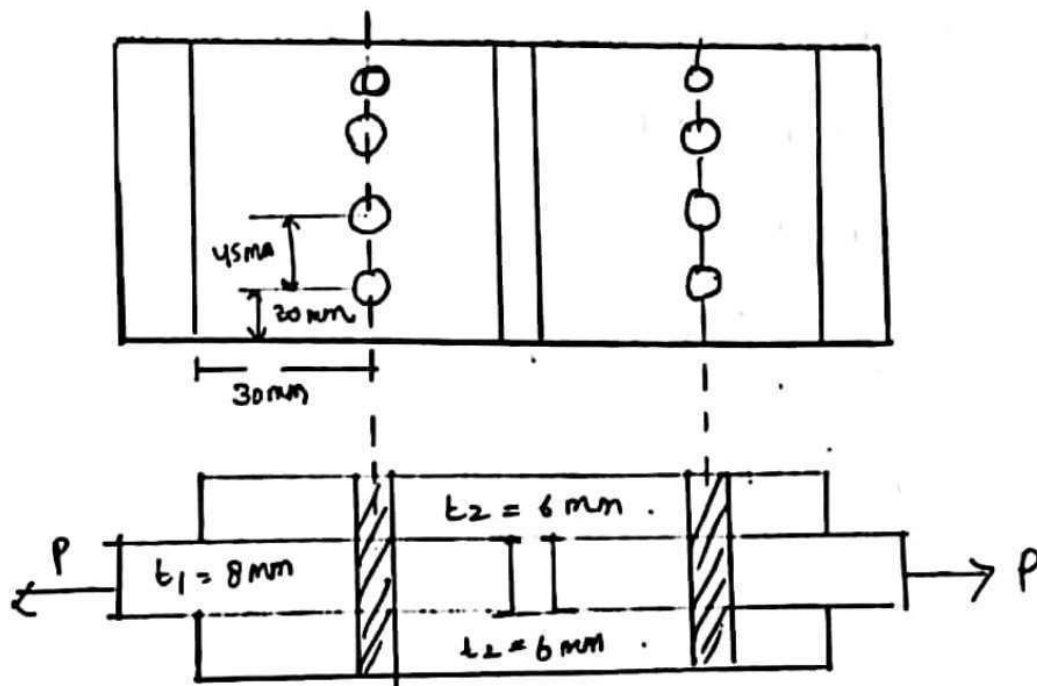
UNWIN'S FORMULA.

$$d_{mm} = 6.05 \sqrt{t_{mm}}$$

t = min. thickness of plate.

d = nominal dia of Bolt.

Q A single bolted double cover butt joint is used to connect 2 plates which are 8 mm thick. Assume M16 bolt of grade 4.6 & cover plate of 6 mm thickness. Compute the strength & efficiency of joint if 4 bolts are provided in a bolt line at a pitch of 45 mm as given. Assume plate of Fe 410 grade.



Sol. (A) Design strength of Bolt (V_{db})

(i) Design shear str. of Bolt

Here bolts are in double shear., assuming $n_n=1, n_s=1$

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} (1 \times A_{nb} + 1 \times A_{sb})$$

$$= \frac{4 \times 100}{\sqrt{3} \times 1.25} \left(1 \times 0.78 \times \frac{\pi}{4} \times 16^2 + 1 \times \frac{\pi}{4} \times 16^2 \right)$$

$$= 66.08 \text{ kN}$$

(ii) Design bearing strength of bolt.

$$V_{dpb} = 2.5 K_b d t \frac{f_u}{1.25}$$

$$\begin{aligned} d_o &= 16 + 2 \\ &= 18 \end{aligned}$$

$$K_b = \begin{cases} \frac{e}{3d_o} = \frac{30}{3 \times 18} = 0.55 \\ \frac{p}{3d_o} - 0.25 = \frac{45}{3 \times 18} - 0.25 = 0.58 \\ \frac{f_{ub}}{f_u} = \frac{400}{410} = 0.97 \\ \min. \left\{ \begin{array}{l} 1.0 \end{array} \right. \end{cases}$$

$$K_b = 0.55$$

$$t = \min \{ 8, 6 \times 2 \} = 8 \text{ mm}$$

$$V_{dpb} = 2.5 \times 0.55 \times 16 \times 8 \times \frac{410}{1.25}$$

$$= 57.72 \text{ kN}$$

$$V_{db} = \min. \begin{cases} V_{dsb} = 66.08 \\ V_{dpb} = 57.72 \end{cases}$$

$$\Rightarrow 57.72 \text{ kN}$$

$$\text{Strength of 4 bolts} = 4 \times 57.72 \\ = 230.912 \text{ kN}.$$

B) Design strength of the plate

(i) Tensile strength of plate in yielding. (T_{dg}).

$$T_{dg} = \frac{f_y A_g}{1.1} = \frac{250 \times [45 \times 3 + 30 \times 2] \times 8}{1.1}$$

$$\Rightarrow 354.54 \text{ kN}$$

(ii) Tensile strength of plate in rupture. (T_{dn}).

$$T_{dn} = \frac{0.9 f_u A_n}{1.25} = \frac{0.9 \times 410 [195 - 4 \times 18] \times 8}{1.25}$$

$$\Rightarrow 290.47 \text{ kN}.$$

Now, strength of Joint $\Rightarrow \min. \{ V_{db}, T_{dn}, T_{dg} \}$.

$$\Rightarrow \min \{ 230.912, 290.47, 354.54 \}$$

$$\Rightarrow \underline{230.912 \text{ kN}}$$

$$\Rightarrow \text{Efficiency of Joint } \eta(\%) = \frac{\text{str. of joint}}{\text{str. of solid plate}} \times 100.$$

$$\Rightarrow \frac{230.912}{354.54} \times 100$$

$$\Rightarrow 65.12\%.$$

Solve the previous using WSM. Lesson 18 Nov 2
Solⁿ By working stress method.

a) Shear strength of bolt (V_{dsb}) = $0.6 V_{nsb}$.

$$V_{dsb} = 0.6 \left[\frac{f_{ub}}{\sqrt{3}} (n_n A_{nb} + n_s A_{sb}) \right]$$

$$= 0.6 \left[\frac{400}{\sqrt{3}} \left(1 \times 0.78 \times \frac{\pi}{4} \times 16^2 + 1 \times \frac{\pi}{4} \times 16^2 \right) \right]$$

$$V_{dsb} = 49.59 \text{ kN}.$$

b) Bearing strength of bolt (V_{dpb}) = $0.6 V_{rpb}$.

$$V_{dpb} = 0.6 \times 2.5 \times 0.55 \times 16 \times 8 \times 410.$$

$$\approx 43.29 \text{ kN}.$$

Strength of bolt $\Rightarrow V_{db} = \min \begin{cases} V_{dsb} \Rightarrow 49.59 \\ V_{dpb} \approx 43.29 \end{cases}$

$$\Rightarrow 43.29 \text{ kN}.$$

Strength of 4 bolts $\Rightarrow 4 \times 43.29 = 173.16 \text{ kN}.$

Strength of plate in yielding $\Rightarrow T_g = 0.6 f_y A_g$
 $= 0.6 \times 250 \times 195 \times 8.$
 $\Rightarrow 234 \text{ kN}.$

Tensile strength of plate in rupture. $T_n = 0.69 T_{dn}.$

$$T_n = 0.69 \left\{ \frac{0.9 f_u}{1.25} (B - n d_o) t \right\}$$

$$\Rightarrow 0.69 \left\{ \frac{0.9 \times 410}{1.25} (.195 - 4 \times .018) \times 8 \right\}$$

$$\Rightarrow 200.42$$

$$\text{Strength of joint} \Rightarrow \min \cdot \{ 4 V_{db}, T_g, T_n \}$$

$$\Rightarrow \min \{ 173.16, 234, 200.42 \}$$

$$\Rightarrow \underline{173.16 \text{ kN}}$$

$$\eta \Rightarrow \frac{\text{str. of joint}}{\text{str. of plate}} \times 100$$

$$\Rightarrow \frac{173.16}{234} \times 100 \Rightarrow 74\%$$

Q Two flats (Fe410 grade of steel) each of 210 mm x 8 mm are to be joined using 20mm dia, 4.6 grade bolt to form a lap joint. The joint is supposed to transfer a factored load of 250 kN. Design the joint & determine the suitable pitch for the bolt.

a) Shear str. of bolt.

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} (n_n A_{nb} + n_s A_{sb})$$

In lap joint, bolts will be in single shear.

$$V_{dsb} = \frac{400}{\sqrt{3} \times 1.25} \left(1 \times 0.78 \times \frac{\pi}{4} \times 20^2 \right)$$

$$\Rightarrow 45.27 \text{ kN}$$

b) Bearing str. of bolt.

$$V_{dpb} = 2.5 K_b \cdot d \cdot t \cdot \frac{f_u}{1.25}$$

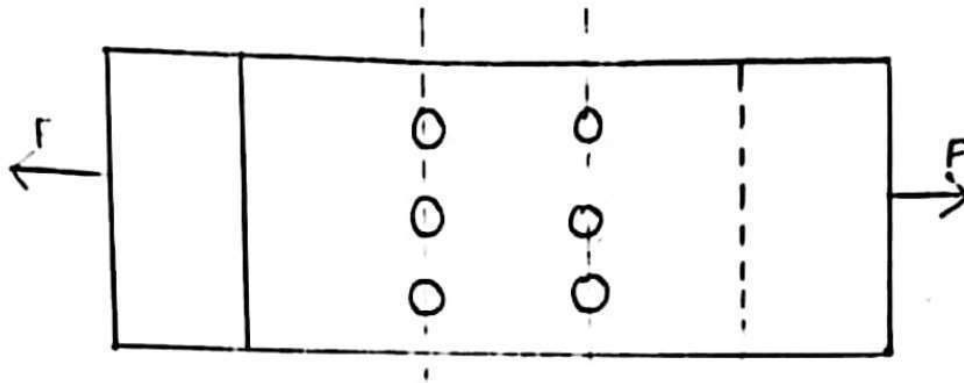
assume shear str. < bearing str. of bolt.

$\therefore$ design str. = shearing str. of bolt $\Rightarrow 45.27 \text{ kN}$

$$\text{no. of bolts reqd. (n)} = \frac{F}{V_{db}} = \frac{250}{45.27} \Rightarrow 5.52 \approx 6$$

$$\text{min. pitch} = 2.5d = 2.5 \times 20 = 50 \text{ mm.}$$

Hence, adopt 6 no. of bolts and provide them in two lines.



- strength of flat joint per pitch length of plate = $2Vdb$
 $= 2 \times 45.27 = 90.54 \text{ kN}$

- Tensile str. of plate per pitch length = $\frac{0.9f_u}{1.25} (p-d_o)t$
 $= \frac{0.9 \times 410}{1.25} (p-22) \times 8 \times 10^{-3}$

Tensile str. of plate per pitch length $\geq 2Vdb$.

$$\frac{0.9 \times 410}{1.25} (p-22) \times 8 \times 10^{-3} \geq 90.54$$

$$p \geq 60.33 \text{ mm.}$$

provide pitch of 65 mm.

Now, edge distance (e) = $\frac{210 - 2 \times 65}{2} = 40 \text{ mm.}$

$$e_{\text{min}} = 1.5d_o = 1.5 \times 22 = 33 \text{ mm.} < \underline{\underline{40 \text{ mm}}}$$

OK

Check, design bearing str. of bolt.

$$V_{dpb} = 2.5 k_b d t \frac{f_u}{1.25}$$

$$K_b = \left\{ \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{t_{ub}}{t_{ub}}, 1.0 \right\}$$

$$\left\{ \frac{40}{3 \times 22}, \frac{65}{3 \times 22} - 0.25, 0.97, 1.0 \right\}$$

$$\{ 0.6, 0.73, 0.97, 1.0 \}$$

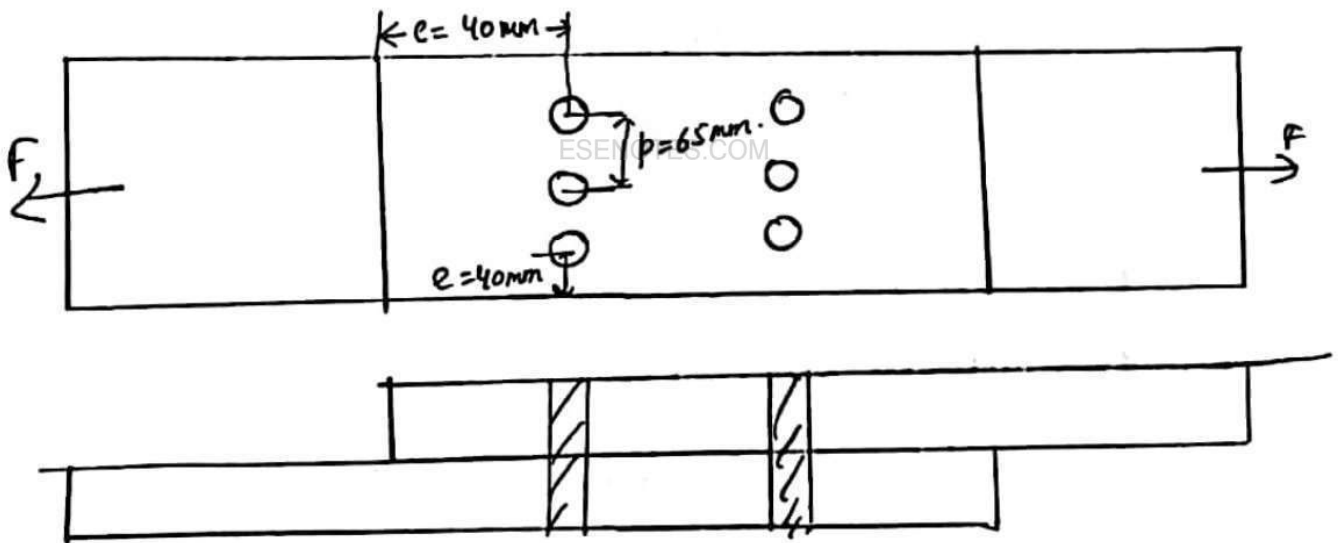
$$K_b = 0.6$$

$$V_{dpb} = 2.5 \times 0.6 \times 20 \times 8 \times \frac{410}{1.25}$$

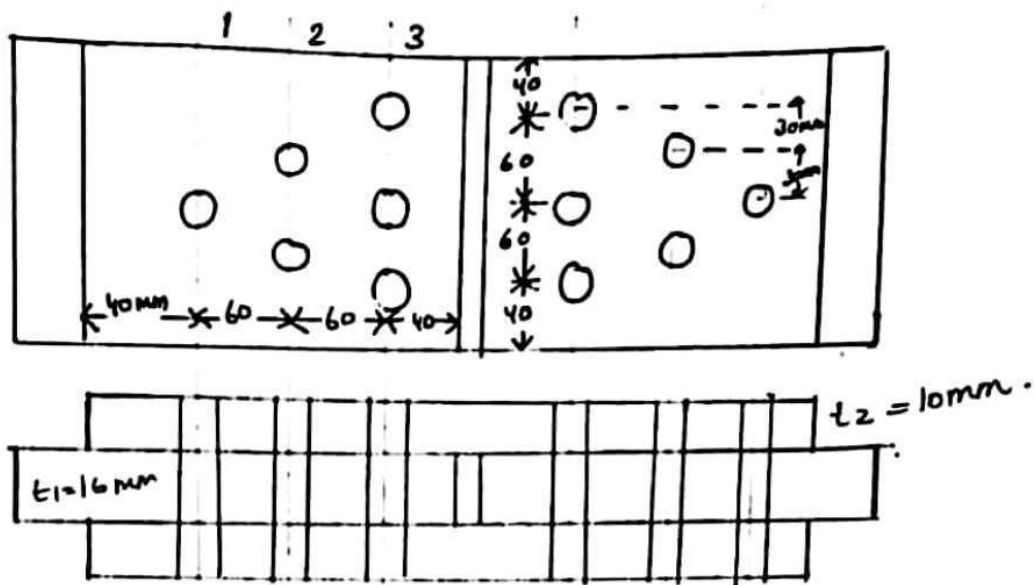
$$\Rightarrow 78.72 \text{ kN} < V_{dsb}$$

Hence OK

Adopt 6 no. of bolts in 2 lines with pitch equal to 65 mm, $e = 40 \text{ mm}$.



Q Find the max. force which can be transferred through the double cover butt joint as given. Also find the efficiency of this joint. Use M20 bolts of grade 4.6 & Fe 410 grade steel for plates.



Solⁿ

a) Design strength of bolt (V_{db})
 Design shear str. of bolt (V_{dsb}) = $\frac{f_{ub}}{\sqrt{3} \times 1.25} (n A_{nb} + n_s A_{sb})$

$$V_{dsb} = \frac{400}{\sqrt{3} \times 1.25} \left(1 \times 0.78 \times \frac{\pi}{4} \times 20^2 + 1 \times \frac{\pi}{4} \times 20^2 \right)$$

$$= 103.314 \text{ kN}$$

Design bearing str. of bolt (V_{dpb}) = $2.5 K_b d t \frac{f_u}{1.25}$

$$K_b = \min. \left\{ \begin{array}{l} \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1.0. \\ \frac{40}{3 \times 22}, \frac{60}{3 \times 22} - 0.25, 0.99, 1.0. \end{array} \right.$$

$$0.6, 0.65, 0.99, 1.0.$$

$$K_b = 0.6$$

$$V_{dpb} = 2.5 \times 0.6 \times 20 \times 16 \times \frac{410}{1.25}$$

$$\Rightarrow 157.44 \text{ kN}$$

$$t = \min \{ t_1, 2t_2 \}$$

$$= 16 \text{ mm}$$

$$V_{db} = \min \begin{cases} V_{dsb} \\ V_{dpsb} \end{cases} \Rightarrow V_{db} = 103.314 \text{ kN.}$$

Strength of 6 bolts $\Rightarrow 6 \times 103.314 \Rightarrow 619.88 \text{ kN.}$

b) Tensile strength of plate in yielding (T_{dy}) = $\frac{f_y A_g}{1.1}$

$$T_{dy} = \frac{250}{1.1} (200 \times 16)$$

$$\Rightarrow 727.27 \text{ kN.}$$

Tensile strength of plate in rupture.

(i) along s/c 1-1.

$$T_{dn1} = \frac{0.9 f_u}{1.25} (B - n d_o) t.$$

$$= \frac{0.9 \times 410}{1.25} (200 - 1 \times 22) \times 16$$

$$\Rightarrow 840.72 \approx 841 \text{ kN.}$$

(ii) along s/c 2-2

$$T_{dn2} = \frac{0.9 \times 410}{1.25} (200 - 2 \times 22) \times 16 + V_{db}.$$

$$\Rightarrow 736.819 + 103.314 \Rightarrow 840.133 \text{ kN.}$$

$$= 840 \text{ kN.}$$

(iii) along s/c 3-3.

$$T_{dn3} = \frac{0.9 \times 410}{1.25} (200 - 3 \times 22) \times 16 + 3V_{db}.$$

$$\Rightarrow 632.90 + 3 \times 103.314.$$

$$\Rightarrow 942.84 \text{ kN.}$$

Hence, tensile strength of plate in rupture.

$$T_{dn} = \min \{ T_{dn1}, T_{dn2}, T_{dn3} \}$$

$$\Rightarrow \underline{\underline{840 \text{ kN.}}}$$

→ Tensile str. of cover plate (T_{dnc}) = $\frac{0.9 f_u (B - 3d_o) \times t_c}{1.25}$

= $\frac{0.9 \times 410}{1.25} (200 - 3 \times 22) \times (10 \times 2)$

= 791.13 kN.

a) Hence, str. of joint = $\min \{ 6V_{db}, T_{dg}, T_{dn}, T_{dnc} \}$.

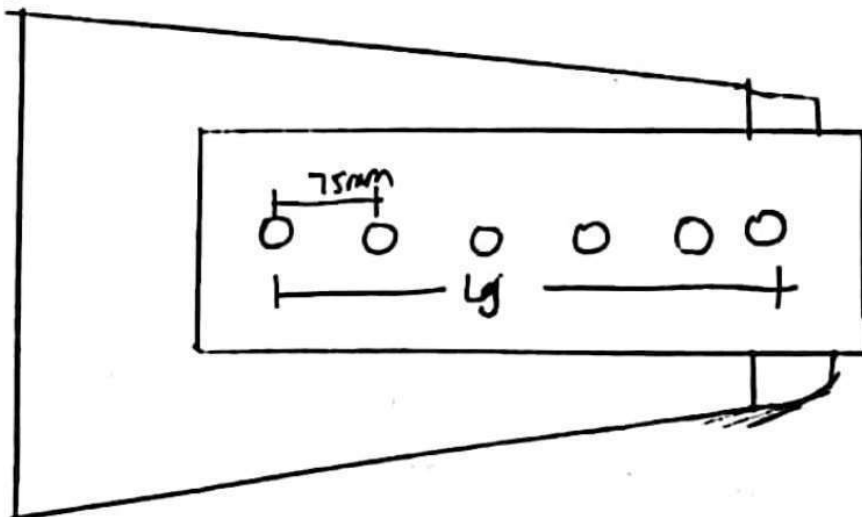
= $\{ 619.88, 727.27, 840, 791.13 \}$

= 619.88 kN.

b) Efficiency of the joint (η) = $\frac{619.88}{727.27} \times 100$.

= 85.23%.

Q A tie member was connected to a gusset plate with 6 bolts of 20mm dia and grade 4.6 along the dirⁿ of the force in the member. If the bolt were provided at a pitch of 75mm. And at edge d/s of 40mm, check whether shear capacity of joint will reduce or not. If yes, then by what %.



$$L_g = 5 \times p = 5 \times 75 = 375 \text{ mm.}$$

$$15d = 15 \times 20 = 300 \text{ mm.}$$

since $L_g > 15d \Rightarrow$ it is long joint.

Hence shear capacity is reduced by β_{Lj} .

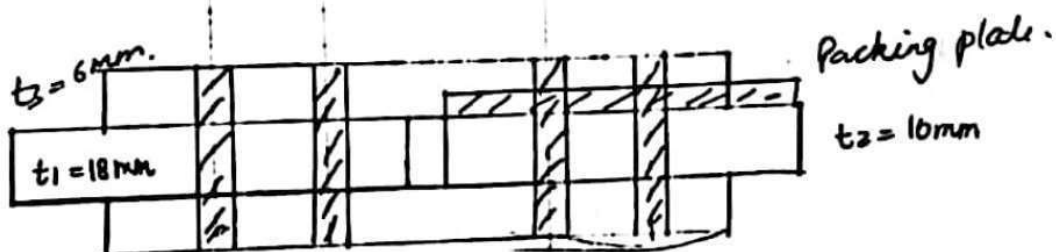
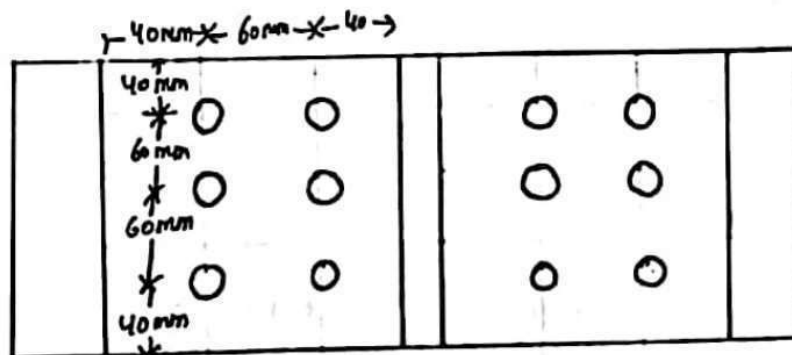
$$\beta_{Lj} = 1.075 - \frac{L_j}{250 \times 20}$$

$$\Rightarrow 0.98125.$$

$$\begin{aligned} \% \text{ reduction in str} &= (1 - \beta_{Lj}) \times 100 \\ &= (1 - 0.98125) \times 100 \\ &= 1.875 \% \end{aligned}$$

Lesson 19 Nov 3.

Q Two plates 10mm & 18mm thick are connected by double cover butt joint using 6mm cover plate as given. Find the strength of the joint. Use Consider M20 bolts of grade 4.6 & Fe 410 grade of steel for plates.



- Design shear strength of bolt.

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} (n_n A_{nb} + n_s A_{sb}) \beta_{pk}$$

$t_{pk} \Rightarrow t_1 - t_2 = 18 - 10 = 8 \text{ mm} > 6 \text{ mm}$. Hence β_{pk} is applicable

$$\begin{aligned} \beta_{pk} &= 1 - 0.0125 \times t_{pk} \\ &= 1 - 0.0125 \times 8 \\ &= 0.9 \end{aligned}$$

$$\begin{aligned} V_{dsb} &= \frac{400}{\sqrt{3} \times 1.25} \left(1 \times 0.78 \times \frac{\pi}{4} \times 20^2 + \frac{\pi}{4} \times 20^2 \right) \times 0.9 \\ &\Rightarrow 92.98 \text{ kN} \end{aligned}$$

- Design bearing str. of bolt

$$V_{dpb} = 2.5 K_b d t \frac{f_u}{1.25}$$

$$K_b = \min \left\{ \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1.0 \right\}$$

$$\Rightarrow \left\{ \frac{40}{3 \times 22}, \frac{60}{3 \times 22} - 0.25, \frac{400}{410}, 1. \right\}$$

$$= \{ 0.6, 0.65, 0.97, 1. \}$$

$$K_b = 0.6$$

$$V_{dpb} = 2.5 \times 0.606 \times 20 \times 10 \times \frac{410}{1.25} \Rightarrow 99.38 \text{ kN}$$

$$t = \min \{ t_1, t_2, 2t_c \}$$

$$= \{ 18, 10, 2 \times 6 \}$$

$$= 10 \text{ mm}$$

$$\text{Strength of bolt } (V_{db} \Rightarrow \min \left\{ \frac{V_{dsb}}{V_{dpsb}} \right\} \Rightarrow 92.98 \text{ kN.}$$

$\Rightarrow$ Tensile strength of plate in yielding

$$T_{dg} = \frac{f_y A_g}{1.1} \Rightarrow \frac{250}{1.1} (200 \times 10).$$

$$T_{dg} \Rightarrow 454.54 \text{ kN.}$$

$$t = \min \{ t_1, t_2, 2t_3 \}$$

$$\Rightarrow \min \{ 18, 10, 2 \times 6 \}$$

$$\Rightarrow 10 \text{ mm}$$

$\Rightarrow$ Tensile strength of plate in rupture

$$T_{dn} = \frac{0.9 f_u}{1.25} (B - n d_o) t.$$

$$\Rightarrow \frac{0.9 \times 410}{1.25} (200 - 3 \times 22) \times 10$$

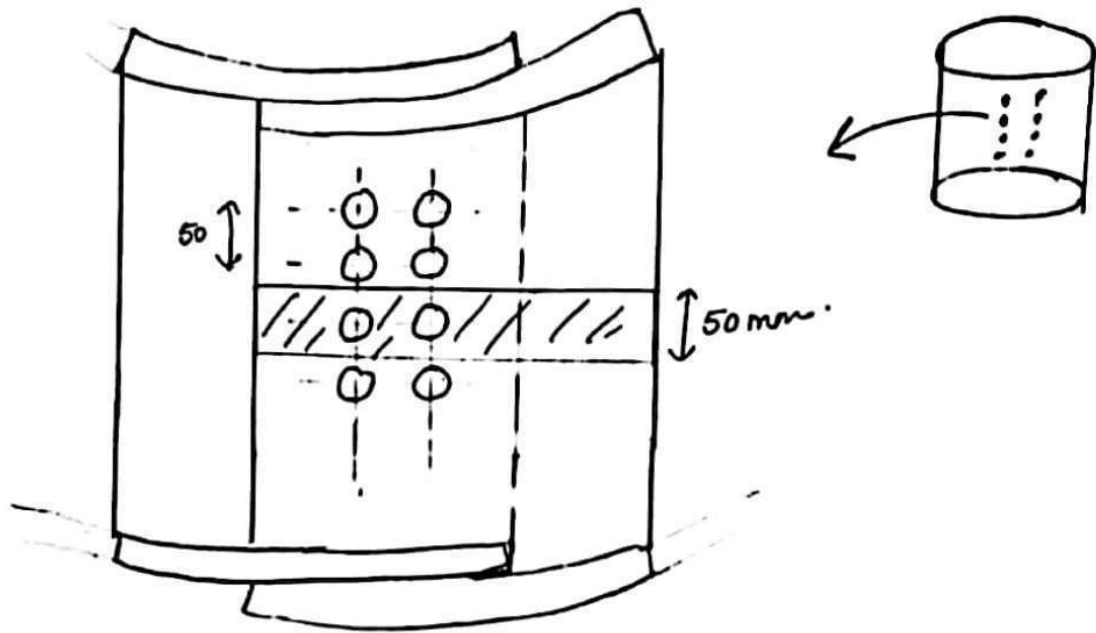
$$\Rightarrow 395.56 \text{ kN.}$$

$$\text{Str. of Joint} \Rightarrow \min. \{ 6V_{db}, T_{dg}, T_{dn} \}.$$

$$\Rightarrow \min. \{ 557.88, 454.54, 395.56 \}.$$

$$\Rightarrow \underline{\underline{395.56 \text{ kN.}}}$$

Q A boiler shell is made up of 14 mm thick Fe 410 plates. If the joint is double belted, Lap joint with M 16 bolts of grade 4.6, at a pitch of 50 mm. determine the design strength of joint per pitch length. Also comment, is it a safe design. if the internal dia. of shell is 1m and factored steam press is 12 MPa.



Solⁿ Design shear strength of bolt.

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} (n_n A_{nb})$$

$$V_{dsb} = \frac{400}{\sqrt{3} \times 1.25} \times (1 \times 0.78 \times \frac{\pi}{4} \times 16^2)$$

$$\Rightarrow 28.97 \text{ kN}$$

Design bearing strength of bolt $V_{dpb} = 2.5 K_b d t \frac{f_u}{1.25}$

$$K_b = \min \left\{ \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1.0 \right\}$$

$$\Rightarrow \left\{ \frac{1.5 \times 18}{3 \times 18}, \frac{50}{3 \times 18} - 0.25, \frac{400}{410}, 1.0 \right\}$$

$$\text{Consider } e \Rightarrow e_{\min} \Rightarrow \min \{ 0.5, 0.67, 0.97, 1.0 \}$$

$$K_b = 0.5$$

$$\approx 1.5d_0$$

$$\approx 1.5 \times 18$$

$$\approx 27 \text{ mm}$$

$$V_{dpb} \Rightarrow 2.5 \times 0.5 \times 16 \times 14 \times \frac{410}{1.25}$$

$$\Rightarrow 91.84 \text{ kN}$$

- Design strength of bolt $V_{db} = \min. \begin{cases} V_{dsb} \\ V_{dpb} \end{cases}$
 $\approx 28.97 \text{ kN}$

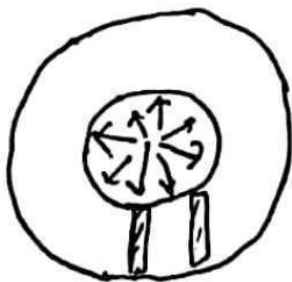
$\Rightarrow$ Design strength of bolt per pitch length $= 2 V_{db}$
 $= 2 \times 28.97$
 $\Rightarrow 57.94 \text{ kN}$

$\Rightarrow$ Design strength of plate in yielding per pitch length
 $T_{dg} = \frac{f_y A_g}{1.1} \approx \frac{250}{1.1} (50 \times 14)$
 $= 159.09 \text{ kN}$

- Design strength of plate per pitch length in rupture (T_{dn}). $\Rightarrow \frac{0.9 f_u}{1.25} (p - n d_o) t$
 $= \frac{0.9 \times 410}{1.25} (50 - 1 \times 18) \times 14$
 $\approx 132.24 \text{ kN}$

Design str. of joint per pitch length $\approx \min \{ 2 V_{db}, T_{dg}, T_{dn} \}$
 $\approx \min \{ 57.94, 159.09, 132.24 \}$
 $\approx 57.94 \text{ kN}$

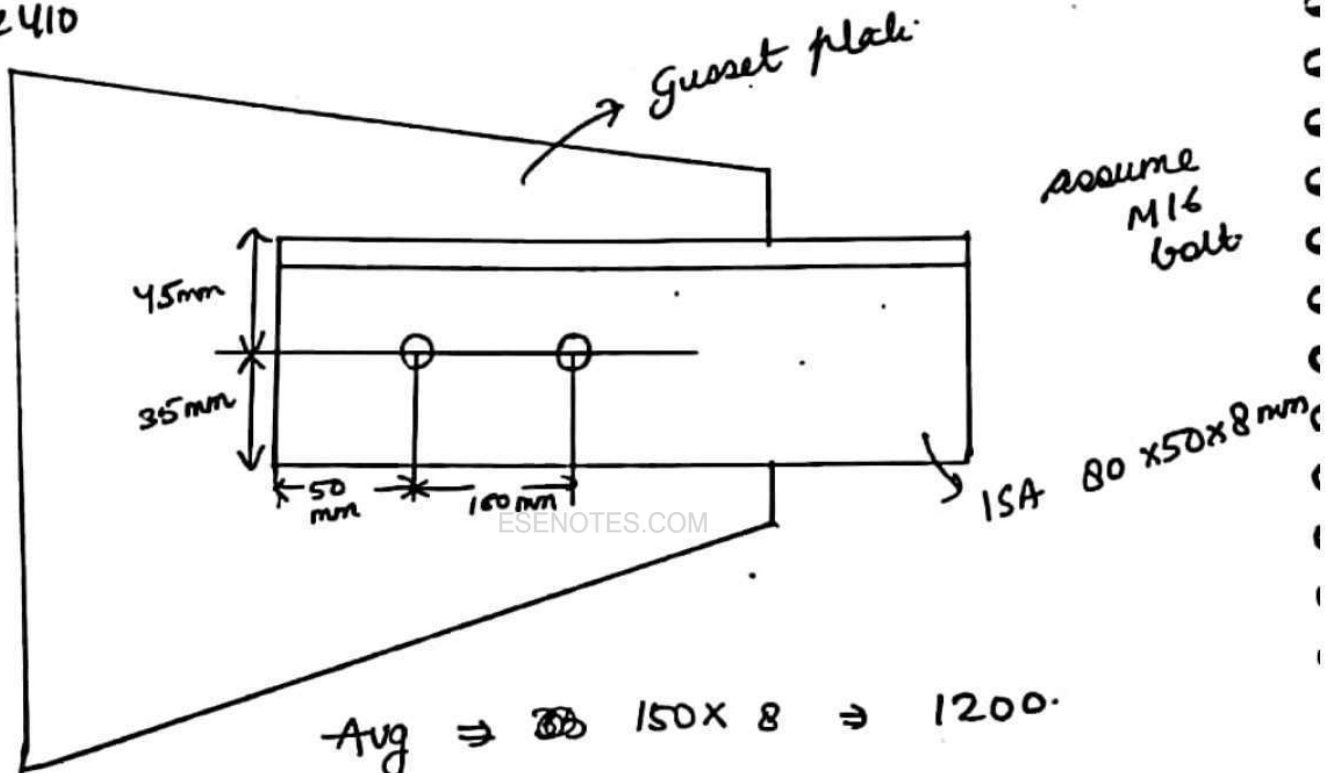
b) Hoop tension in the plate $\sigma_h = \frac{PD}{2t} = \frac{12 \times 1 \times 10^3}{2 \times 14}$
 $= 428.57 \text{ MPa}$



Force on the plate per pitch length due to hoop tension,
 $= \sigma_h p t$
 $= 428.57 \times 50 \times 14 \times 10^{-3}$
 $\Rightarrow 300 \text{ kN} > 57.94 \text{ kN}$ Not Safe

Q Determine the block shear strength of the tension member as given. Steel used is of grade

Fe410



$$A_{vg} \Rightarrow 150 \times 8 \Rightarrow 1200$$

$$A_{tn} \Rightarrow \left(35 - \frac{18}{2} \right) \times 8 \Rightarrow 208$$

$$A_{tg} \Rightarrow 35 \times 8 \Rightarrow 280$$

$$A_{vn} \Rightarrow \left(150 - \frac{1.5 \times 18}{2} \right) \times 8 \Rightarrow 984$$

$$T_{db} \Rightarrow \frac{f_y A_{vg}}{\sqrt{3} \times 1.1} + \frac{0.9 f_u A_{tn}}{1.25}$$

$$\Rightarrow \frac{250 \times 1200}{\sqrt{3} \times 1.1} + \frac{0.9 \times 410 \times 208}{1.25}$$

$$\Rightarrow 157.459 + 61.40 \Rightarrow 218.86 \text{ kN}$$

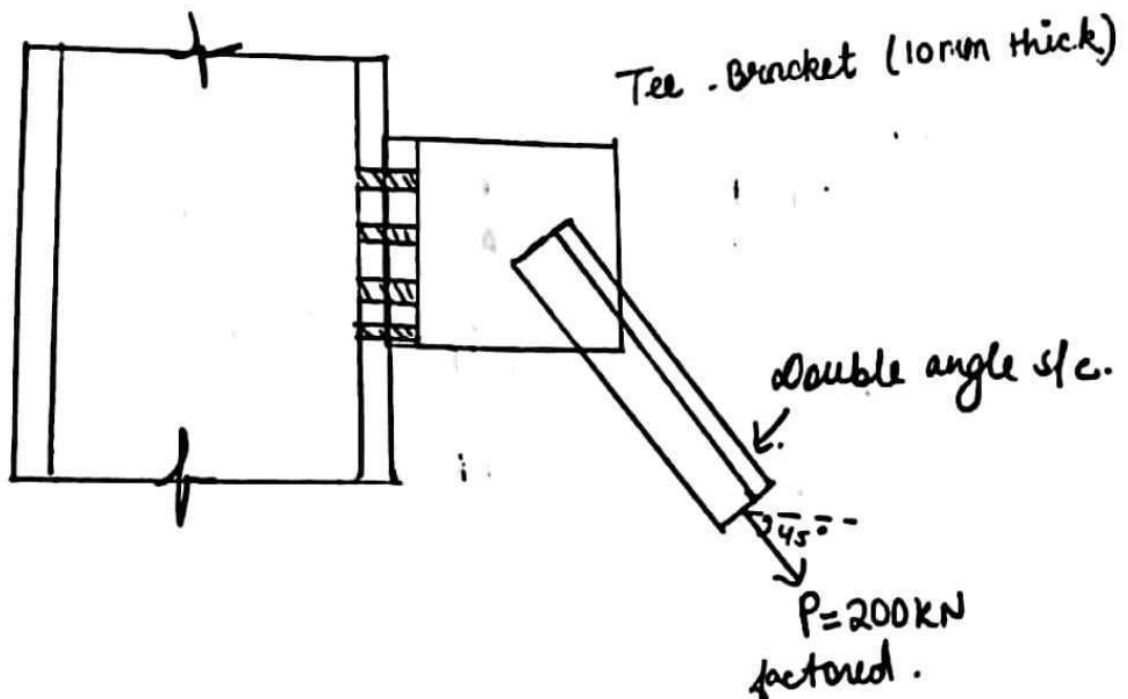
$$\begin{aligned}
 T_{dB} &\Rightarrow \frac{0.9 f_u A_{vn}}{\sqrt{3} \times 1.25} + \frac{f_y A_{tg}}{1.1} \\
 &\Rightarrow \frac{0.9 \times 410 \times 984}{\sqrt{3} \times 1.25} + \frac{250 \times 280}{1.1} \\
 &167.706 + 63.63 \\
 &\Rightarrow 231.34 \text{ kN}
 \end{aligned}$$

$$T_{dB} = \underline{\underline{218.8 \text{ kN}}}$$

Lesson 22 Nov 6.

Q. Determine whether the joint given below is safe or not. 8 M16 bolts of grade 4.6. Neglect the effect of prying forces.

Also compute the no. of 16mm dia. bolts of grade 4.6 to connect the double angle section (8mm thick each) member with the web of the T-bracket



$$\left(\frac{V_{sb}}{V_{db}}\right)^2 + \left(\frac{T_b}{T_{db}}\right)^2 \leq 1.$$

$V_{db} \Rightarrow$ design strength of bolt $\Rightarrow \min. \begin{cases} V_{dsb} \\ V_{dpb} \end{cases}$

Factored tension force in 1 bolt. $(T_b) = \frac{P \cos 45^\circ}{8}$

$$\Rightarrow \frac{200 \cos 45^\circ}{8} = 17.67.$$

Factored shear force in 1 bolt $= \frac{P \sin 45^\circ}{8}$

$$\Rightarrow \frac{200 \sin 45^\circ}{8} = 17.67$$

$V_{dsb} \Rightarrow \frac{f_{ub}}{\sqrt{3} \times 1.25} (n_n A_{nb} + n_s A_{sb})$

$$\Rightarrow \frac{400}{\sqrt{3} \times 1.25} (1 \times 0.78 \times \frac{\pi}{4} \times 16^2)$$

$$\Rightarrow 28.974 \text{ kN}.$$

$V_{dpb} \Rightarrow 2.5 K_b d t \frac{f_u}{1.25}$

$K_b \Rightarrow \min \left\{ \frac{e}{3d_o}, \frac{p}{3d_o} - 0.25, \frac{f_{ub}}{f_u}, 1 \right\}$

$$\left\{ \frac{1.5d_o}{3d_o}, \frac{2.5}{3d_o} - 0.25, \frac{400}{410}, 1 \right\}$$

$\min \{ 0.5, 0.49, 0.97, 1 \}$

$K_b \Rightarrow 0.49$

$$V_{dpb} = 2.5 \times 0.49 \times 16 \times 10 \times \frac{410}{1.25}$$

$$\Rightarrow 64.28 \text{ kN.}$$

$$V_{db} = \min. \left\{ \begin{array}{l} 28.97 \\ 64.28 \end{array} \right\} \Rightarrow 28.97 \text{ kN.}$$

$$T_{db} = \frac{0.9 f_{ub}}{1.25} A_{nb}.$$

$$\Rightarrow \frac{0.9 \times 400}{1.25} \times 0.78 \times \frac{\pi}{4} (16)^2 = 45.16 \text{ kN.}$$

$$T_{db} \leq \frac{f_{yb}}{1.1} \times A_{sb} = \frac{400 \times 0.6}{1.1} \times \frac{\pi}{4} (16)^2$$

$$\Rightarrow 43.86 \text{ kN}$$

$$\text{Hence } T_{db} = \underline{43.86 \text{ kN}}$$

for safety in Combined tension & shear.

$$\left(\frac{V_{db}}{V_{dpb}} \right)^2 + \left(\frac{T_{db}}{T_{db}} \right)^2 \leq 1.$$

$$\left(\frac{17.67}{28.97} \right)^2 + \left(\frac{17.67}{43.86} \right)^2 \Rightarrow$$

$$0.372 + 0.162 \Rightarrow 0.534 \leq 1$$

Hence OK.

b) for double angle section: bolts are in double shear

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} (n_n A_{nb} + n_s A_{sb})$$

$$\Rightarrow \frac{400}{\sqrt{3} \times 1.25} \left(1.78 \times \frac{\pi}{4} 16^2 \right) = 66.1 \text{ kN}$$

$$V_{dpb} = 2.5 K_b d t \frac{f_u}{1.25}$$

Assume, $e = e_{min} = 1.5 d_o = 1.5 \times 18 \Rightarrow 27 \text{ mm}$.

$p \approx p_{min} \Rightarrow 2.5 d = 2.5 \times 16 \Rightarrow 40 \text{ mm}$

$$K_b = \min \left\{ \frac{e}{3d_o}, \frac{p}{3d_o} - 0.25, \frac{f_{ub}}{f_u} \right\}, 1$$

$K_b = 0.49$.

$t = \frac{2 \times 8}{2} = 16$
 $\frac{10}{10} = 10 \text{ mm}$.

$$V_{dpb} = 2.5 \times 0.49 \times 16 \times 10 \times \frac{410}{1.25}$$

$\Rightarrow 64.288 \text{ kN}$.

$$V_{db} = \min \begin{cases} V_{dsb} \\ V_{dpb} \end{cases}$$

$\Rightarrow 64.28 \text{ kN}$.

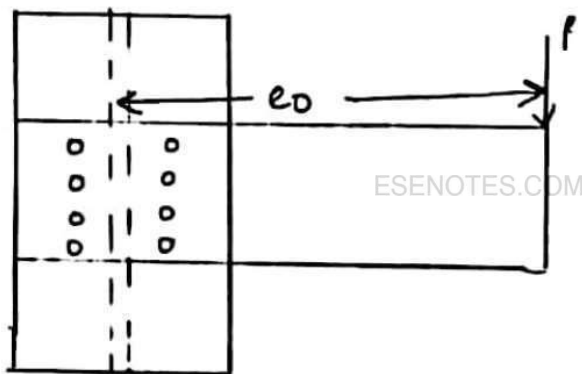
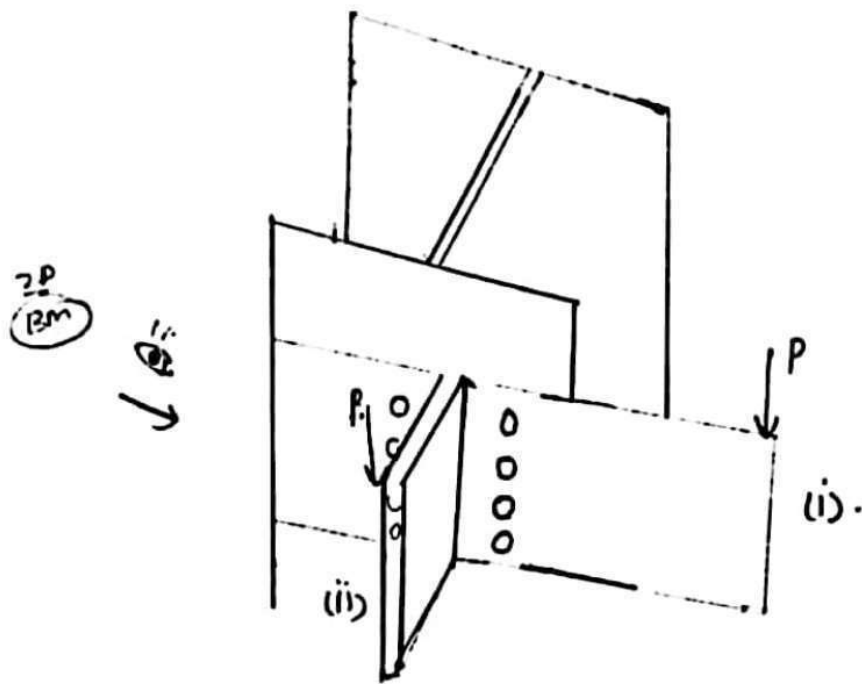
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no. of $16 \text{ mm } \phi$ bolt req. = $\frac{P}{V_{db}} = \frac{200}{64.28} = 3.11 \approx 4 \text{ bolts}$

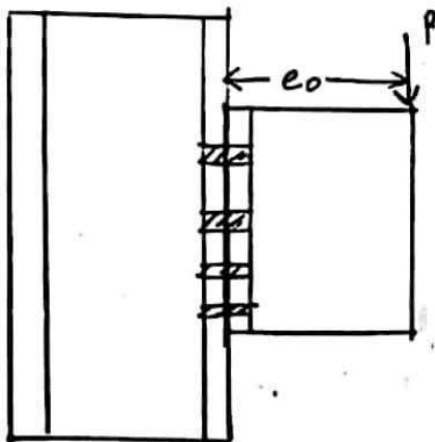
Eccentrically loaded Connections.

$\Rightarrow$ depending on how the bolts are placed in the connection following two types of eccentric connections can be obtained.

- DIRECT LOAD & TWISTING MOMENT.
- DIRECT LOAD & BENDING MOMENT

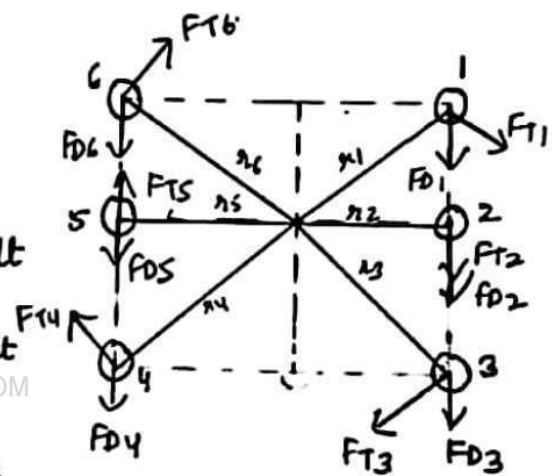
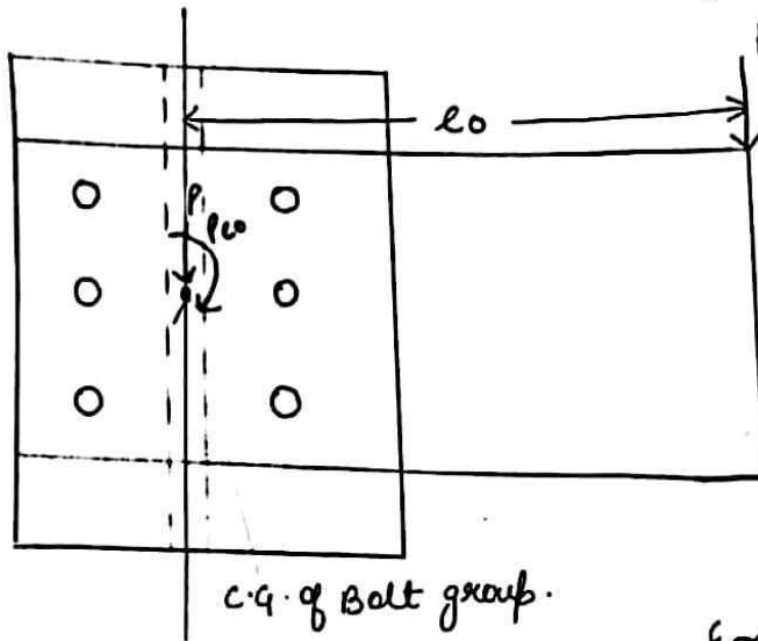


a) Direct load & Twisting moment. ($T = Pe_o$).



b) Direct load & Bending moment. ($M = Pe_o$)

a) Bolts subjected to direct loading & Twisting moment.
 P (factored load).



F_{Di} = direct shear force in i^{th} bolt

F_{Ti} = torsional shear force in i^{th} bolt

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$$F_{Di} = \frac{P}{n}; \text{ where } n = \text{no. of bolts.}$$

and its line of action is parallel to the line of action of applied force.

$$\left[\frac{T}{J} = \frac{\tau}{x} \Rightarrow \tau = \frac{T x}{J} \right]$$

$$F_{Ti} = \frac{T x_i}{J} \cdot A_i$$

T = Twisting moment. ($P e_0$)

x_i $\Rightarrow$ distance of i^{th} bolt from C.G. of bolt group.

J = polar moment of inertia of bolt group.

$$= \sum A_i x_i^2$$

A_i = area of i^{th} bolt.

Hence,
$$F_{Ti} = \frac{P e_0 x_i \cdot A_i}{\sum A_i x_i^2}$$

→ If area of all the bolts are same; then.

$$F_{Ti} = \frac{P_{e0} x_i}{\sum x_i^2}$$

and its line of action is perpendicular to the line joining the C.G. of the bolt group with the center of bolt under consideration (and its sense must be same as that of torsional moment)

Hence, resultant force on i^{th} bolt. F_{Ri}

$$F_{Ri} = \sqrt{F_{Di}^2 + F_{Ti}^2 + 2 F_{Di} F_{Ti} \cos \theta}$$

and for safety. $F_{Ri} \leq V_{db}$

Note: Most critical bolt.

- Instead of finding out the forces in all the bolts it would be sufficient to check the safety of that bolt which is carrying max. resultant force.
 - This bolt is termed as "most critical bolt."
 - The most critical bolts are those which are farthest from the C.G. of the bolt group and in which angle between F_{Di} & F_{Ti} is least. (for eg. bolt ① & ③).
- ⇒ or. a general rule can be followed that a bolt which is farthest from the C.G. of bolt group and nearest to the applied load line is most critical bolt.
- ⇒ To design the eccentric connection we will have to choose the number and arrangement of the bolts.
- ⇒ Here no. of bolts are given by "n"

$$n = \sqrt{\frac{6M}{m P V_{db}}}$$

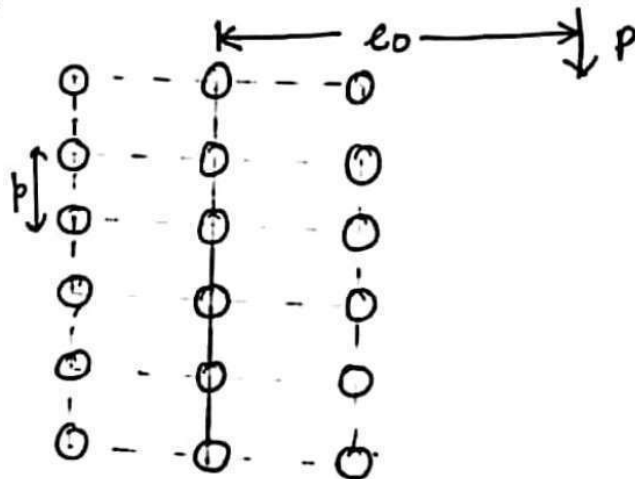
$M \Rightarrow$ Twisting moment. (P20)

$P \Rightarrow$ pitch

$V_{db} \Rightarrow$ design strength of bolt

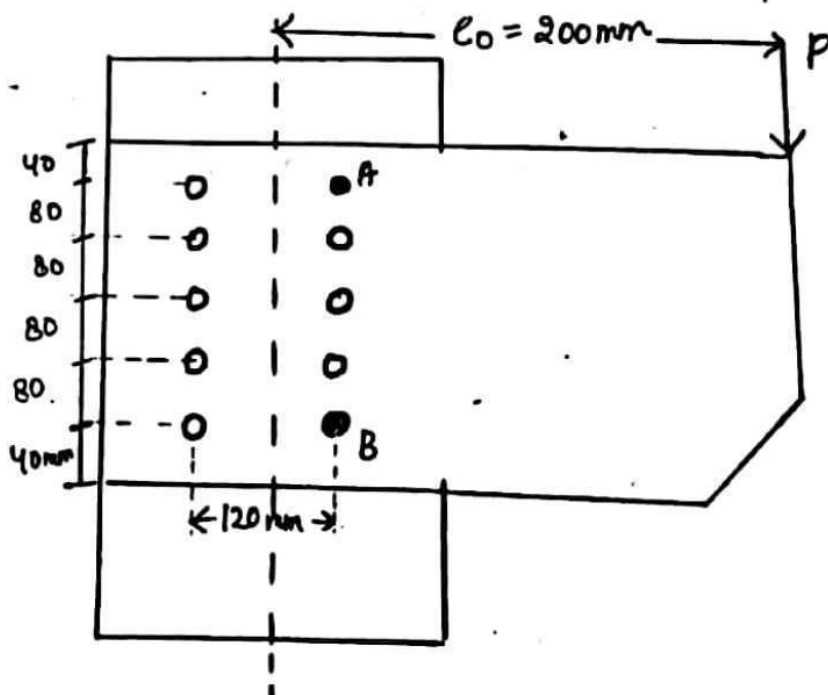
$n =$ no. of bolts in one line. (6).

$m =$ no. of bolt lines. (3)



Lesson 30 Nov-16

Q Determine the factored load "P" that can be carried by the Joint as shown below. The bolt used are 20 mm dia of grade 4.6. The thickness of the flange of I-section is 9.1 mm. & that of bracket plate is 10 mm. Assume steel of Fe 410 grade.



Solⁿ - Design shear str. of bolt. $V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} [n_n A_{nb} + n_s A_{sb}] \beta_l$

$$n_n = 1 \quad n_s = 0$$

$$l_j = 4 \times 80 = 320 \text{ mm.}$$

$$\text{also } l_{sd} = 15 \times 20 = 300 \text{ mm.}$$

Since $l_j > l_{sd} \Rightarrow \beta_l$ is to be applied.

$$\beta_l = 1.075 - \frac{l_j}{200d}$$

$$= 1.075 - \frac{320}{200 \times 20} = 0.995$$

$$V_{dsb} = \frac{400}{\sqrt{3} \times 1.25} \left(1 \times 0.78 \times \frac{\pi \times 20^2}{4} \right) \times 0.995$$

$$= 45.03 \text{ kN.}$$

- Design bearing str. of bolt. $V_{dpb} = 2.5 K_b d t \frac{f_u}{1.25}$

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$$K_b = \min \left\{ \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1.0 \right\}$$

$$\Rightarrow \min \left\{ \frac{40}{3 \times (20+2)}, \frac{80}{3 \times 22} - 0.25, \frac{400}{410}, 1 \right\}$$

$$\Rightarrow 0.606$$

$$t = \min \{ 9.1, 10 \} \Rightarrow 9.1 \text{ mm.}$$

$$V_{dpb} = 2.5 \times 0.606 \times 20 \times 9.1 \times \frac{410}{1.25}$$

$$\Rightarrow 90.44 \text{ kN.}$$

Design strength of the bolt. $V_{db} = \min \begin{cases} V_{dsb} \\ V_{dpb} \end{cases}$

$$\Rightarrow 45.03 \text{ kN.}$$

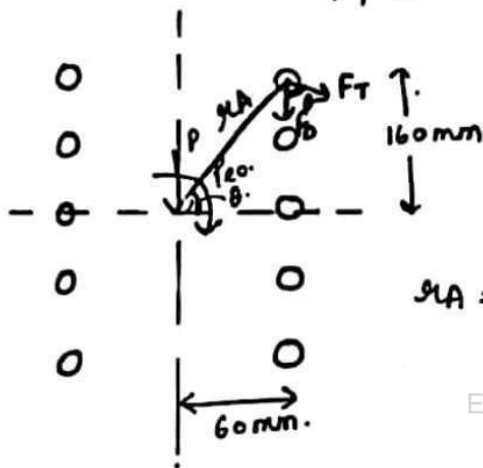
- Bolts A & B are the most critical bolts, having resultant force.

$$F_R = \sqrt{F_D^2 + F_T^2 + 2 F_D F_T \cos \theta}$$

$$F_D = \frac{P}{n} = \frac{P}{10} \quad - (i).$$

$$F_T = \frac{P e_0 \cdot x_A}{\sum x_i^2} = \frac{P \times 200 \times \sqrt{160^2 + 60^2}}{2 \times 60^2 + 4(\sqrt{60^2 + 160^2})^2 + 4(\sqrt{60^2 + 80^2})^2}$$

$$F_T = 0.2085 P \quad - (ii).$$



$$x_A = \sqrt{160^2 + 60^2} = 170.88$$

$$\cos \theta = \frac{60}{\sqrt{160^2 + 60^2}} = 0.3511$$

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$$F_R = \sqrt{\left(\frac{P}{10}\right)^2 + (0.2085 P)^2 + 2 \times \frac{P}{10} \times 0.2085 P \times 0.3511}$$

$$F_R = 0.261 P$$

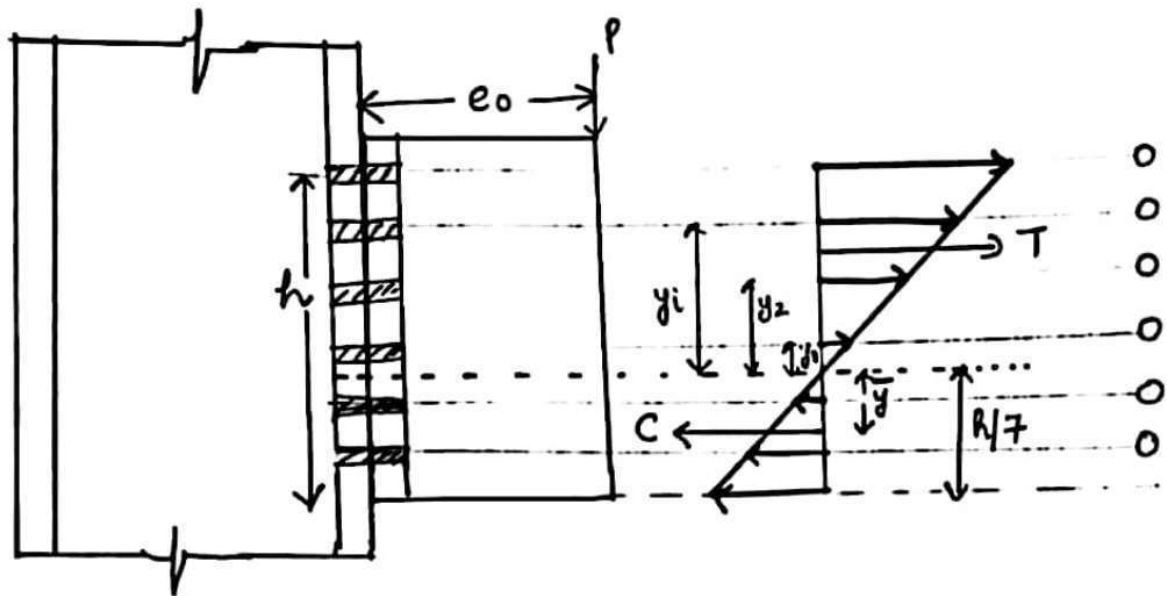
for safety $F_R \leq V d b$

$$0.261 P \leq 45.03$$

$$P \leq 172.66 \text{ kN}$$

$$P_{\max} = 172.66 \text{ kN}$$

b) Bolts subjected to direct loading & Bending moment.



- Here centre of rotation is not assumed at the centre of bolt group as area of the bracket section below the centre of rotation will be in compression & is more than the area of bolt in tension.
- centre of rotation lies b/w the foot of bracket and the centre of bolt group. and is assumed at " $\frac{h}{7}$ " from the base of bracket.
- Here "h" is the depth of bracket measured from the bottom of the bracket to the C.G. of topmost bolt in the connection.

Hence, shear force in i^{th} bolt due to direct loading $(V_{bi}) = \frac{P}{n}$

also, Tension force in i^{th} bolt due to Bending moment $(T_{bi}) = \frac{M' y_i}{I} \times A_i$

Note $\Rightarrow \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \Rightarrow \sigma = \frac{M}{I} \cdot y$

M' = moment resisted in tension.

y_i = distance of the concerned i^{th} bolt in tension from centre of rotation.

A_i $\Rightarrow$ area of the i^{th} bolt in tension

$I \Rightarrow$ moment of inertia of bolt group. $= \sum A_i y_i^2$.

If all the bolts are of same size.

$$T_{bi} = \frac{M' y_i A_i}{\sum A_i y_i^2}$$

$$\frac{M' y_i A_i}{A_i \sum y_i^2} = \frac{M' y_i}{\sum y_i^2}$$

$$T_{bi} = \frac{M' y_i}{\sum y_i^2}$$

Total tension force in bolts $T = \sum T_{bi} = \sum \frac{M' y_i}{\sum y_i^2}$

$$= \frac{M' \sum y_i}{\sum y_i^2}$$

- For Equilibrium, Total compressive force. = Total tensile force (T)

(C)

$$C = \frac{M' \sum y_i}{\sum y_i^2}$$

- Total external moment = moment resisted by belts in tension + moment resisted in comp.

$$P_{eo} = M' + c\bar{y}$$

$$P_{eo} = M' + M' \frac{\sum y_i^2}{\sum y_i^2} \cdot \bar{y}$$

$$P_{eo} = M' \left[1 + \frac{\sum y_i^2}{\sum y_i^2} \cdot \bar{y} \right]$$

$$M' = \frac{P_{eo}}{1 + \frac{\sum y_i^2}{\sum y_i^2} \cdot \bar{y}}$$

$$\text{Here } \bar{y} = \frac{2}{3} \cdot \frac{h}{7}$$

$$\bar{y} = \frac{2h}{21}$$

Using M' , we can calculate tension in most critical bolt. i.e. bolt for which y_i is max.

$$T_b = \frac{M' y_m}{\sum y_i^2}$$

Now, for safety of bolt subjected to combined shear & tension, Interaction Equation must be satisfied.

$$\left(\frac{V_{sb}}{V_{db}} \right)^2 + \left(\frac{T_b}{T_{db}} \right)^2 \leq 1$$

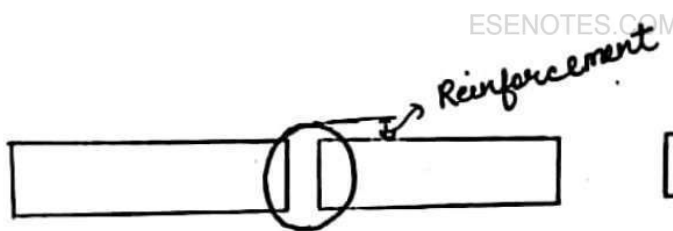
(C) WELDED CONNECTION

- several types of weld can be used for making the connections as follows.

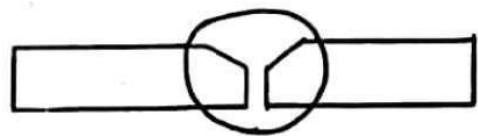
- (i) Butt / Groove weld
- (ii) Fillet weld
- (iii) Plug weld
- (iv) slot weld
- (v) ~~Stitch~~ seam weld.

(i) Butt / Groove Weld.

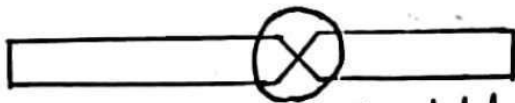
- It is also termed as groove weld
- It is provided when the members to be joined are lined up (एक साथ)
- Various types of Butt weld are as follows:→



Square Butt weld.



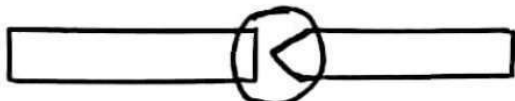
Single V-Butt weld.



Double V-Butt Weld.



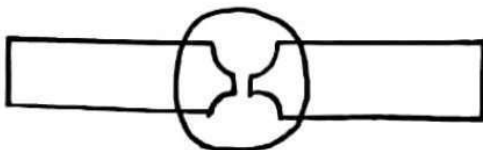
Single Bevel Butt weld



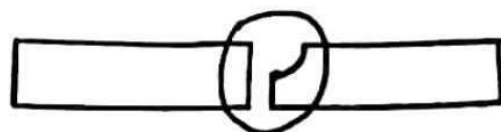
Double Bevel Butt weld.



Single U-Butt weld



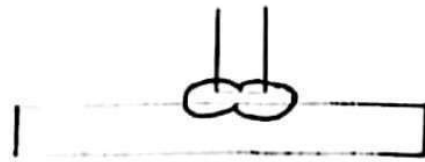
Double U-Butt Weld



Single J-Butt Weld.



Double J-Butt Joint



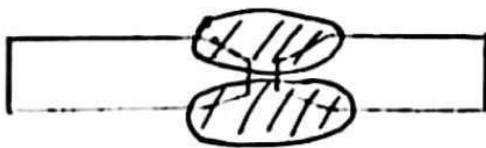
Double Tee Butt Weld.



Single Tee Butt Weld.

- A square butt weld is provided for section having thickness less than 8 mm, above this single U, V, J, T and double U, V, J, T etc. butt weld are provided
- Butt weld is most suitable for alternating stresses provided that complete penetration is insured.
- Whenever complete penetration is ensured, strength of weld is taken same as that of parent metal, but not in case of partial penetration

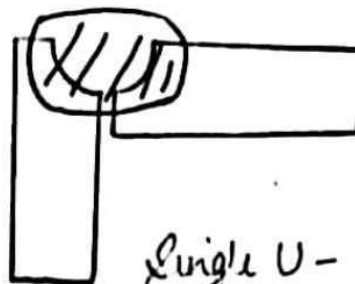
Partial Penetration Groove Weld



Double V-Butt Weld

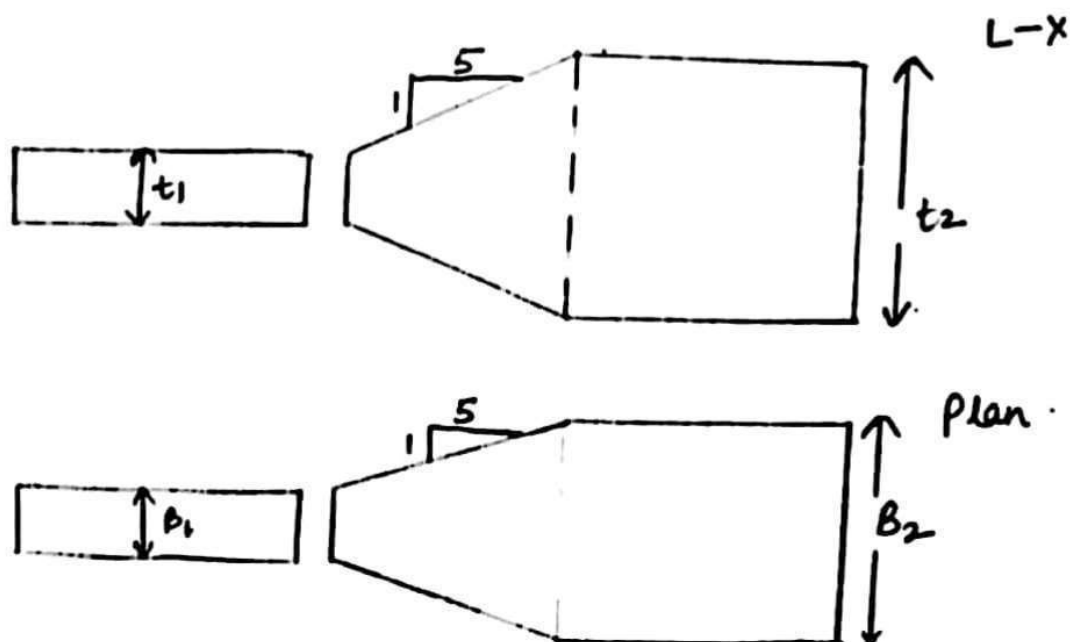


Square Butt Weld.



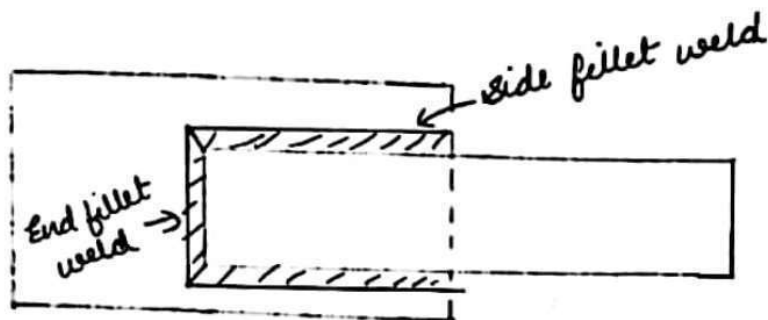
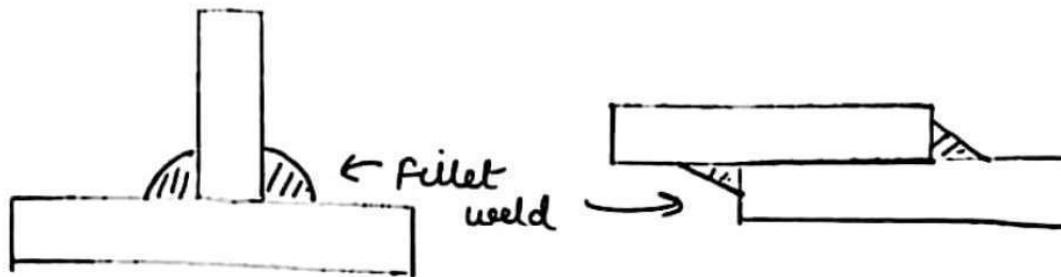
Single U-Butt Weld.

- Reinforcement is the extra weld metal which makes the throat dimension at least 10% greater than the thickness of the welded parts.
- It is provided to increase the efficiency of joints.
- Any reinforcement on the weld is not considered in the calculation of strength.
- Reinforcement makes the weld stronger for static load but in case of repeated / alternating & vibrating loads, stress concentration develops in it leading to earlier failure; hence it is not considered in such cases.
- In any case reinforcement must not exceed 3mm
- If the difference of thickness of 2 plates to be joined i.e. $\left| t_2 - t_1 \right| > \frac{t_1}{4}$ or 3mm whichever is greater, then, we provide tapering of thicker plate and tapering should not be greater than 1 in 5
- This criteria is applicable for width of plate



(ii) FILLET WELD.

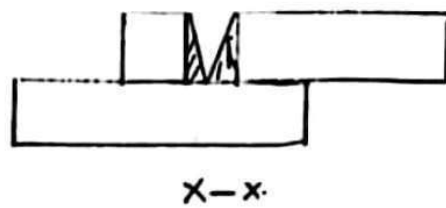
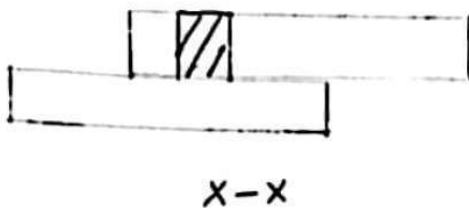
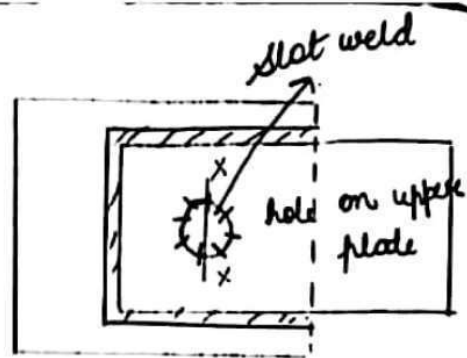
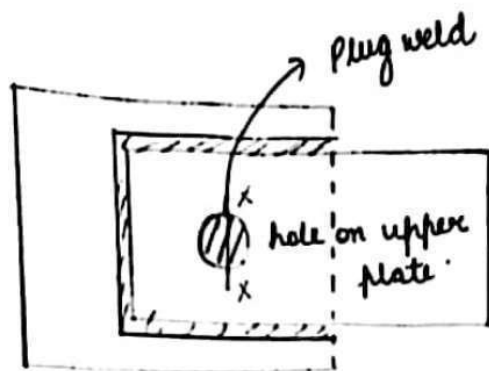
- It is provided when the members to be joined are in different plane.



- End fillet weld is generally stronger than the side fillet weld but our code (IS 800:2007) assumes no difference between the two.
- It is always assumed that fillet weld resist the load by shearing action on its throat.

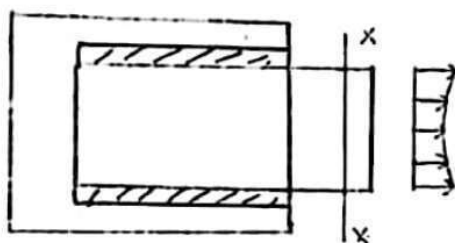
(iii) & (iv) PLUG & SLOT WELD

- If the length of the weld required is more than that can be provided in limited overlap length; we provide plug or slot fillet weld.
- In plug weld small holes are made in one of the plate to be joined & is kept over the another plate & then entire hole is filled with filler material while in slot weld, welding is done along the periphery of hole.

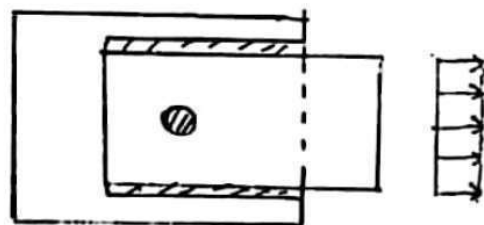


- as per code, plug weld is not used for strength calculation.
- It is provided to make stress uniform

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Tensile stress distribution without plug weld



Tensile stress distribution with plug weld.

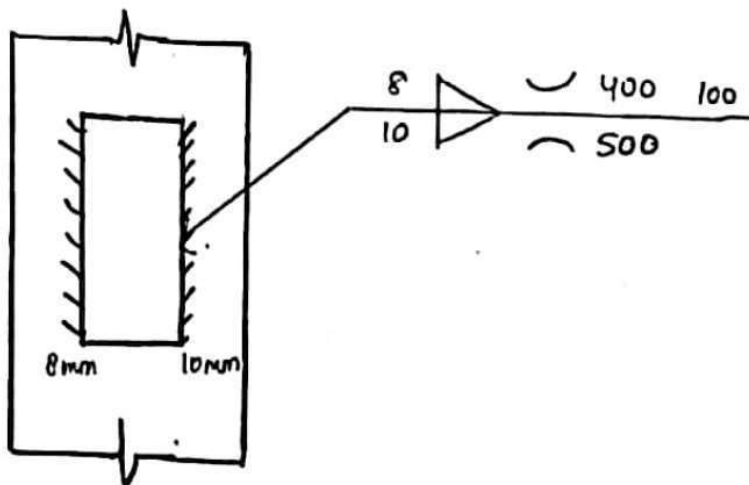
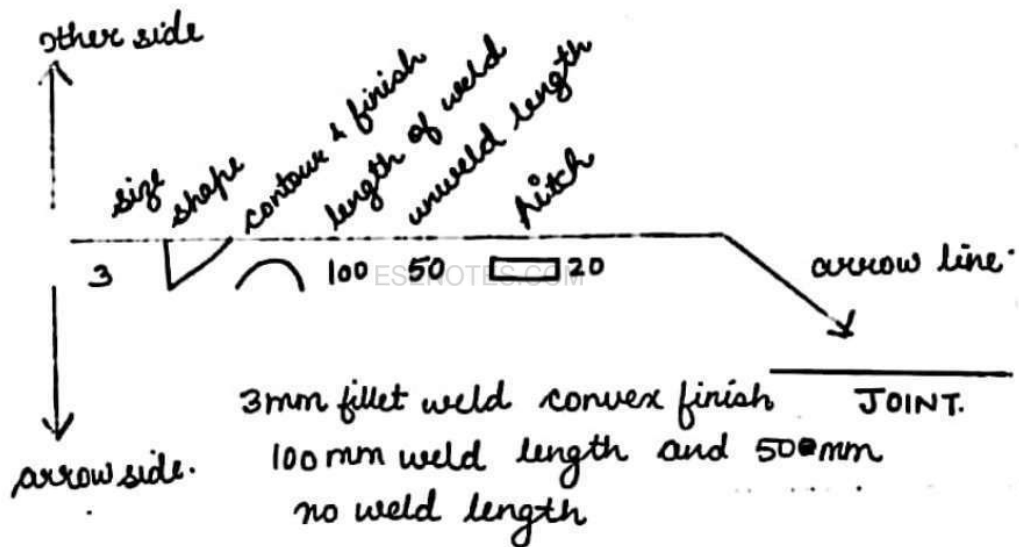
(V) SEAM WELD

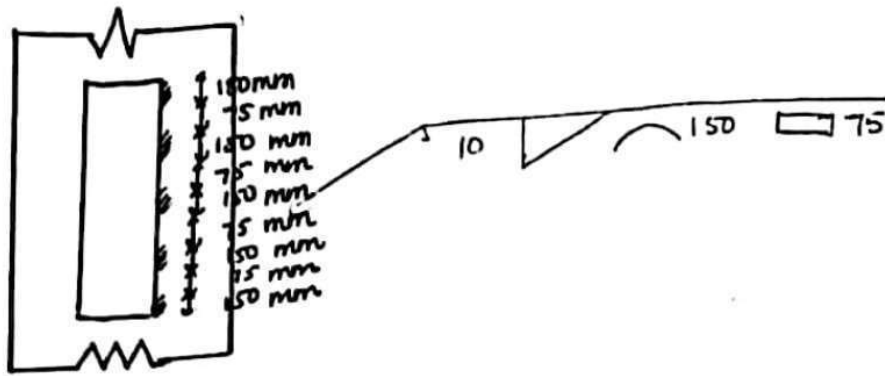
- It is used for applications requiring high air and water tightness.
- A current is passed while the electrodes are rotated & the heat generated by electrical resistance joints weld the metal continuously.

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Description of Type of Weld and its Representation

Type of Weld	Representation	Type of Weld	Representation
(i) Fillet		(iii) SPOT	
(ii) Groove		(iv) PLUG	
a) Square		(v) SEAM	
b) V		(vi) FIELD.	
c) Bevel		(vii) Convex Surface	
d) U		(viii) Concave "	
e) J		(ix) Flat	





Design of BUTT WELD.

- The design strength of groove weld in tension or compression is governed by yielding.

$$T_{dw} = \frac{f_y l_w t_e}{\gamma_{mw}}$$

f_y = minimum of yield strength of weld (f_{yw}) or yield str. of parent material (f_y).

l_w = effective length of weld.

t_e = effective throat thickness

γ_{mw} = partial safety factor.

1.25 for shop weld

1.5 for field weld.

Notes → 1) for complete penetration " t_e " is thickness of thinner plate.

and, for partial penetration " t_e " is taken as thickness of weld common to the two plates.

2) If this value is not given, then for partial penetration

$$t_e = \frac{5}{8} \times \text{thickness of thinner plate}$$

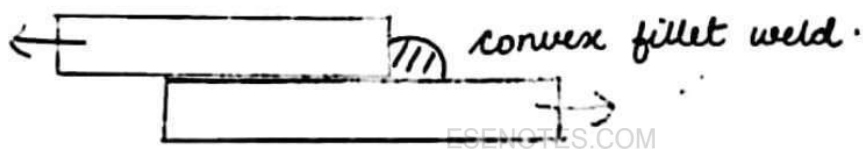
- The design strength of Butt weld in shear is also governed by yield.

$$V_{dw} = \frac{f_y l_w t_e}{\sqrt{3} \gamma_{mw}}$$

Note 3) IS code does not mention the strength formula for butt weld, but it simply says that butt weld shall be treated as parent metal with thickness equals to throat thickness & stress shall not exceed those permitted in the parent metal

Design of Fillet Weld.

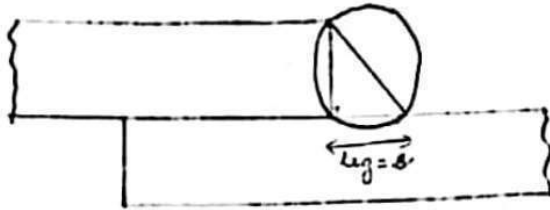
- Generally concave or convex shape of fillet weld is provided



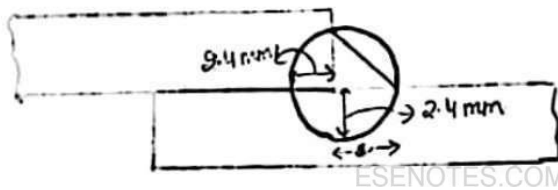
- convex weld have larger throat thickness, hence is stronger than concave weld even though it may have less deposited metal.
- concave weld are most suitable for alternating stresses as it offer smooth path of smooth flow of stresses.
- In concave fillet weld there is a chance of cracking on cooling hence they are made in two or three passes, ~~where~~ where the first pass is slightly convex & the other passes are built up to form a concave weld.

SPECIFICATIONS

- a) Size: The largest right angled triangle that can be inscribed is found out and for normal fillet weld the weld size is taken as the minimum leg length of weld.



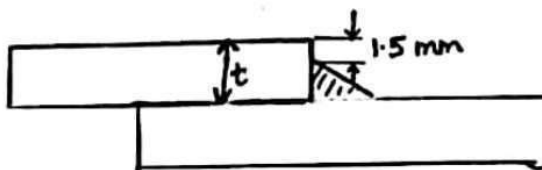
- for deep penetration weld where the depth of the penetration beyond the root run is minimum ~~2.4 mm~~ 2.4 mm then size of weld is taken $\cdot 2.4 \text{ mm} + \text{min leg size } (s)$



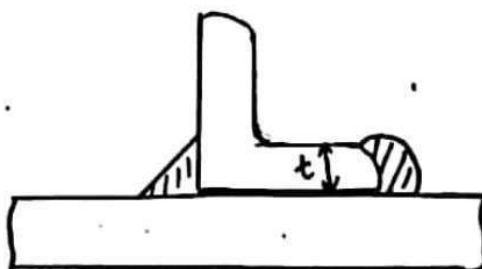
$$\text{Size of weld} = 2.4 + s$$

b) Maximum size of fillet weld

- for square edge, the maximum size is $t - 1.5 \text{ mm}$
where t = thickness of thinner part



- for Round edge, the maximum size is $\frac{3}{4}t$.
where t = thickness of round edge.



This specification limits the size of the fillet weld, so that total strength may be developed, without overstressing the adjacent metal.

c) Minimum size of fillet weld

- minimum size of fillet weld is related to thickness of thicker member as follows

[Note: other than this specifications all are linked with thickness of thinner member]

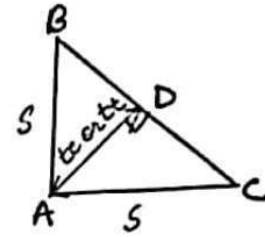
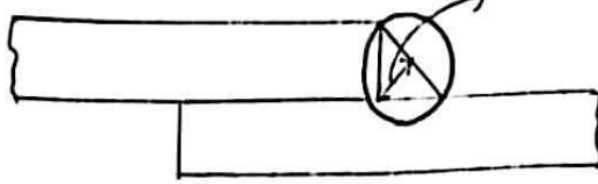
Thickness of thicker member (mm)	Minimum size (mm).
0-10	3
10-20	5
20-32	6
32-50	8 mm first run & 10 mm final run.

- For thickness $> 50\text{mm}$, special precautions are to be taken like pre-heating must be done.
- The min. size of weld should not exceed the thickness of thinner part to be joined.
- The weld size closer to the minimum size is generally adopted for the following reasons.

a) Large size welds require more than 1 run of welding hence the cost increases because of chipping & cleaning which will be required for proper bond of successive weld runs.

b) A smaller size weld will be cheaper than a larger one for the same strength (since volume of welding required is less)

d) Effective Throat Thickness



from right angled $\triangle ADC$

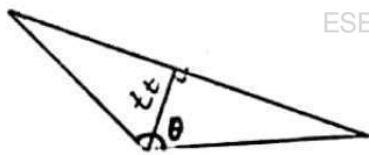
$$2t_e^2 = S^2$$

$$t_e = \frac{S}{\sqrt{2}} = 0.7 S$$

$S = \text{size of weld.}$

In general, $t_e = K S$

where K is a constant, that depends on the angle b/w the fusion faces as follows.



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θ	K
60-90	0.7
91-100	0.65
101-106	0.6
107-113	0.55
114-120	0.5

- Throat thickness is the min. thickness on which shearing will occur.
- If θ is less than 60° or greater than 120° , fillet weld is not recommended.

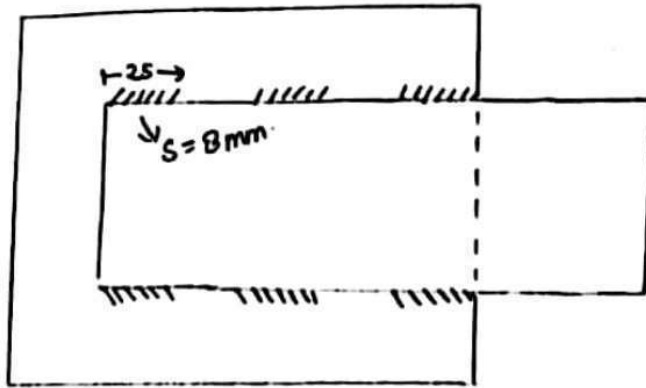
e) Effective Length (l_e)

$$\text{Effective length } (l_e) = \text{Overall length} - 2S$$

& 4S

where $S = \text{size of weld.}$

- On the structural drawing only effective length is shown
- Min. length required for a weld to be 100% effective is $4S$, if min. length is less than $4S$, then effective size is taken as $\frac{1}{4}$ th of the effective length.



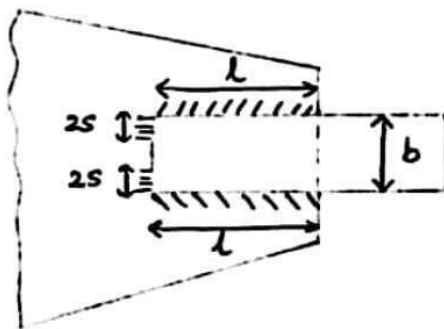
$$S_{\text{provided}} = 8 \text{ mm}$$

$$l_{\text{min provided}} = 25 \text{ mm.}$$

$$l_{\text{min reqd}} = 4S.$$

$$= 4 \times 8 = 32 \text{ mm}$$

$$S_{\text{actual}} = \frac{25}{4} = 6.25 \text{ mm}$$



$$- \text{minimum end return} = 25$$

$$- l \neq b$$

$$- b \neq 16t$$

t = thickness of thinner plate.

- End returns are provided to release the weld from high stress concentrations at their ends.
- It must always be provided, if the joints are subjected to stress reversals, impact and eccentricity.
- $l \neq b$, length of side fillet should not be less than the width of plate.
- $b \neq 16t$, ~~as~~ because as "b" increases, non-uniformity of stress in the plate also increases.
- Hence, if plate is wider than $16t$, slot or plug weld, must be introduced.

- If the length of joint is greater than $150t_t$, the strength of the weld is reduced by a factor.

$$\beta_{lj} = 1.2 - \frac{0.2 l_j}{150 t_t} \leq 1.0$$

l_j = length of welded joint in the direction of load transfer.

t_t = effective throat thickness

f) Effective Area (A_e)

- It is the area (mm) over which shear stress develops.

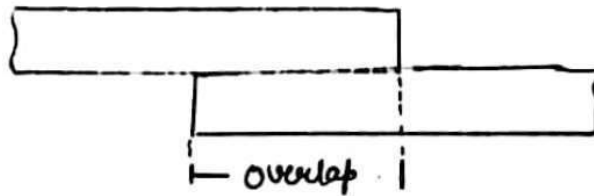
$$A_e = l_e \times t_t$$

g) Overlap.

Overlap $\nless (4t \text{ or } 40 \text{ mm}) \text{ max.}$

t = thickness of thinner part to be jointed.

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h) Design strength

- The design strength of fillet weld is based on effective area.

Design strength of fillet weld.

$$P_{dw} = \frac{f_u l_e t_t}{\sqrt{3} \gamma_{mw}} = \frac{f_u l_e k \cdot s}{\sqrt{3} \gamma_{mw}}$$

l_e = effective length of weld.

t_t = effective throat thickness

s = size of weld

f_u = smaller of ultimate strength of weld or parent metal.

γ_{mw} = 1.25 for shop welding
= 1.5 for field welding.

Q A flat tension member $75\text{ mm} \times 8\text{ mm}$ is to transmit a factored load of 145 kN . Design shop fillet weld and necessary overlap when the are provided along the following cases.

a) Two edges (sides of the member)

b) End and 2 edges of the member.

c) all around the joint

steel is of grade Fe 410 & gusset plate is 12 mm thick.

solⁿ (i) Min. size of weld for 12 mm plate = 5 mm

max. size of weld for 8 mm plate = $8 - 1.5 = 6.5\text{ mm}$

Adopt size of weld = 5 mm .

(ii) Effective throat thickness $t_t = k_s = 0.7s = 0.7 \times 5$
 $= 3.5\text{ mm}$.

(iii) strength of weld = $\frac{f_u l_e t_t}{\sqrt{3} \gamma_{mw}} \geq 145\text{ kN}$.

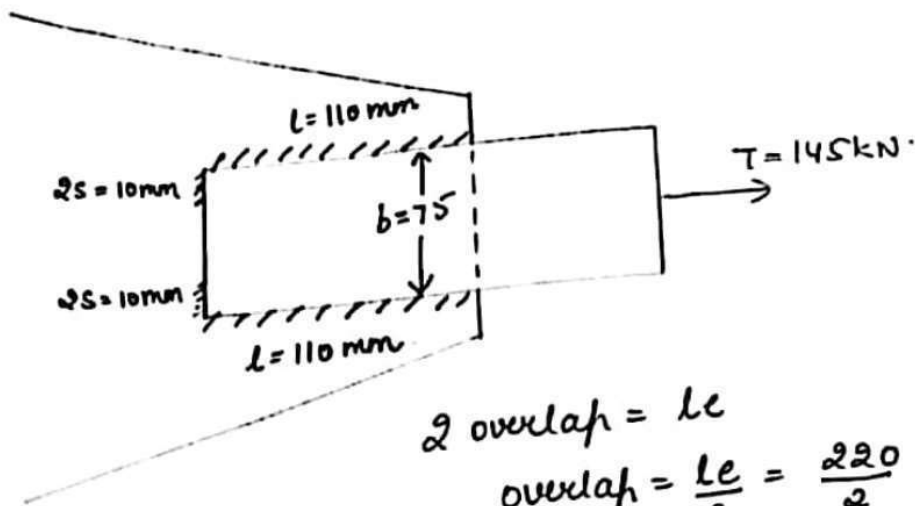
$$\gamma_{mw} = 1.25$$

$$\frac{410 \times l_e \times 3.5}{\sqrt{3} \times 1.25} \geq 145 \times 10^3$$

$$l_e \geq 218.77\text{ mm}$$

provide $l_e = 220\text{ mm}$.

a)



$$2 \text{ overlap} = l_e$$

$$\text{overlap} = \frac{l_e}{2} = \frac{220}{2} = 110 \text{ mm}$$

(1) Overlap $\nless 4t$ or 40 mm. ($4 \times 8 = 32$ or 40)

(1) $\nless b$. (75 mm).

Hence OK.

Thus provide 5 mm side fillet weld of length 110 mm on the two sides as shown in previous figure with end return of 2S (10 mm) on both edges.

$$\text{Hence eff. length provided} = 2 \times 110 + 2 \times 10 = 240 \text{ mm}$$

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b) weld provided along end & two sides.

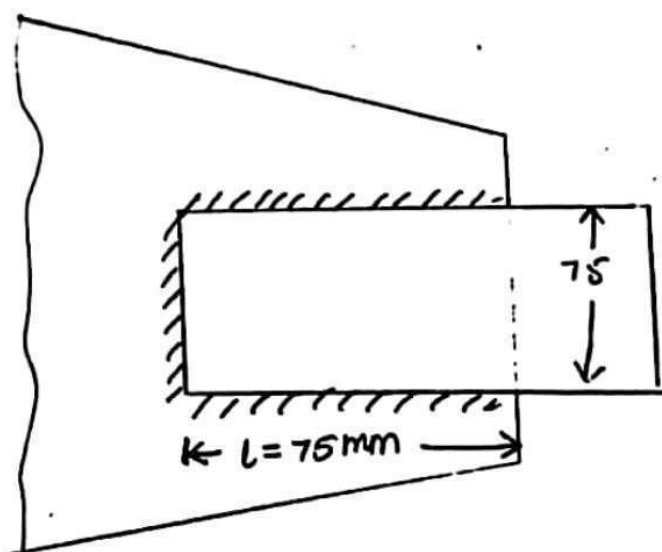
- length of end fillet weld = 75 mm.

- from previous cal.
 $l_w = 220$ mm.

$$\text{Hence } 2l + 75 = 220.$$

$$l = 72.5 \text{ mm}$$

provide $l = 73$ mm.



Overlap. $l = 73 \text{ mm}$. $\nless (4t \text{ or } 40 \text{ mm})_{\text{max}}$.
 $4 \times 8 = 32 \text{ mm or } 40 \text{ mm}$
OK

$l \nless 75 \text{ mm}$. Not OK.

so provide $l = 75 \text{ mm}$.

provide 5 mm size fillet weld of length

$$l_w = 75 + 2 \times 75 = 225 \text{ mm}$$

c) weld is provided all around the joint.

$$2(l + 75) = l_w = 220$$

$$l = 35 \text{ mm}$$

$$l \nless (4t \text{ or } 40 \text{ mm})_{\text{max}}$$

$$l \nless (32 \text{ or } 40 \text{ mm})$$

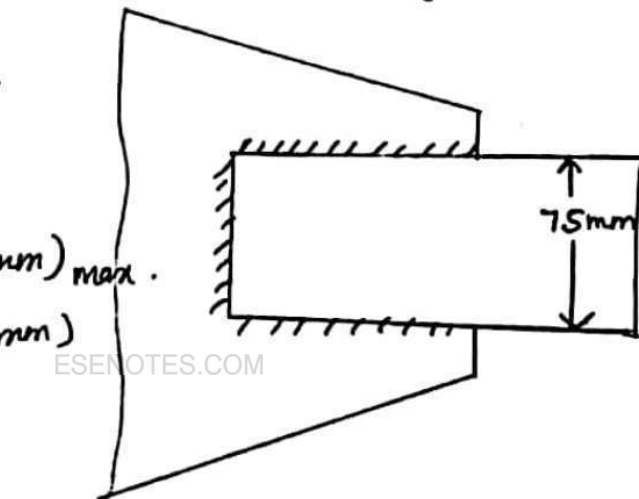
Not OK

also,

$$l \nless b$$

$$l \nless 75 \text{ mm}$$

Not OK

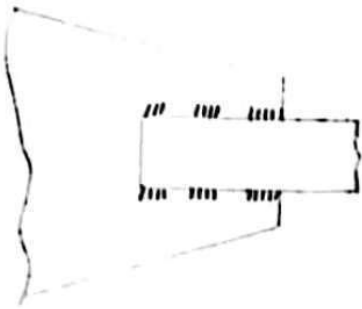


Provide $l = 75 \text{ mm}$.

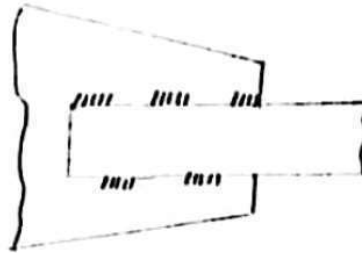
$$\text{Effective length of weld} = 2(75 + 75) = 300 \text{ mm}$$

Design of Intermittent fillet weld.

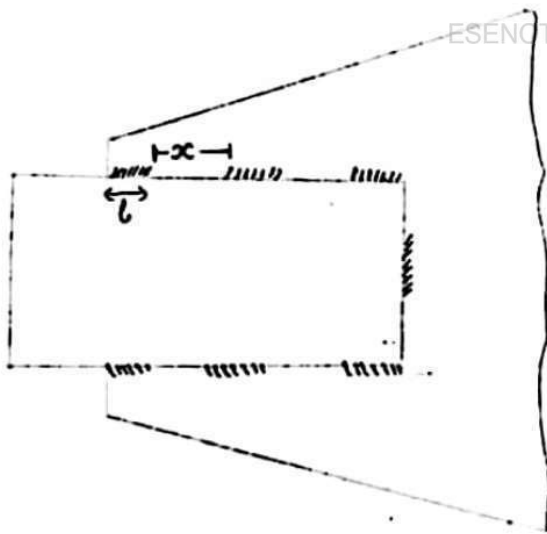
- When the length of smallest size fillet weld required to transmit the design load is less than the continuous length of the joint, Intermittent fillet weld will be provided.
- But, it is not suitable for dynamic or repetitive loads.



Chain pattern
(preferred because of symmetry)



Staggered pattern

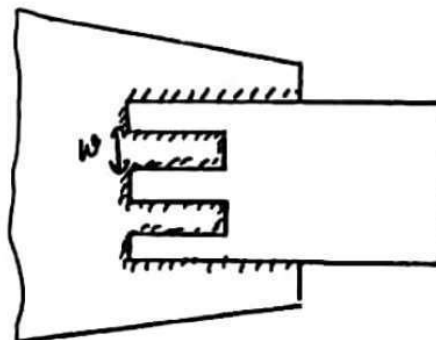


- $l \nless (4t \text{ or } 40\text{mm}) \text{ max}$
 - $x \nless (16t \text{ or } 200\text{mm}) \text{ min}$ for tension member.
 - $x \nless (12t \text{ or } 200\text{mm}) \text{ min}$ for compression member
- t = thickness of thinner part to be joined.

PLUG & SLOT WELD

$w \text{ or } D \nless (3t \text{ or } 25\text{mm})$

t = thickness of plate in which slot is made.



IF WORKING STRESS METHOD.

- The provision of limit state design also apply to the design by working stress except for allowable stress calculation.

a) Actual stress in the throat of fillet weld should be less than or equal to permissible shear stress of the weld.

$$f_{aw} = 0.4 f_y.$$

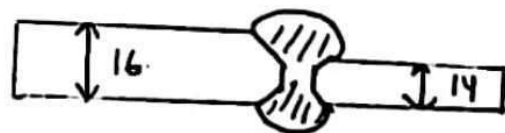
b) actual stress in the butt weld should be less than or equal to the permissible stress governed by the parent metal to be welded.

Q Two plates of 16mm & 14mm are to be joined by groove weld as given. The joint is subjected to a factored tensile force of 430kN, due to some reasons, effective length of weld that could be provided was 175mm only. Check the safety of weld if.

a) single V groove weld is provided.

b) Double V groove weld is provided.

assume shop weld and Fe 410 steel.



a) Single V groove weld has partial penetration

$$t_e = \frac{5}{8} t = \frac{5}{8} \times 14 = 8.75 \text{ mm.}$$

$$l_w = 175 \text{ mm}$$

$$\text{Tensile strength of weld. } (T_{dw}) = \frac{f_y l_w t_e}{\gamma_{mw}}$$

$$= \frac{250 \times 175 \times 8.75}{1.25}$$

$$= 306.25 < 430 \text{ kN}$$

Joint is not safe.

b) Double V groove weld with complete penetration

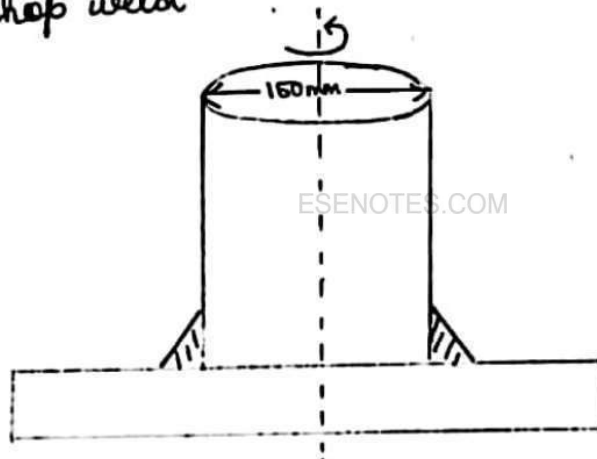
$$t_e = \min(16, 14) = 14 \text{ mm}$$

$$l_w = 175 \text{ mm}$$

$$T_{dw} = \frac{f_y l_w t_e}{\gamma_{mw}} = \frac{250 \times 175 \times 14}{1.25} = 490 \text{ kN} > 430$$

Hence, joint is safe.

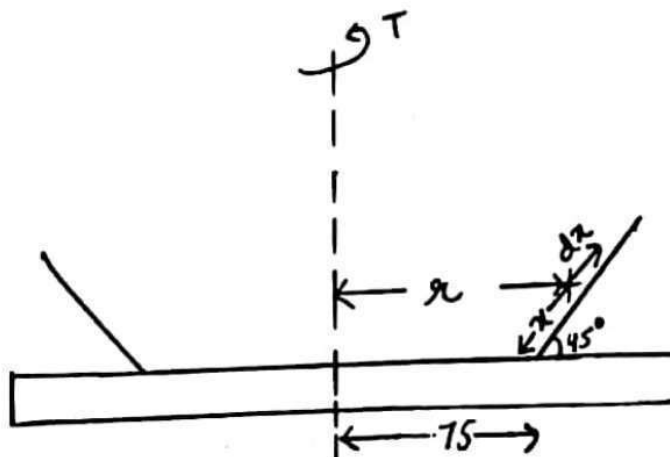
Q. A circular shaft of diameter 150 mm is welded to a rigid plate by means of 6 mm all around fillet weld. Compute the ultimate twisting moment that can be resisted by the weld. Use steel of grade Fe 410 and shop weld.



$$S = 6 \text{ mm}$$

$$t_t = K S = 0.7 \times S = 4.2 \text{ mm}$$

$$\text{Design shear stress of weld: } f_{dw} = \frac{f_u}{\sqrt{3} \gamma_{mw}} = \frac{410}{\sqrt{3} \times 1.25} = 189.37 \text{ N/mm}^2$$



$$\text{Twisting moment } (T) = \int dT = \int_0^{t_t} [f d w (2\pi r dx)] r. \quad \text{or } r \text{ area } \times \text{ distance}$$

$$T = 189.37 \times 2\pi \int_0^{t_t} r^2 dr$$

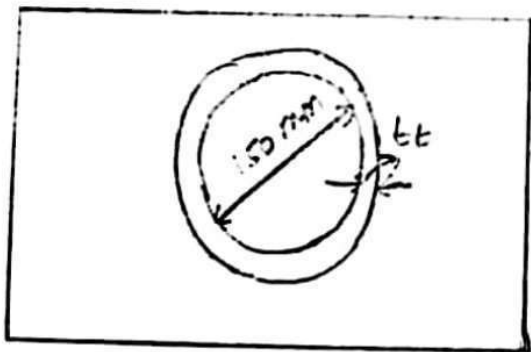
$$= 189.37 \times 2\pi \int_0^{t_t} \left[75 + \frac{r}{\sqrt{2}} \right]^2 dr \quad \begin{aligned} r &= 75 + r \cos 45^\circ \\ &= 75 + \frac{r}{\sqrt{2}} \end{aligned}$$

$$= 189.37 \times 2\pi \left| \frac{\left(75 + \frac{r}{\sqrt{2}} \right)^3}{3 \times \frac{1}{\sqrt{2}}} \right|_0^{4.2} \times 10^{-6}$$

$$T = 29 \text{ kNm}$$

C we can also consider weld as line area of thickness equal to throat thickness

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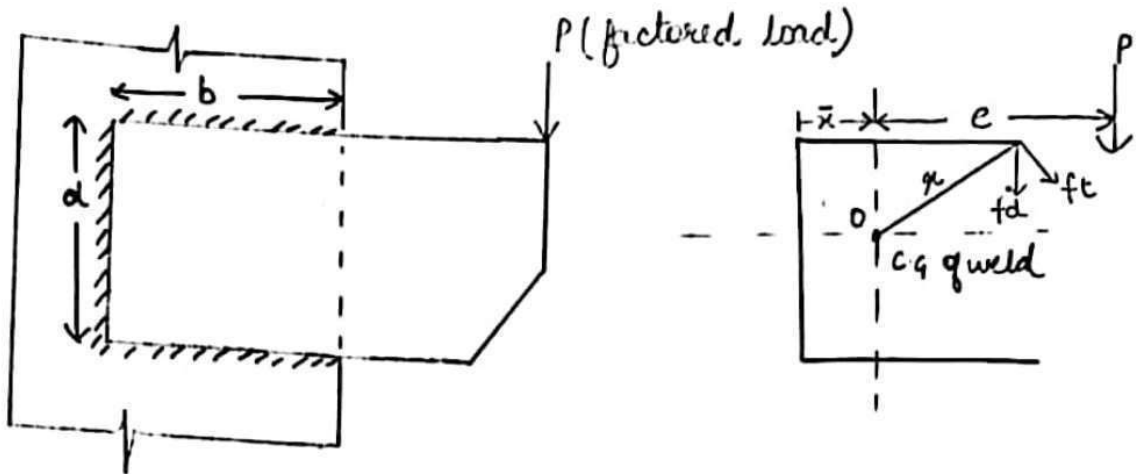
$$\text{Hence } T = f_w d \times 2\pi r t_t \times r$$

$$\begin{aligned} T &= 189.37 \times 2\pi \times 75 \times 4.2 \times 75 \\ &\Rightarrow 28.11 \text{ kNm} \end{aligned}$$

Lesson 51 Sec 10.

Eccentric connections using welding

(A) Fillet Weld Subjected to direct load & torsional moment.



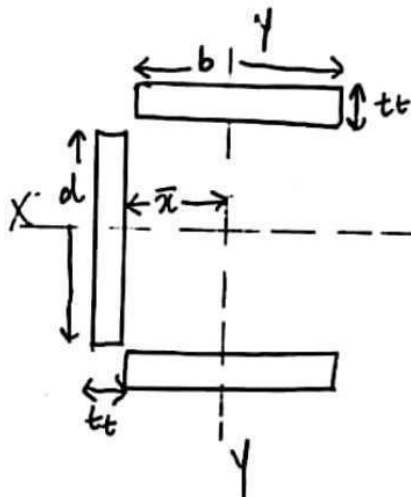
- Direct shear stress $f_d = \frac{P}{d \times t_t + 2 \times b \times t_t} = \frac{P}{(d + 2b)t_t}$

- Its line of action is parallel to applied direct load.

- Torsional shear stress $f_t = \frac{Tx}{J}$, where $T = Pe$

- Its line of action is perpendicular to the line joining the C.G. of weld with the concerned point and it will be in same sense as that of applied torsion.

- Now, $I = I_{xx} +$



$$I_{xx} = 2 \times b \times t_t \left(\frac{d}{2}\right)^2 + \frac{t_t d^3}{12}$$

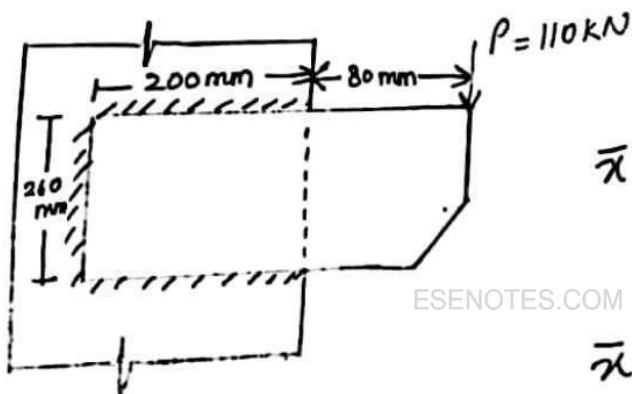
$$I_{yy} = \overset{\text{neglected}}{\frac{d t_t^3}{12}} + d t_t \bar{x}^2 + \left[\frac{t_t \cdot b^3}{12} + b t_t \left(\frac{b}{2} - \bar{x}\right)^2 \right]$$

Now, resultant shear stress τ_r .

$$\tau_r = \sqrt{f_d^2 + f_t^2 + 2f_d f_t \cos \theta}$$

for safety, $\tau_r \leq \frac{f_u}{\sqrt{3} \gamma_{mw}}$

2. A bracket plate is welded to the flange of a column section ISHB: 300 as given. Calculate the size of the weld required to support a factored load of 110 kN. Assume shop welding & Fe 410 grade of steel.



$$\bar{x} = \frac{2 \times 200 \times t_t \times 100 + 260 \times t_t \times 0}{2 \times 200 \times t_t + 260 \times t_t}$$

$$\bar{x} = 60.6 \text{ mm}$$

$$e = 200 + 80 - \bar{x} = 219.4 \text{ mm}$$

$$f_d = \frac{P}{t_t (d + 2b)} = \frac{110 \times 10^3}{t_t (260 + 2 \times 200)} = \frac{166.67}{t_t}$$

$$f_t = \frac{P e \cdot x}{J}$$

$$T = P e = 110 \times 10^3 \times 219.4 = 2.41 \times 10^7 \text{ Nmm}$$

$$x = \sqrt{(200 - 60.6)^2 + \left(\frac{260}{2}\right)^2}$$

$$x = 190.6 \text{ mm}$$

$$\cos \theta = \frac{260 - 60.6}{190.6} = \frac{199.4}{190.6} = 1.046 \approx 0.73$$

$$J = I_{xx} + I_{yy}$$

$$I_{xx} = 2 \times 200 \times t \times \left(\frac{260}{2} \right)^2 + \frac{t (260)^3}{12} = 8.224 \times 10^6 \times t \text{ mm}^4$$

$$I_{yy} = 260 \times t (60.6)^2 + 2 \left[\frac{t \times 200^3}{12} + 200 \times t \left(\frac{200}{2} - 60.6 \right)^2 \right]$$

$$I_{yy} = 2.9 \times 10^6 \times t \text{ mm}^4$$

$$J = (8.224 + 2.9) \times 10^6 \times t = 11.124 \times 10^6 t$$

$$f_t = \frac{2.41 \times 10^7 \times 190.6}{11.124 \times 10^6 \times t} = \frac{412.93}{t}$$

$$f_x \leq \frac{f_u}{\sqrt{3} \gamma_{mw}}$$

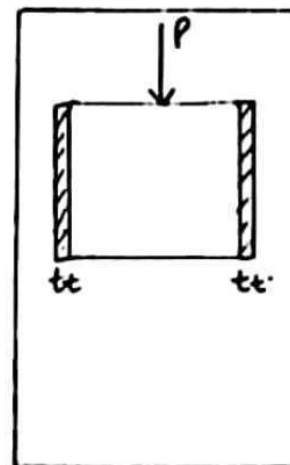
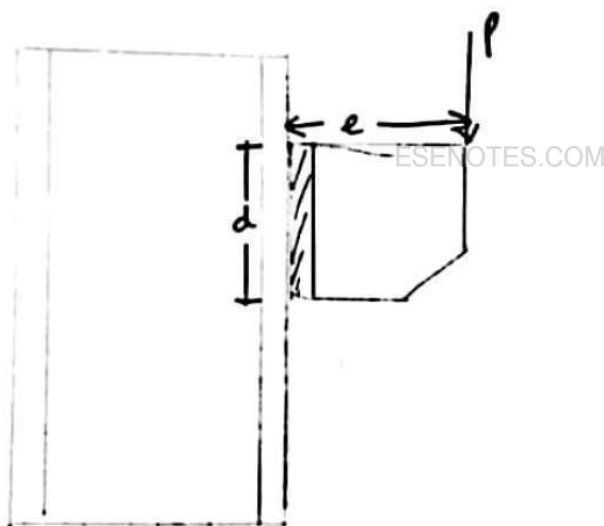
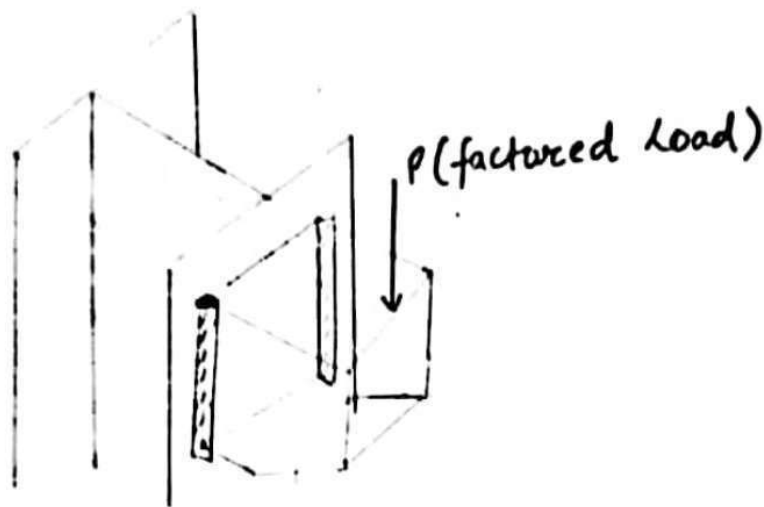
$$\sqrt{f_d^2 + f_t^2 + 2 f_d f_t \cos \theta} \leq \frac{f_u}{\sqrt{3} \gamma_{mw}}$$

$$\sqrt{\left(\frac{166.67}{0.75} \right)^2 + \left(\frac{412.93}{0.75} \right)^2} \times 2 \times \frac{166.67}{0.75} \times \frac{412.93}{0.75} \times 0.73 \leq \frac{410}{\sqrt{3} \times 1.25}$$

$$S \geq 4.123 \text{ mm}$$

$$\text{adopt } S = 5 \text{ mm}$$

B) Fillet weld subjected to direct load + Bending moment



- Direct shear stress $q = \frac{P}{2 \times d \times t}$
- Its line of action is parallel to the line of applied direct load.
- Normal stress in weld due to bending $f_b = \frac{My}{I}$

$f_b =$ Here, $M = Pe$
 $y = d/2$

$$I = \frac{2 \times t \times d^3}{12}$$

- Equivalent shear stress as per code

$$f_e = \sqrt{f_b^2 + 3q^2}$$

- for safety, $f_e \leq \frac{f_u}{\sqrt{3} \gamma_{mw}}$

Note: Equivalent stress is computed by using the concept of distortion energy.

$$f_e^2 = \sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2$$

Here, $\sigma_x = f_b$, $\sigma_y = 0$, $\tau_{xy} = q$

$$f_e = \sqrt{f_b^2 + 0 - 0 + 3q^2}$$

$$f_e = \sqrt{f_b^2 + 3q^2}$$

For design depth^(d) of bracket plate in fillet weld; use

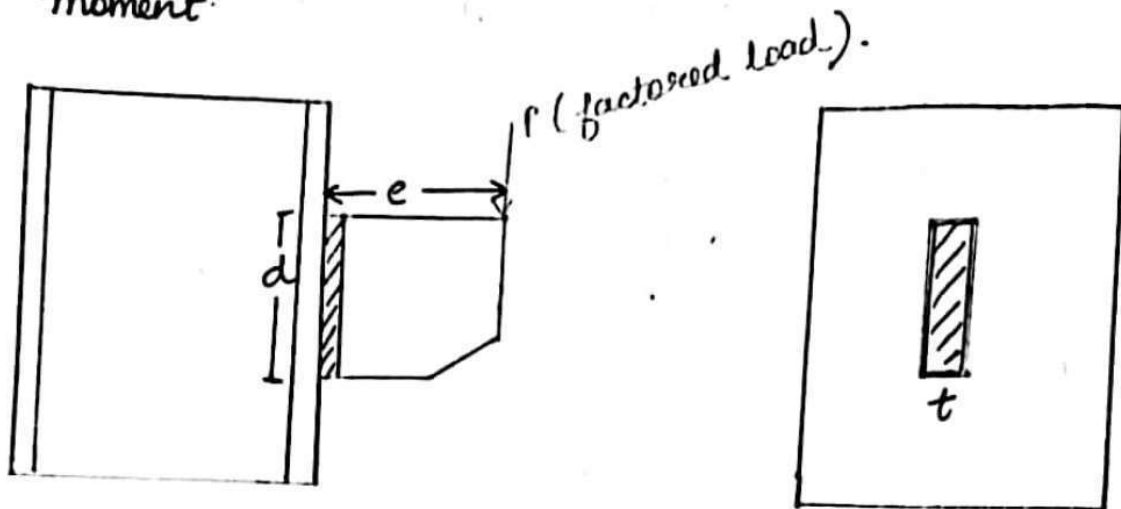
$$d = \sqrt{\frac{6M}{2tt f_{wd}}}$$

$$f_{wd} = \frac{0.8 f_u}{\sqrt{3} \gamma_{mw}}, \quad tt = ks = 0.7S$$

- A reduced value of about 80% should be used to account for direct shear stress.

Lesson 52 Dec 11

C) BUTT WELD subjected to Direct load & Bending moment.



- Direct shear stress $q = \frac{P}{d \times t}$

- normal stress due to bending moment $f_b = \frac{P M y}{I}$

$$M = P e$$

$$y = d/2$$

$$I = \frac{t d^3}{12}$$

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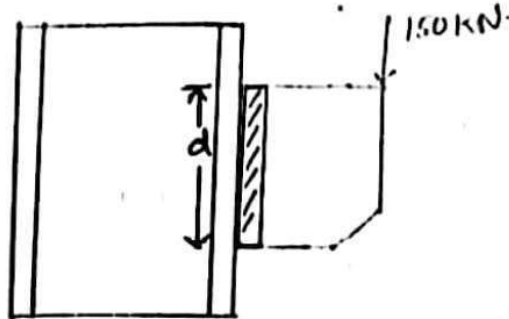
- Equivalent stress $f_e = \sqrt{f_b^2 + 3q^2}$

for safety ; $f_e \leq \frac{f_y}{\gamma_{mo}}$

Design depth for bracket plate $d = \sqrt{\frac{6M}{t f_b}}$

$$f_b = \frac{f_y}{\gamma_{mo}}$$

- Q A bracket plate 12 mm thick is used to transmit factored load of 150 kN at an eccentricity of 120 mm from the column flange as given. Design shop fillet weld on two sides. grade of steel is Fe410.



- Min. size of weld = 5 mm (for 12 mm thick plate)
- Max. size of weld = $12 - 1.5 = 10.5$ mm.

Adopt weld size $s = 5$ mm.

$$t_t = 0.7s = 0.7 \times 5 = 3.5 \text{ mm.}$$

$$\text{Now, } d = \sqrt{\frac{6M}{2t_t \cdot f_{wd}}} \quad \text{ESENOTES.COM}$$

$$\text{Here, } f_{wd} = \frac{0.8f_u}{\sqrt{3} \gamma_{mw}} = \frac{0.8 \times 410}{\sqrt{3} \times 1.25} = 151.49 \text{ N/mm}^2$$

$$d = \sqrt{\frac{6 \times 150 \times 10^3 \times 120}{2 \times 3.5 \times 151.49}}$$

$$d = 319.125 \text{ mm.}$$

provide 320 mm.

$$\text{Now, direct shear stress } q = \frac{P}{2 \cdot d \cdot t_t} = \frac{150 \times 10^3}{2 \times 320 \times 3.5} = 67 \text{ N/mm}^2.$$

$$\text{normal stress due to B.M. } f_b = \frac{My}{I} = \frac{150 \times 10^3 \times 120 \times 320}{\frac{2 \times 3.5 \times (320)^3 \times 2}{12}}$$

$$f_b = 150.6 \text{ N/mm}^2$$

Equivalent stress, $f_e = \sqrt{f_b^2 + 3q^2}$

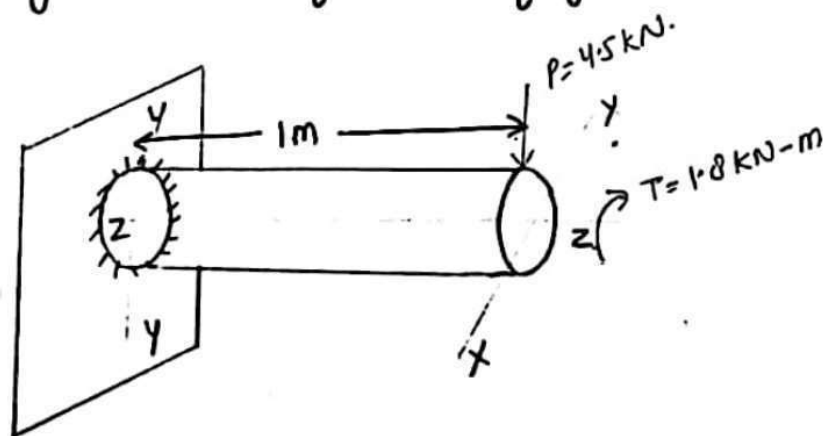
$$f_e = \sqrt{150.6^2 + 3 \times 67^2}$$

$$f_e = 190 \text{ N/mm}^2$$

for safety: $f_e \leq \frac{f_u}{\sqrt{3} \times \gamma_{mw}} = \frac{410}{\sqrt{3} \times 1.25} = 190 \text{ N/mm}^2$ OK

Note: If the above condition would have not been satisfied redesigning is done such that value of 's' considered is more than 5 mm.

Q A 120 mm diameter and 6 mm thick pipe is fillet welded to a 8 mm thick plate. It is subjected to vertical factored load of 4.5 kN at 1 m from the welded end & a twisting moment of 1.8 kNm (factored) as given. Design the joint assuming shop welding & sheet of steel of grade Fe 410.



Soln at the point of max. bending stress.

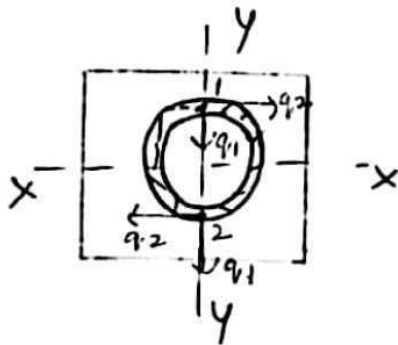
a) shear stress due to torsion acts horizontally in transverse direction

b) shear stress due to direct loading acts vertically downward.

$$\text{Hence, direct shear stress } q_1 = \frac{P}{2\pi r t} = \frac{4.5 \times 10^3}{2 \times \pi \times \frac{120}{2} t}$$

$$q_1 = \frac{11.936}{t} \text{ N/mm}^2.$$

Shear stress due to torsion $q_2 = \frac{T}{J} \cdot r$



$$J = A r^2 = (2\pi r t) \cdot r^2 \\ = 2\pi r^3 t$$

$$q_2 = \frac{T r}{2\pi r^3 t} = \frac{T}{2\pi r^2 t}$$

$$q_2 = \frac{1.8 \times 10^6}{2\pi \times \left(\frac{120}{2}\right)^2 \times t}$$

$$q_2 = \frac{79.57}{t} \text{ N/mm}^2.$$

Resultant shear stress $q = \sqrt{q_1^2 + q_2^2}$

$$= \sqrt{\left(\frac{11.936}{t}\right)^2 + \left(\frac{79.57}{t}\right)^2}$$

$$q = \frac{80.5}{t} \text{ N/mm}^2.$$

Normal stress due to B.M. $f_b = \frac{M y}{I_{xx}}$

$$\text{also, } I_{xx} + I_{yy} = I_{zz} \quad (i) \quad I_{xx} = I_{yy} \quad (ii)$$

$$I_{xx} = \frac{I_{zz}}{2} = \frac{J}{2} = \frac{2\pi r^3 t}{2}$$

$$I_{xx} = \pi r^3 t t.$$

$$f_b = \frac{4.5 \times 10^3 \times 1 \times 10^3 \times 120/2}{\pi \times \left(\frac{120}{2}\right)^3 \times t t} = \frac{397.88}{t t} \text{ N/mm}^2.$$

Now, equivalent stress $f_e = \sqrt{f_b^2 + 3q^2}$

$$= \sqrt{\left(\frac{397.88}{t t}\right)^2 + 3 \left(\frac{80.5}{t t}\right)^2}$$

$$= \frac{421.6}{t t} \text{ N/mm}^2.$$

for safety ; $f_e \leq \frac{f_u}{\sqrt{3} \gamma_{mw}}.$

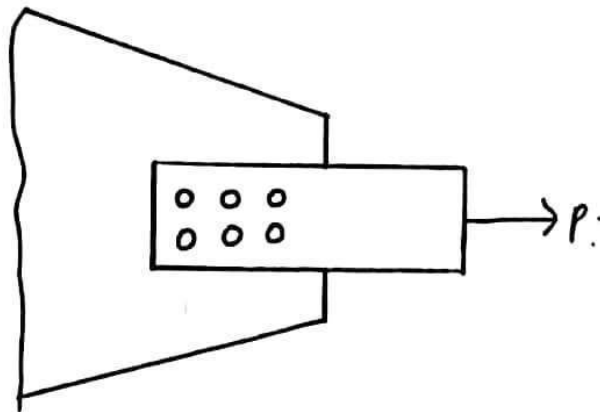
$$\frac{421.6}{0.7 S} \leq \frac{410}{\sqrt{3} \times 1.25}.$$

$$S \geq 3.18 \text{ mm}.$$

adopt $S = 4 \text{ mm}.$

TENSION MEMBERS. OR TIE MEMBERS

- A structural member subjected to axial tension is called as tension / Tie member.
- The members and connectors are so arranged that eccentricity in the connection and bending stresses are not developed.



- Tension members may fail in any of following modes
 - a) Gross section yielding.
 - considerable deformation of the member in longitudinal direction may take place before it fractures, making the structure unserviceable.
 - b) Net section Rupture
 - The fracture of the member occurs when the net cross-section of the member reaches the ultimate stress.
 - c) Block shear Failure

Design strength of Tension member.

a) for gross section yielding.

$$T_{dg} = \frac{f_y A_g}{\gamma_{mo}}$$

where, $\gamma_{mo} = 1.1$, $A_g = \text{gross area}$.

b) for net section rupture

(i) In plates & flats

$$T_{dn} = \frac{0.9 f_u A_n}{\gamma_{m1}}$$

where $\gamma_{m1} = 1.25$, $A_n = \text{net area}$.

$$A_n = \left[B - n d_o + \sum_{i=1}^m \frac{p_i^2}{4g_i} \right] t$$

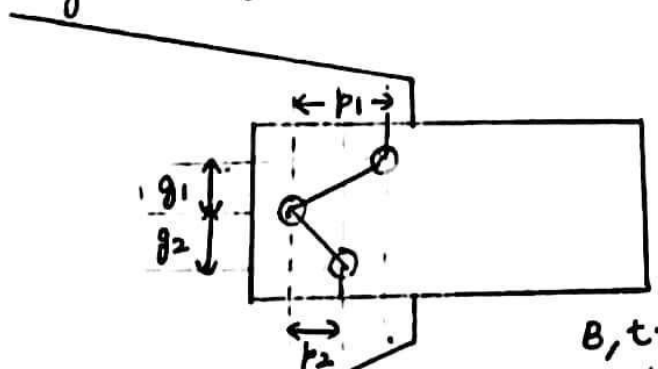
$B = \text{width of plate or flat}$

$n = \text{no. of bolts along failure line}$

$m = \text{no. of inclined lines}$

$p = \text{pitch distance}$

$g = \text{gauge distance}$



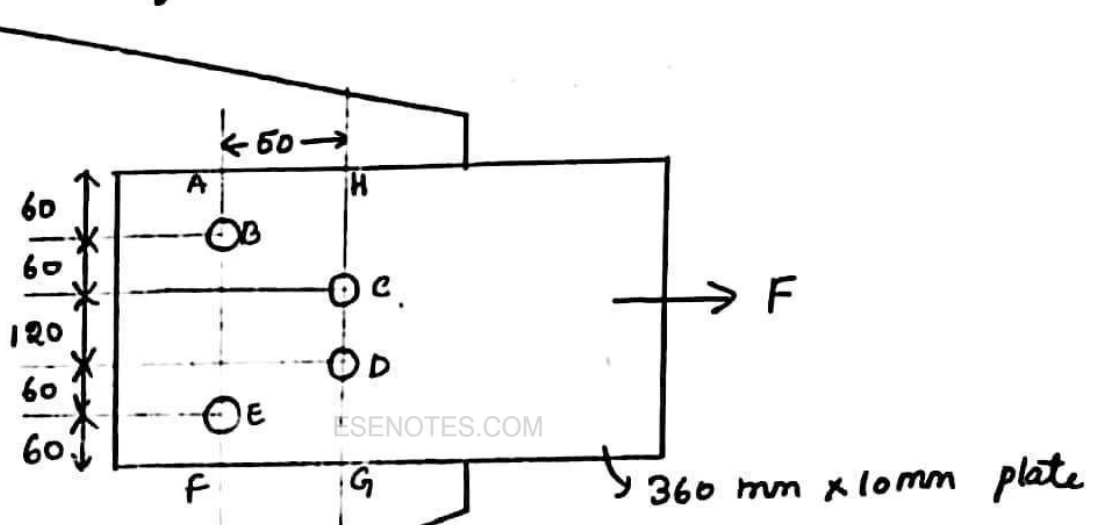
$$A_n = \left[B - 3d_o + \frac{p_1^2}{4g_1} + \frac{p_2^2}{4g_2} \right] t$$

- at critical section, net area is minimum.
- Hence, maximum load carried by tension member is

$$F = \min \{ Tdg, Tdn \}$$

Part 2. Lesson 1 Dec 13.

Q If the hole dia is 20mm; calculate the maximum force that can be applied to the plate. Assume Fe 410 grade of steel.



Solⁿ

$$d_o = 20 \text{ mm.}$$

a) for section ABCDEF.

$$A_n = \left[360 - 4 \times 20 + \frac{2(50)^2}{4 \times 60} \right] \times 10 = 3008.83 \text{ mm}^2$$

b) for section ABCDG.

$$A_n = \left[360 - 3 \times 20 + \frac{1 \times 50^2}{4 \times 60} \right] \times 10 = 3104.167 \text{ mm}^2$$

c) for section ABCEF

$$A_n = \left[360 - 3 \times 20 + \frac{50^2}{4 \times 60} + \frac{50^2}{4 \times 180} \right] \times 10$$

$$= 3138.88 \text{ mm}^2$$

d) for section AB EF / HCDG.

$$A_n = (360 - 2 \times 20) \times 10 = 3200 \text{ mm}^2.$$

$$A_n = \min \{ A_b, A_c, A_d \} = 3008.33 \text{ mm}^2$$

$$T_{dn} = \frac{0.9 f_u A_n}{1.25} = \frac{0.9 \times 410 \times 3008.33}{1.25}$$

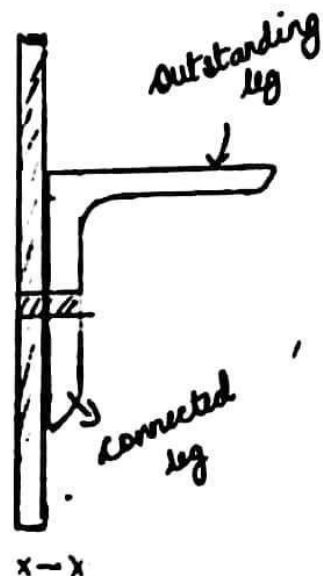
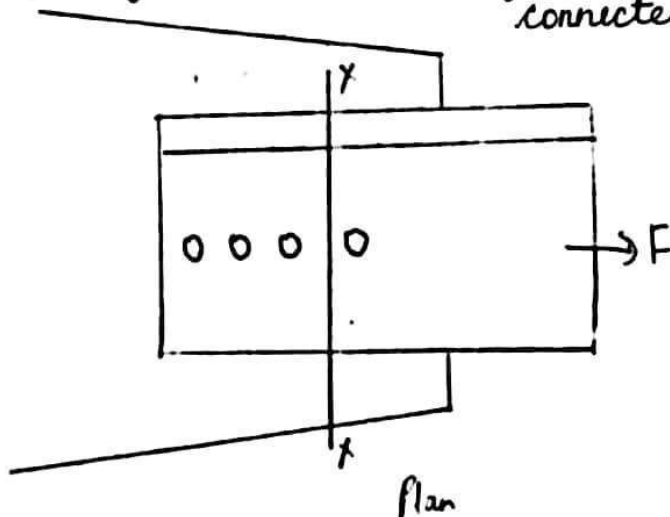
$$= 888.05 \text{ kN}$$

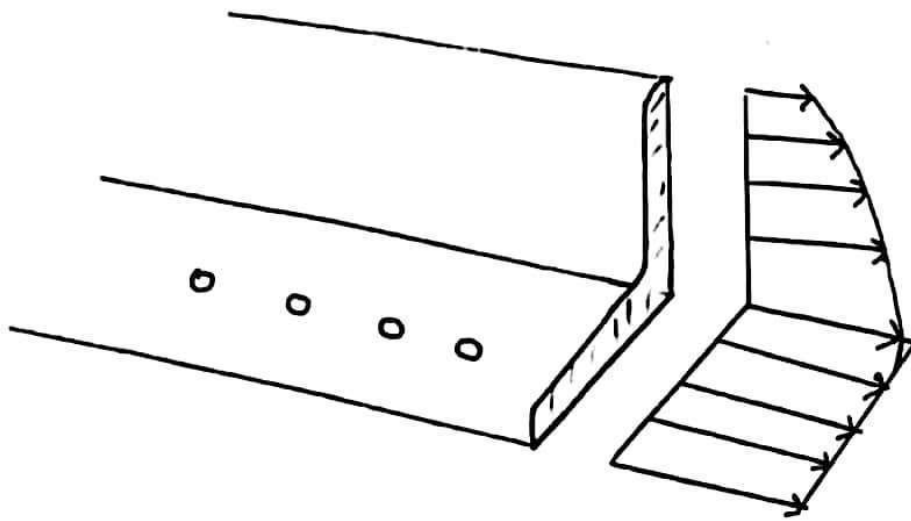
$$T_{dg} = \frac{f_y A_g}{1.1} = \frac{250 \times 360 \times 10}{1.1} = 818.18 \text{ kN}.$$

$$F = \min \{ T_{dg}, T_{dn} \}$$

$$= \underline{818.18 \text{ kN}}$$

(ii) In angles. ① when only 1 leg is connected.





- Force transferred to one leg (connected leg) is by end connection which gets locally transferred as Tensile stress over the entire cross-section by shear.
- The connected leg will have higher stress than the outstanding leg, thus one part of angle lags behind the other part of in stress. The phenomenon is called "shear lag".
- Since shear lag reduces the effectiveness of the outstanding leg, unequal angles are connected through longer leg.
- The code has recommended the following relationship for computation of design rupture strength T_{dn} in this case.

$$T_{dn} = \frac{0.9 f_u A_{nc}}{\gamma_{m1}} + \frac{\beta f_y A_{go}}{\gamma_{m0}}$$

$$\text{where } \beta = 1.4 - 0.076 \frac{w}{t} \times \frac{b_f}{f_u} \times \frac{b_s}{l_c}$$

$$\beta \leq \frac{0.9 f_u / \gamma_{m1}}{f_y / \gamma_{m0}}, \quad \beta \geq 0.7$$

A_{nc} = net area of connected leg
 A_{go} = gross area of outstanding leg.

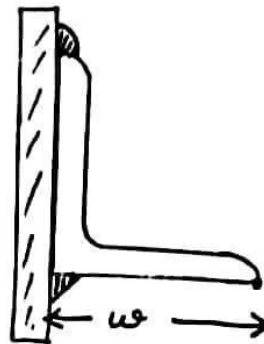
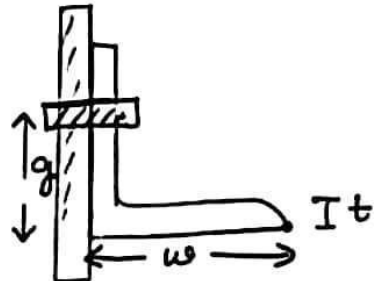
w = width of outstanding leg.

t = thickness of angle.

b_s = shear lag width

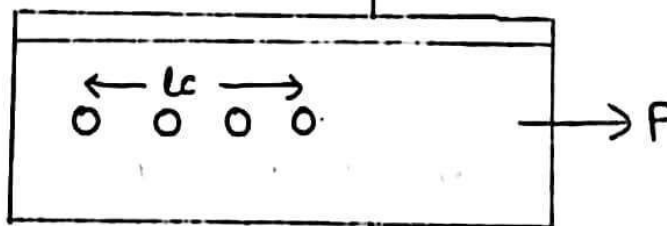
$b_s = w + g - t$
for bolted connection

$b_s = w$; for welded connection



— b_s is the distance of the farthest point on the outstanding leg from the nearest bolt or weld

l_c = length of the connection in the direction of load transfer. i.e. centre to centre distance b/w the extreme bolts.



- As the above expression has l_c in it which cannot be determined unless design is complete, hence for preliminary sizing the rupture strength of net section approximated result may be used.

$$T_{dn} = \frac{\alpha f_u A_n}{\gamma_{m1}}$$

$$\begin{aligned}\alpha &= 0.6 ; \text{ for 1 or 2 bolts} \\ &= 0.7 ; \text{ for 3 bolts} \\ &= 0.8 ; \text{ for 4 or more bolts} \\ &= 0.8 ; \text{ for welded connection.}\end{aligned}$$

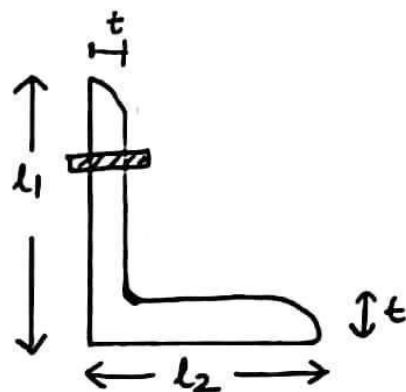
Here " α " is the reduction factor considering shear lag and depends upon the length of the connection i.e. the no. of bolts per line in the direction of applied load.

- Hence, shear lag can be reduced by increasing the length of end connection.

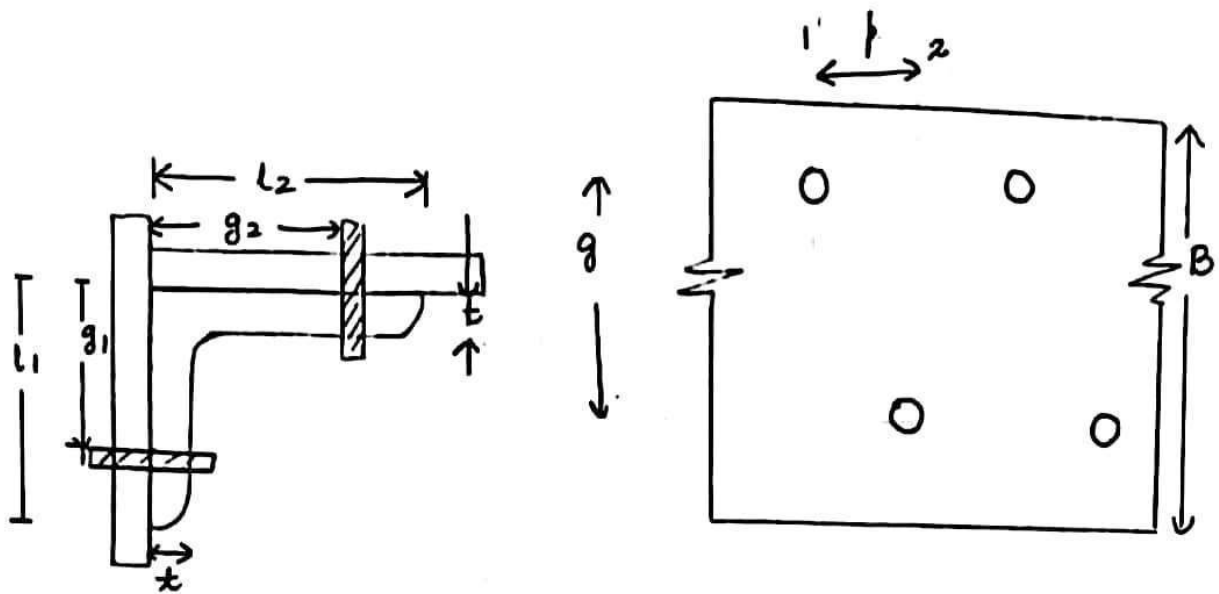
for eg. $A_{g0} = \left(l_2 - \frac{t}{2}\right)t$

$$A_{nc} = \left(l_1 - d_0 - \frac{t}{2}\right)t$$

$$A_g = (l_1 + l_2 - t)t$$



(2) When both the legs of the angle are connected.



$$B = l_1 + l_2 - t$$

$$g = g_1 + g_2 - t$$

- When both the legs of the angles are connected; the angle can be opened and be treated as plate.

$$A_{n1} = (B - d_o)t$$

$$A_{n2} = (B - 2d_o + \frac{p^2}{4g})t$$

$$A_n = \min \{ A_{n1}, A_{n2} \}$$

$$T_{dn} = \frac{0.9 f_u A_n}{\gamma_{m1}}, \quad \gamma_{m1} = 1.25$$

c) for Block shear failure.

$$T_{db} = \min. \begin{cases} \text{shear yielding + Tension rupture} \\ \frac{f_y A_{vg}}{\sqrt{3} \gamma_{m0}} + \frac{0.9 f_u A_{tn}}{\gamma_{m1}} \\ \text{shear rupture + tension yielding} \\ \frac{0.9 f_u A_{vn}}{\sqrt{3} \cdot \gamma_{m1}} + \frac{f_y A_{tg}}{\gamma_{m0}} \end{cases}$$

T_{db} = Block shear strength

Note $\Rightarrow$ In case of welded angle; tension member A_{tn} and A_{vn} is to be replaced A_{tg} & A_{vg} respectively.

Hence; design Tensile strength (T_d) = $\min. \{ T_{dg}, T_{dn}, T_{db} \}$

Lesson 2 Dec 14.

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SLENDERNESS RATIO

- It is the ratio of unsupported length to the least radius of gyration.
- Although, there is no stability problem in tension member; yet maximum slenderness ratio (λ_{max}) is defined to safeguard against buckling due to load reversals during erection or transportation etc.

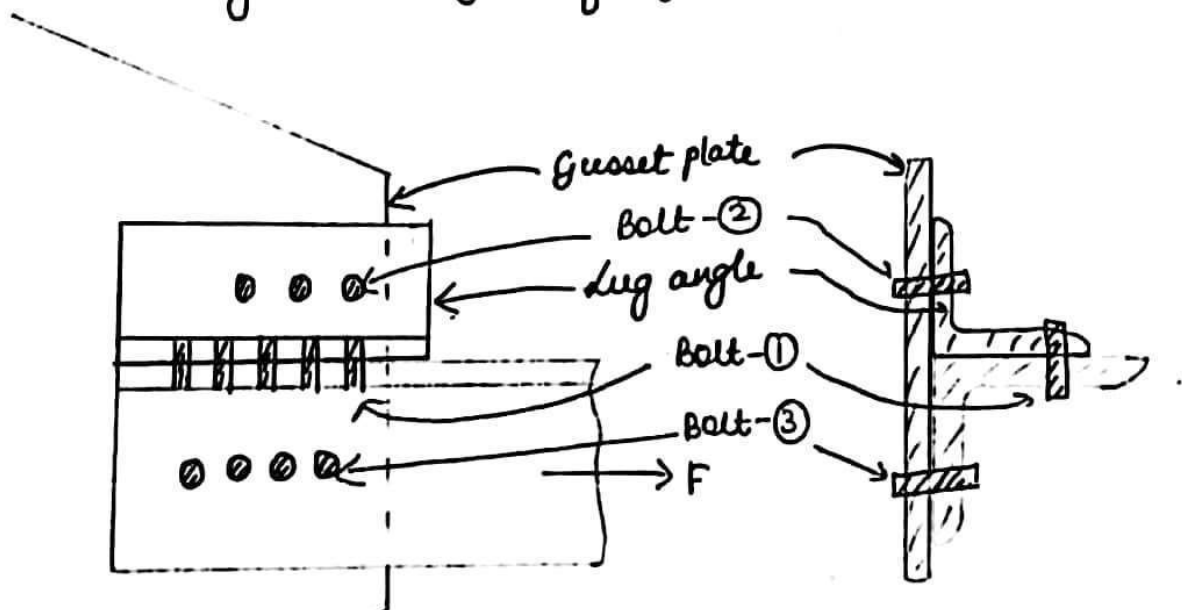
$$\lambda = \frac{L_{eff}}{r_{min.}}$$

- Maximum slenderness ratio for tension members are as follows:

Description	λ_{max} .
1. A tension member in which reversal of direct stress is due to load other than wind or earthquake (i.e. DL & LL).	180
2. A member normally acting as tension member but in which stress reversal is due to wind or earthquake	350
3. A member always under tension except pretensioned member.	400

LUG ANGLES

- Lug angle is the short length of angle section at joint to connect the outstanding leg of a member thereby reducing the length of joint.

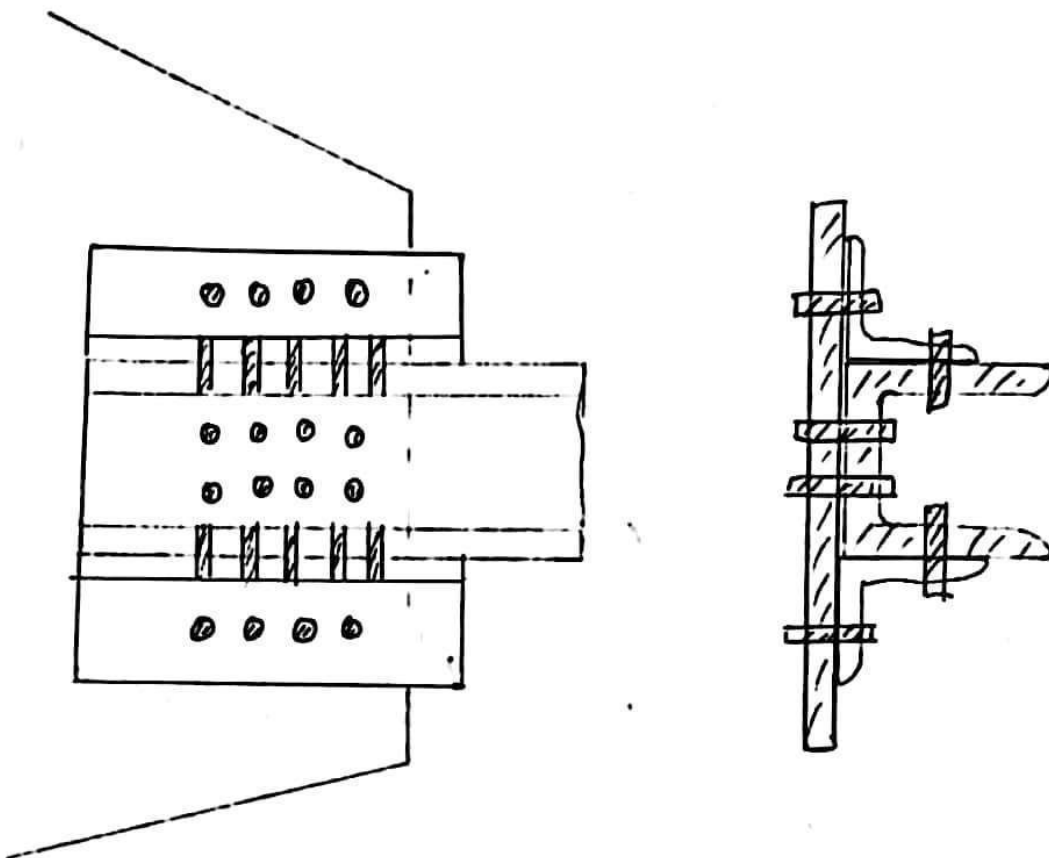


- Bolts connecting the outstanding leg of main member with lug angle (bolt-1) must start in advance of all the other bolts as this will

ensure that force in the outstanding leg is effectively transferred in the lug angle.

- where lug angles are used to connect the angle members; the whole area of the member shall be taken as effective area i.e. shear lag is not considered.
- At least two bolts should be used for attaching the lug angle to the gusset plate.
- when angle member is the main member, check
 - a) Lug angle and their connection with the gusset plate (Bolt-②) should be designed for force > 1.2 times the force in the outstanding leg of main member.
 - b) Connection of outstanding leg of the main member with the lug angle (Bolt-①) should be designed for a force > 1.4 times the force in the outstanding leg of main member.

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- When the channel member is the main member; check
 - a) Lug angles and their connection with the gusset plate should be designed for force ≥ 1.1 times the force in the outstanding leg of main member.
 - b) Connection of lug angle with outstanding leg of main member should be designed for force ≥ 1.2 times the force in the outstanding leg of main member.

Design of Tension Members.

STEPS

$$1) A_n \text{ req} = \frac{T}{0.9 f_u / \gamma_{m1}} \quad (\text{for Plates})$$

or.

$$A_n \text{ req} = \frac{T}{\alpha f_u / \gamma_{m1}} \quad (\text{for angles}) \text{ with one leg connected.}$$

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Here T = factored or design load.

- 2) " A_n " so computed in previous step is increased by 10-25% to compute the gross-sectional area (A_g) and a trial section is chosen.
- 3) For the chosen section, check gross section yielding.

$$A_g = \frac{T}{f_y / \gamma_{m0}}$$

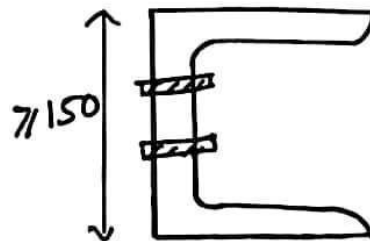
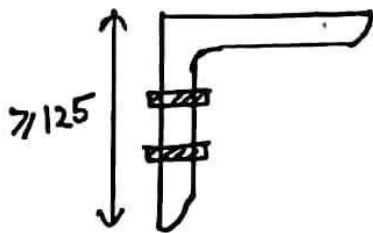
and $A_g \text{ provided} \geq A_g \text{ yielding.}$

- 4) Connection is designed and checked for
 - (i) Gross section yielding (already done in step ③)
 - (ii) Net section rupture
 - (iii) Block shear failure

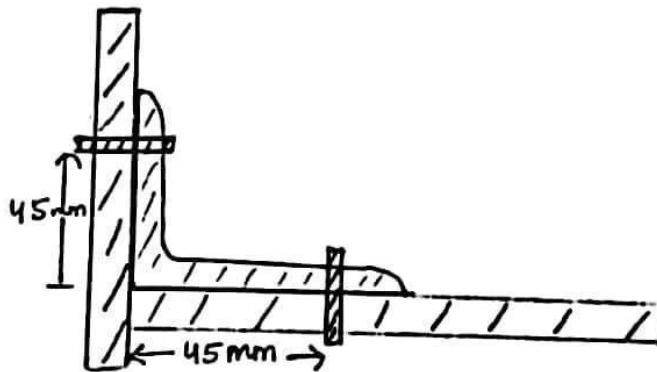
5) Check for slenderness ratio i.e $\lambda \neq \lambda_{max}$.

Note :- a) On one leg of angle section two line of bolt can be accommodated only when length of leg ≥ 125 mm.

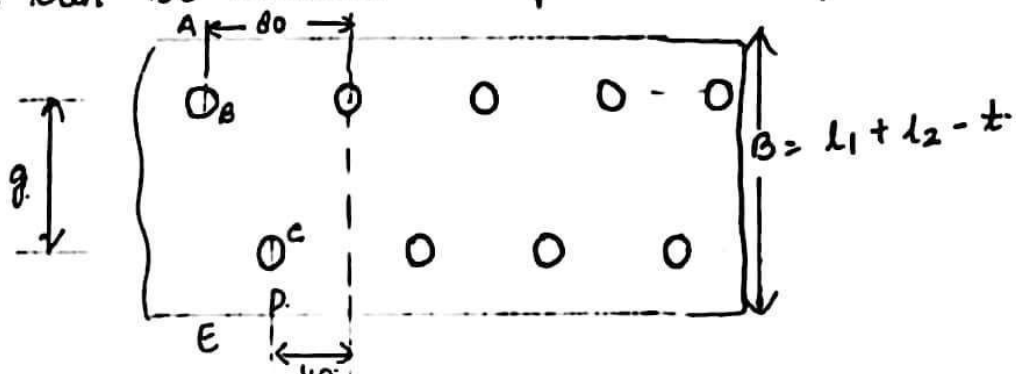
b) Two lines of bolts can be accommodated in the web of the channel section only if the width of web ≥ 150 mm.



Q An ISA 75x75x10 mm is connected to the gusset plate by 16 mm dia bolts through both legs, the pitch on one leg is 80 mm and bolts on one leg is staggered by 40 mm w.r.t. those in the other. Determine the design rupture strength of the angle. Assume Fe 410 grade of steel.



Solⁿ Since both the legs of angles are connected, so it can be treated as plate when opened up.



$$B = 75 + 75 - 10 = 140 \text{ mm}$$

$$d_o = 16 + 2 = 18 \text{ mm}$$

$$g = 45 + 45 - 10 = 80 \text{ mm}$$

for section ABE, $A_{n1} = (B - d_o)t = (140 - 18) \times 10 = 1220 \text{ mm}^2$.

for section ABCD, $A_{n2} = \left(B - 2d_o + \frac{p^2}{4g} \right) t$
 $= \left(140 - 2 \times 18 + \frac{40^2}{4 \times 80} \right) \times 10$
 $= 1090 \text{ mm}^2$.

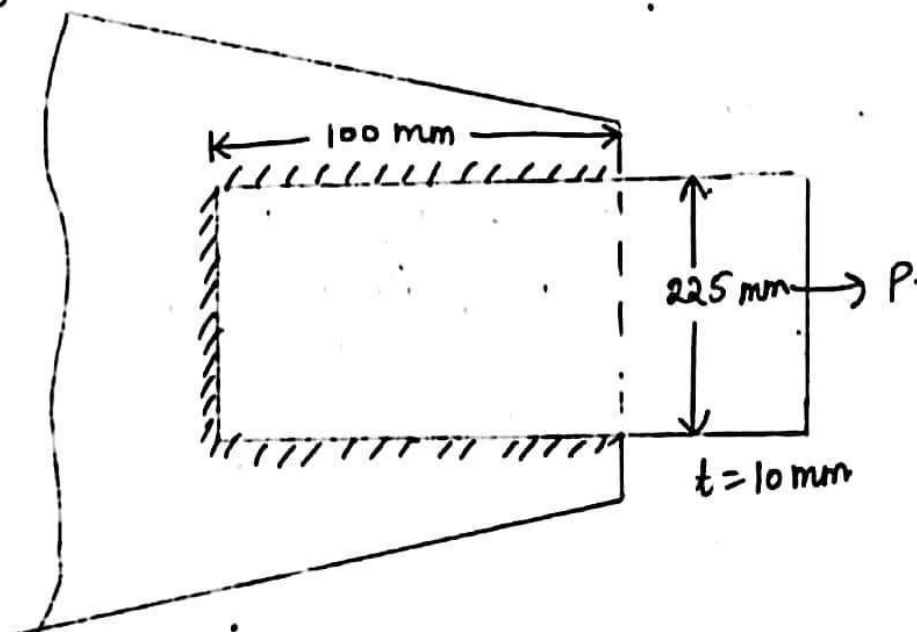
$$A_n = \min. \{ A_{n1}, A_{n2} \}$$

$$= 1090 \text{ mm}^2$$

$$T_{dn} = \frac{0.9 f_u A_n}{\gamma_{m1}} = \frac{0.9 \times 410 \times 1090 \times 10^{-3}}{1.25} = 321.76 \text{ kN}$$

Lesson 3 Dec 15.

Q. Determine the Block shear strength of the welded tension member as shown. Use Fe 410 grade of steel.



Solⁿ

$$T_{db} = \min \left\{ \begin{aligned} &\frac{f_y A_{vg}}{\sqrt{3} \times \gamma_{m0}} + \frac{0.9 f_u A_{tg}}{\gamma_{m1}} \\ &\frac{0.9 f_u A_{vg}}{\sqrt{3} \gamma_{m1}} + \frac{f_y A_{tg}}{\gamma_{m0}} \end{aligned} \right.$$

$$\begin{aligned} A_{vn} = A_{vg} &= 2 \times 100 \times 10 = 2000 \text{ mm}^2 \\ A_{tn} = A_{tg} &= 225 \times 10 = 2250 \text{ mm}^2 \end{aligned} \quad \left\{ \begin{array}{l} \text{welded} \\ \text{connection} \end{array} \right.$$

$$T_{db} = \min \left\{ \begin{aligned} &\frac{250 \times 2000}{\sqrt{3} \times 1.1} + \frac{0.9 \times 410 \times 2250}{1.25} = 926.63 \text{ kN} \\ &\frac{0.9 \times 410 \times 2000}{\sqrt{3} \times 1.25} + \frac{250 \times 2250}{1.1} = 852.22 \text{ kN} \end{aligned} \right.$$

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$$\therefore, T_{db} = 852.22 \text{ kN}$$

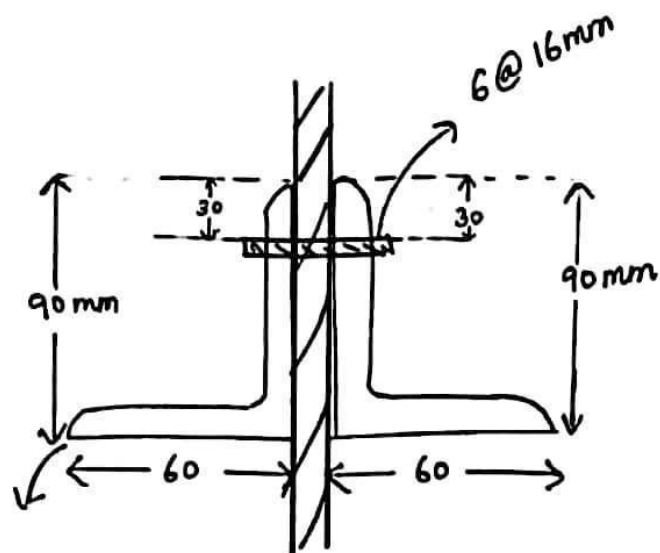
Q. Determine the tensile capacity of the section shown, if

- angles are placed on the opposite side of gusset plate (Tack bolted).
- angles are placed on the same side of gusset plate (Tack bolted).
- angles are not Tack bolted.

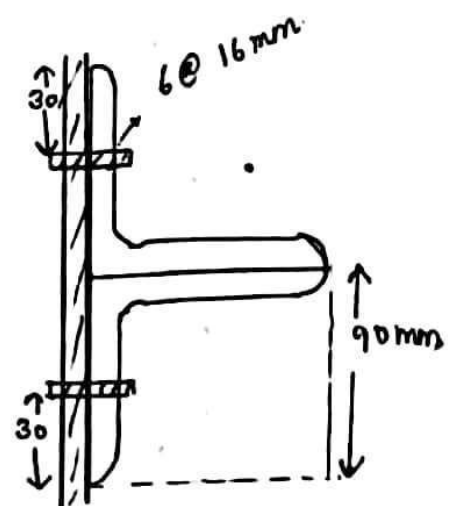
Assume Fe 410 grade steel ; pitch of bolts = 40 mm.

Edge / end distance of bolts = 30 mm.

for ISA : 90 x 60 x 8 $\Rightarrow A_g = 1136 \text{ mm}^2$.



ISA : 90x60x8.



Solⁿ $d_o = 16 + 2 = 18 \text{ mm}$.

- Design tensile strength of angle in yielding (T_{dg}).

$$T_{dg} = \frac{f_y A_g}{\gamma_{m0}} = \frac{250 \times 1136 \times 2 \times 10^{-3}}{1.1} = 516.36 \text{ kN}$$

↑
2 angle s/c.

- Design tensile strength in rupture (T_{dn}).

$$T_{dn} = \frac{0.9 f_u A_{nc}}{\gamma_{m1}} + \frac{\beta f_y A_{go}}{\gamma_{m0}}$$

$$\text{here } \beta = 1.4 - 0.076 \frac{w}{t} \cdot \frac{f_y}{f_u} \cdot \frac{b_s}{l_c}$$

$$\leq \frac{0.9 f_u / \gamma_{m1}}{f_y / \gamma_{m0}}$$

$$\gg 0.7$$

$$w = 60 \text{ mm}$$

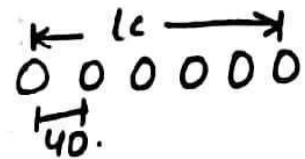
$$t = 8 \text{ mm.}$$

$$b_s = w + g - t$$

$$= 60 + 60 - 8 = 112 \text{ mm}$$

$$l_c = 5 \times 40 = 200 \text{ mm.}$$

$$g = 90 - 30 = 60$$



$$\beta = 1.4 - 0.076 \times \frac{60}{8} \times \frac{250}{410} \times \frac{112}{200}$$

$$= 1.205$$

$$\beta \leq \frac{0.9 \times 410 / 1.25}{250 / 1.1}$$

$$\leq 1.2988 \text{ (OK).}$$

$$\beta > 0.7 \text{ (OK).}$$

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$$T_{dn1} = \frac{0.9 \times 410 \times 544}{1.25} + \frac{1.205 \times 250 \times 448}{1.1}$$

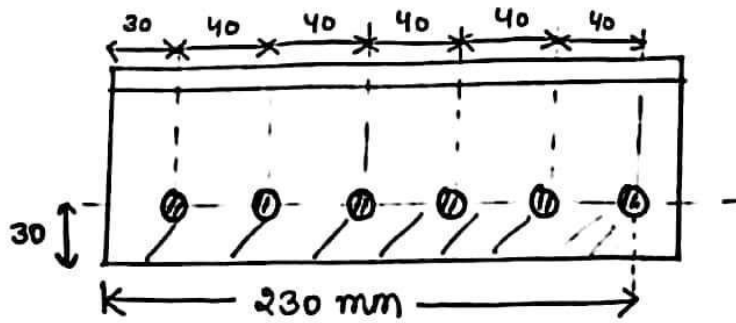
$$\Rightarrow 283.27 \text{ kN.}$$

$$A_{nc} = \left(90 - 18 - \frac{8}{2}\right) \times 8 = 544 \text{ mm}^2$$

$$A_{go} = \left(60 - \frac{8}{2}\right) \times 8 = 448 \text{ mm}^2$$

$$T_{dn} = 2 \times 283.27 = 566.56 \text{ kN.}$$

- Block shear strength (T_{db}).



$$T_{db1} = \begin{cases} \frac{f_y A_{vg}}{\sqrt{3} \gamma_{m0}} + \frac{0.9 f_u A_{tn}}{\gamma_{m1}} \\ \min \left\{ \frac{0.9 f_u A_{vn}}{\sqrt{3} \gamma_{m1}} + \frac{f_y A_{tg}}{\gamma_{m0}} \right\} \end{cases}$$

$$A_{vg} = 230 \times 8 = 1840 \text{ mm}^2$$

$$A_{vn} = (230 - 5.5 \times 18) \times 8 = 1048 \text{ mm}^2$$

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$$A_{tg} = 30 \times 8 = 240 \text{ mm}^2$$

$$A_{tn} = \left(30 - \frac{18}{2} \right) \times 8 = 168 \text{ mm}^2$$

$$T_{db1} = \begin{cases} \frac{250 \times 1840}{\sqrt{3} \times 1.1} + \frac{0.9 \times 410 \times 168}{1.25} = 291.1 \text{ kN} \\ \min \left\{ \frac{0.9 \times 410 \times 1048}{\sqrt{3} \times 1.25} + \frac{250 \times 240}{1.1} = 233.16 \text{ kN} \right\} \end{cases}$$

$$T_{db1} = 233.16 \text{ kN}$$

$$T_{db} = 2 \times 233.16 = 466.32 \text{ kN}$$

Design tensile strength of member. $T_d = \min \{T_{dg}, T_{dn}, T_{db}\}$

$$T_d = 466.32 \text{ kN.}$$

b, c) as per IS 800: 2007, it makes no difference whether angles are connected on opposite side or on same side of gusset plate., and whether are tack bolted or not:

Q. Design a bridge truss diagonal subjected to factored tensile load of 300 kN. The length of the diagonal is 3m. The tension member is connected to a gusset plate 16 mm thick with one line of 20 mm dia bolts of grade 8.8. steel section is Fe 410 grade. and the available sections are

ISA: 100 x 75 x 8. $r_{uu}: 34.8 \text{ mm}$ $r_{vv}: 15.9 \text{ mm}$	ISA: 100 x 65 x 8 $r_{uu}: 33.8 \text{ mm}$ $r_{vv}: 13.9 \text{ mm}$	ISA: 125 x 75 x 8 $r_{uu}: 42.1 \text{ mm}$ $r_{vv}: 16.1 \text{ mm}$
Sol ⁿ $A_g = (100 + 75 - 8) \times 8$ $= 1336 \text{ mm}^2$	$A_g = (100 + 65 - 8) \times 8$ $= 1256 \text{ mm}^2$	$A_g = (125 + 75 - 8) \times 8$ $= 1536 \text{ mm}^2$

(A) Design strength of plate Bolt (V_{db}).

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} \left(0.78 \times 1 \times \frac{\pi d^2}{4} \right)$$

$$= \frac{800}{\sqrt{3} \times 1.25} \left(0.78 \times \pi \times \frac{20^2}{4} \right) \times 10^{-3} = 90.54 \text{ kN.}$$

(as bolts are in single shear)

$$V_{dpb} = 2.5 k_b d + \frac{f_u}{1.25}$$

$$k_b = \min \left\{ \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u} - 1.0 \right\}$$

$$e_{\min} = 1.5 d_0 = 1.5 (20+2) = 33 \text{ mm.}$$

$$p_{\min} = 2.5 d = 2.5 \times 20 = 50 \text{ mm.}$$

Note:- Since high strength bolts are used; e & p is chosen such that $V_{dsb} = V_{dpb}$.

$$\frac{2.5 K_b d \cdot t f_u}{1.25} = 90.54 \times 10^3.$$

$$\frac{2.5 K_b \times 20 \times 8 \times 410}{1.25} = 90.54 \times 10^3$$

$$K_b = 0.69.$$

$$\text{Now. } \frac{e}{3 \times 22} = 0.69 \Rightarrow e = 45.54 \text{ mm}$$

$$\text{adopt } e = 50 \text{ mm.}$$

$$\text{also. } \frac{p}{3 d_0} - 0.25 = 0.69.$$

$$\frac{p}{3 \times 22} - 0.25 = 0.69.$$

$$p = 62.04 \text{ mm.}$$

$$\text{adopt } p = 65 \text{ mm.}$$

$$\therefore V_{db} = 90.54 \text{ kN.}$$

$$\text{No. of bolts} = \frac{T}{V_{db}} = \frac{300}{90.54} = 3.31$$

$$\text{Adopt 4 no. of bolts.}$$

B) Design of angle section.

(i) for one leg connected of angle.

$$A_{n \text{ req}} = \frac{T}{\alpha f_u / \gamma_{m1}}$$

$$\text{for 4 bolts, } \alpha = 0.8.$$

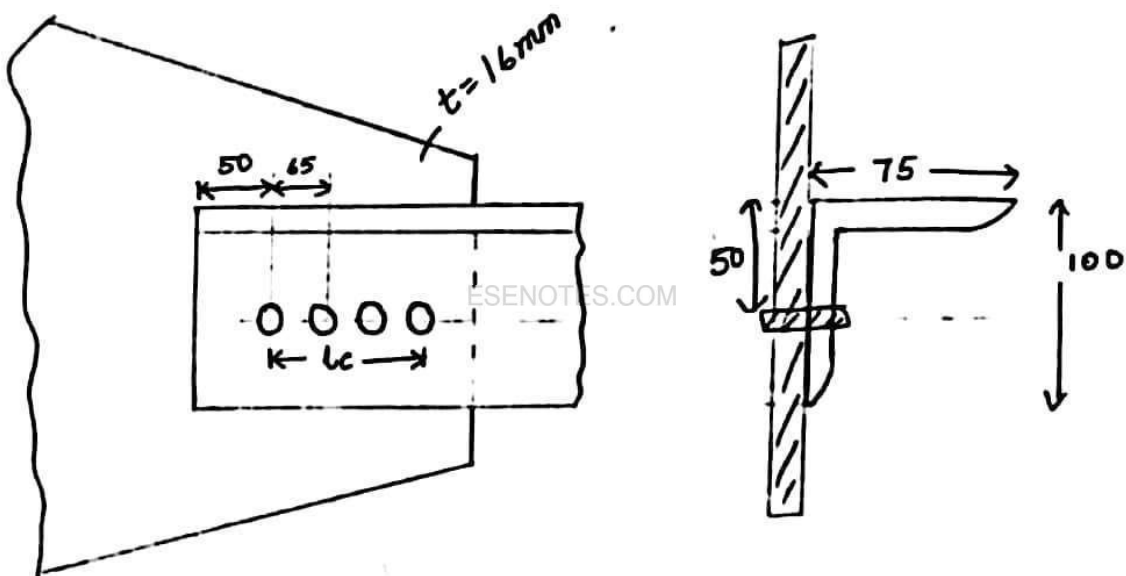
$$A_{n \text{ req}} = \frac{300 \times 10^3}{0.8 \times \frac{410}{1.25}} = 1143.3 \text{ mm}^2.$$

Let $A_{n \text{ req}}$ is increased by 15%. $\Rightarrow A_g = 1.15 \times 1143.3$
 $A_g = 1315 \text{ mm}^2.$

(ii) A_g from gross yielding criteria

$$A_g = \frac{T}{f_y / \gamma_{m0}} = \frac{300 \times 10^3}{250 / 1.1} = 1320 \text{ mm}^2.$$

Adopt ISA : $100 \times 75 \times 8$; $A_g \text{ provided} = 1336 \text{ mm}^2.$
 $A_g \text{ provided} \geq A_g \text{ yielding}.$



Check for rupture

$$T_{dn} = \frac{0.9 f_u A_{nc}}{\gamma_{m1}} + \frac{\beta f_y A_{go}}{\gamma_{m0}}.$$

$$\beta = 1.4 - 0.076 \frac{w}{t} \frac{b_f}{f_u} \times \frac{b_s}{l_c}.$$

$$\leq \frac{0.9 f_u / \gamma_{m1}}{b_f / \gamma_{m0}} = 1.29.$$

$$\geq 0.7$$

$$w = 75 \text{ mm}$$

$$t = 8 \text{ mm}$$

$$b_s = w + g - t$$

$$= 75 + 50 - 8 = 117 \text{ mm.}$$

$$d_c = 65 \times 3 = 195 \text{ mm.}$$

$$\beta = 1.4 - 0.076 \times \frac{75}{8} \times \frac{250}{410} \times \frac{117}{195}$$

$$= 1.14 \geq 0.7 \text{ OK.}$$

$$\leq 1.29 \text{ OK.}$$

$$A_{nc} = \left(100 - 22 - \frac{8}{2}\right) 8 = 592 \text{ mm}^2$$

$$A_{g0} = \left(75 - \frac{8}{2}\right) \times 8 = 568$$

$$T_{dn} = \frac{0.9 \times 410 \times 592}{1.25} + \frac{1.14 \times 250 \times 568}{1.1}$$

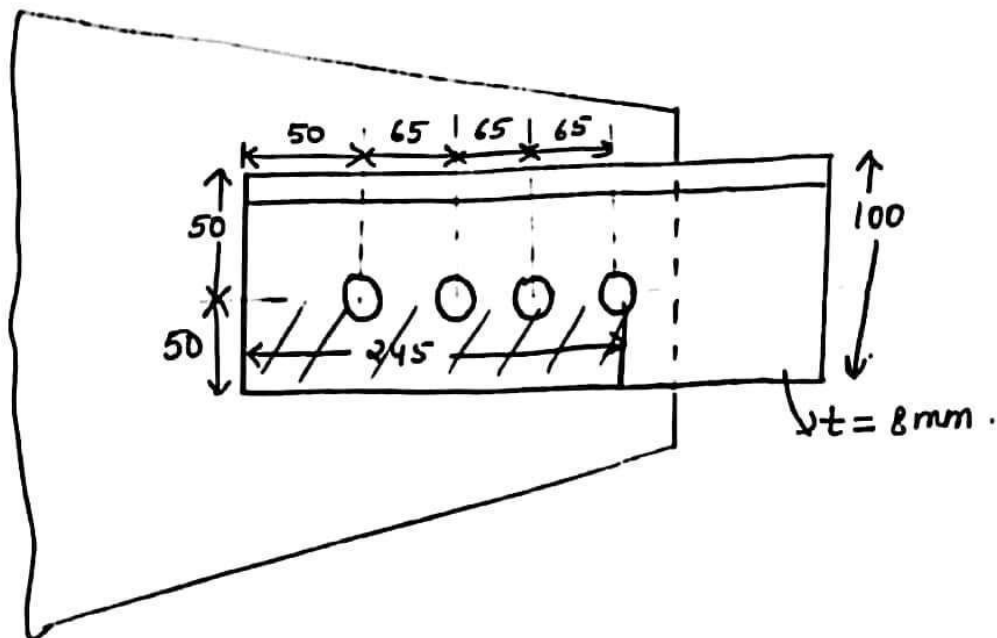
$$= 321.9 \text{ kN.}$$

$$T_{dn} \geq T (300 \text{ kN}) \therefore \text{Safe in Rupture.}$$

Lesson 4 Dec 16.

- Check for Block shear strength.

$$T_{db} = \begin{cases} \frac{f_y A_{vg}}{\sqrt{3} \gamma_{m0}} + \frac{0.9 f_u A_{tn}}{\gamma_{m1}} \\ \min. \left\{ \frac{0.9 f_u A_{vn}}{\sqrt{3} \gamma_{m1}} + \frac{f_y A_{tg}}{\gamma_{m0}} \right\} \end{cases}$$



$$A_{vg} = 245 \times 8 = 1960 \text{ mm}^2$$

$$A_{vn} = (245 - 3.5 \times 22) \times 8 = 1344 \text{ mm}^2$$

$$A_{tg} = 50 \times 8 = 400 \text{ mm}^2$$

$$A_{tn} = \left(50 - \frac{22}{2}\right) \times 8 = 312 \text{ mm}^2$$

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$$T_{db} = \begin{cases} \frac{250 \times 1960}{\sqrt{3} \times 1.1} + \frac{0.9 \times 410 \times 312}{1.25} = 349.9 \text{ kN.} \\ \text{min. } \frac{0.9 \times 410 \times 1344}{\sqrt{3} \times 1.25} + \frac{250 \times 400}{1.1} = 319.97 \text{ kN.} \end{cases}$$

$$T_{db} = 319.97 \text{ kN.} > 300 \text{ (kN).}$$

Safe in Block Shear.

- Check for Slenderness

$$\lambda = \frac{l_{eff}}{r_{min}} = \frac{3 \times 10^3}{15.9} = 188.67 < 350 \quad \underline{\underline{OK.}}$$

Note: Check the same bridge truss diagonal, using Working stress Method.

Design load $T = 300 \text{ kN}$

$$\text{service load } T_s = \frac{T}{1.5} = \frac{300}{1.5} = 200 \text{ kN.}$$

Permissible tensile stress is least of following.

a) Yielding stress

$$f_{at} = 0.6 f_y = 0.6 \times 250 = 150 \text{ N/mm}^2$$

b) Rupture stress

$$f_{at} = 0.69 \frac{T_{dn}}{A_g} = 0.69 \times \frac{321.9}{1336} \\ = 166.25 \text{ N/mm}^2$$

T_{dn} : tensile strength in rupture as calculated in LSM.

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c) Block shear stress

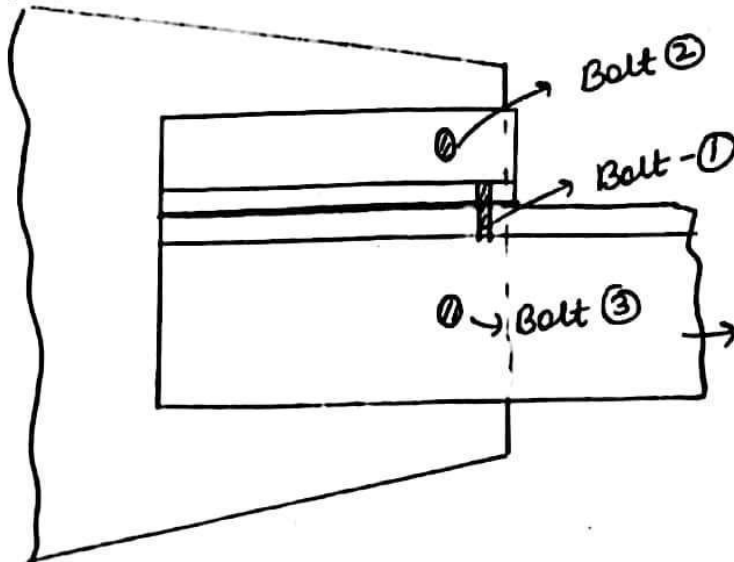
$$f_{at} = 0.69 \frac{T_{db}}{A_g} = 0.69 \times \frac{319.97}{1336} \\ = 165.2 \text{ N/mm}^2.$$

$$\text{Permissible stress } f_{at} = \min \{ 150, 166.25, 165.2 \} \\ = 150 \text{ N/mm}^2.$$

$$\text{Actual tensile stress } f_t = \frac{T_s}{A_g} = \frac{200 \times 10^3}{1336} = 149.7 \text{ N/mm}^2$$

$$f_t < 150 \text{ N/mm}^2 \quad \underline{\underline{\text{OK}}}.$$

Q An ISA : 100 x 75 x 10 mm is used as a tension member with longer leg connected to 10 mm thick gusset plate. The connection is made with the help of lug angle. Assume M20 bolts of grade 4.6 & steel of grade Fe 410. Design the connection if sections available for lug angle are ISA : 60 x 60 x 8 , ISA : 60 x 60 x 10 ; ISA : 70 x 70 x 8



$$\begin{aligned} \text{ISA : } 100 \times 75 \times 10 \\ A_g = (100 + 75 - 10) \times 10 \\ = 1650 \text{ mm}^2 \end{aligned}$$

Solⁿ $A_g = (60 + 60 - 8) \times 8$
 $= 896 \text{ mm}^2$

$A_g = (60 + 60 - 10) \times 10$
 $= 1100 \text{ mm}^2$

$A_g = (70 + 70 - 8) \times 8$
 $= 1056 \text{ mm}^2$

$d = 20 \text{ mm}$, $d_o = 20 + 2 = 22 \text{ mm}$

Since force for which design is to be done is not given ; it is computed first assuming one bolt line is provided in main member.

— For gross section yielding .

$$T_{dg} = \frac{f_y A_g}{\gamma_{mo}} = \frac{250 \times 1650}{1.1} = 375 \text{ kN} .$$

— for net section rupture

$$\begin{aligned} T_{dn} &= \frac{0.9 f_u A_n}{\gamma_{m1}} = \frac{0.9 \times 410 \times (100 + 75 - 10 - 22) \times 10}{1.25} \\ &= 422.13 \text{ kN} . \end{aligned}$$

This formula is used because both legs are connected

- Design factored load $T_d = \min. \{ T_{dg}, T_{dn} \}$
 $= 375 \text{ kN}.$

Let F_1 and F_2 be the force in outstanding leg & connected leg of main member.

$$F_1 + F_2 = T = 375 \text{ kN} - (i).$$

Now, gross area of outstanding leg $A_1 = \left(75 - \frac{10}{2} \right) 10$
 $= 700 \text{ mm}^2.$

gross area of connected leg $A_2 = \left(100 - \frac{10}{2} \right) \times 10 = 950 \text{ mm}^2$

$$F_1 = \frac{T A_1}{A_1 + A_2} = \frac{375 \times 700}{1650} = 159.1 \text{ kN}.$$

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$$F_2 = \frac{T A_2}{A_1 + A_2} = \frac{375 \times 950}{1650} = 215.9 \text{ kN}$$

Design force for lug angle and Bolt 2 ; $T_1 = 1.2 \times 159.1$
 $= 190.9 \text{ kN}$

Design force for lug angle & Bolt ① ; $T_2 = 1.4 \times 159.1$
 $= 222.7 \text{ kN}.$

Design force for Bolt ③ $T_3 = 215.9 \text{ kN}.$

Now, design of lug angle.

$$(i) A_{n \text{ req}} = \frac{T_1}{0.9 f_u / \gamma_{m1}} = \frac{190.9 \times 10^3}{0.9 \times 410 / 1.25} = 646.68 \text{ mm}^2$$

$$(ii) A_{g \text{ req}} = \frac{T_1}{f_y / \gamma_{m0}} = \frac{190.9 \times 10^3}{250 / 1.1} = 840 \text{ mm}^2$$

So provide ISA : 60x60x8 mm , $A_g = 896 \text{ mm}^2$

- Design of Bolt

$$V_{dab} = \frac{f_{ub}}{\sqrt{3} \times 1.25} \times 0.78 \times \frac{\pi d^2}{4}$$

$$= \frac{400}{\sqrt{3} \times 1.25} \times 0.78 \times \frac{\pi \times 20^2}{4} = 45.27 \text{ kN}$$

$$V_{dpb} = 2.5 K_b d t \frac{f_u}{1.25}$$

$$K_b = \min \left\{ \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1.0 \right\}$$

$$e = e_{\min} = 1.5d_0 = 1.5 \times 22 = 33 \text{ mm}$$

$$p = p_{\min} = 2.5d = 2.5 \times 20 = 50 \text{ mm}$$

$$K_b = \min \left\{ \frac{33}{3 \times 22}, \frac{50}{3 \times 22} - 0.25, \frac{400}{410}, 1.0 \right\}$$

$$= \min \{ 0.5, 0.507, 0.97, 1 \}$$

$$= 0.5$$

$$t = \min \{ 10, 8 \} = 8 \text{ mm}$$

$$V_{dpb} = 2.5 \times 0.5 \times 20 \times 8 \times \frac{410}{1.25} = 65.6 \text{ kN}$$

$$V_{db} = \min \{ V_{dab}, V_{dpb} \}$$

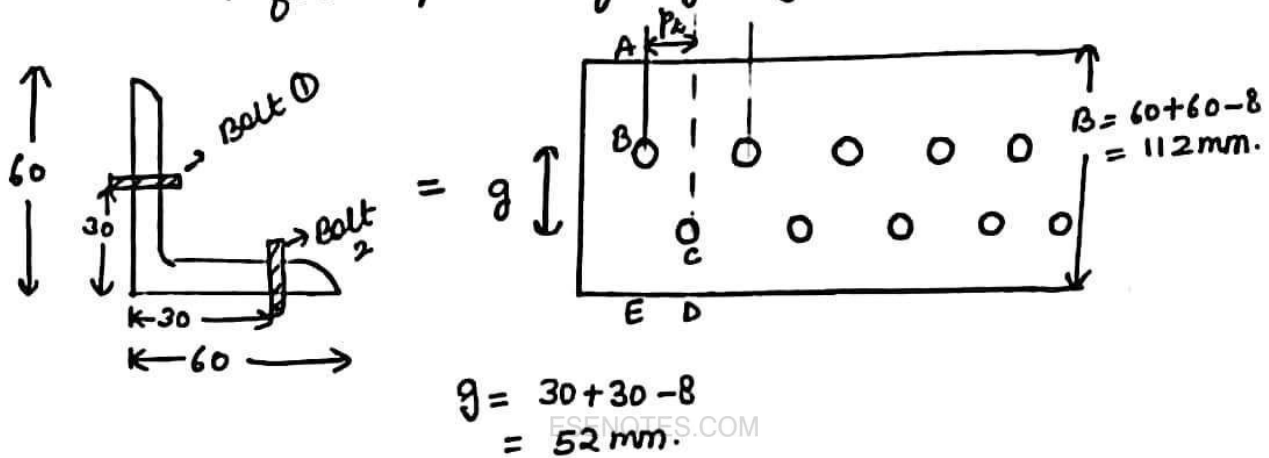
$$= 45.27 \text{ kN}$$

$$\text{No. of bolt } \textcircled{1} = \frac{T_2}{V_{db}} = \frac{222.7}{45.27} = 4.91 \approx 5$$

$$\text{" " } \textcircled{2} = \frac{T_1}{V_{db}} = \frac{190.9}{45.27} = 4.2 \approx 5$$

$$\text{" " } \textcircled{3} = \frac{T_3}{V_{db}} = \frac{215.9}{45.27} = 4.76 \approx 5$$

⇒ Check for rupture of lug angle.

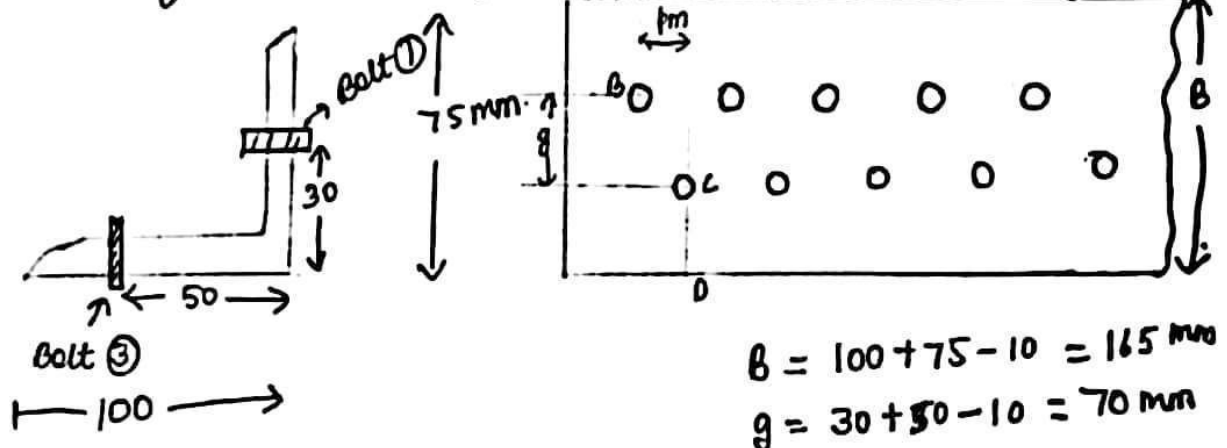


- along section ABE; $A_n = (112 - 22)8 = 720 \text{ mm}^2 > A_{n \text{ req}} (646.6 \text{ mm}^2)$

- along section ABCD, $A_n = \left(112 - 2 \times 22 + \frac{p_t^2}{4 \times 52}\right) \times 8 > A_{n \text{ req}} = 646.6$

$$p_t \geq 51.64 \text{ mm}$$

- Check for rupture of main angle.

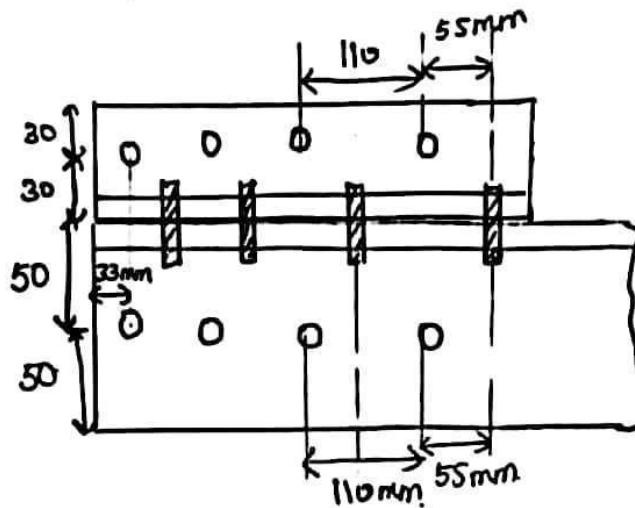


Along s/c ABCD.

$$A_n = \left(165 - 2 \times 22 + \frac{p m^2}{4 \times 70} \right) \times 10 \geq \frac{T}{0.9 f_u / \gamma_m} = \frac{375 \times 10^3}{0.9 \times 410 / 1.2}$$

$$p m \geq 41.098 \text{ mm}$$

Adopt $p_l = p_m = 55 \text{ mm}$.



- check main angle for block shear & if it comes to be safe no need to check lug angle for block shear.

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COMPRESSION MEMBERS

- A structural member which is subjected to compressive forces along its axis is termed as Compression member.
- It is further classified as.

A) Stanction

- The vertical compression member is termed as Stanction.
(it is equivalent to column in RCC structures)

B) Strut

- Horizontal or inclined compression member is termed as strut.
- It is generally used in roof truss and bracings

C) BOOM

- It is the principal compression members used in crane.

Modes of failure of Compression Member

(i) SQUASHING

- When the length is relatively small & the component plate elements are prevented from local buckling, then the column will be able to attain its full strength or squash load.
- If the loading is more than squash load, the failure is termed as Squashing.

(ii) Local Buckling

- Failure occurs by buckling of one or more individual plate elements of column is termed as local buckling.

- Eg.: Buckling of web or flange prior to the buckling of the column.

(iii) FLEXURAL BUCKLING

- In this mode, failure of the member occurs by the excessive deflection caused by bending or flexure about the axis corresponding to the largest slenderness ratio. (usually the minor principal axis)

(iv) TORSIONAL BUCKLING

- This type of failure is caused by twisting about the longitudinal axis of member.
- It can occur only in doubly symmetrical cross-section with very slender cross-sectional elements.

(v) FLEXURAL TORSIONAL BUCKLING

- It is caused by combination of flexure and torsional buckling.
- The member bends & twist simultaneously in this case.
- This takes place with unsymmetrical sections or singly symmetrical cross-sections.

Effective Length (l_e)

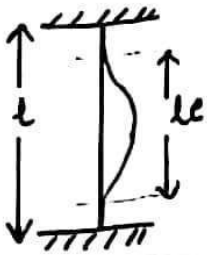
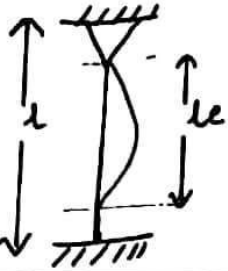
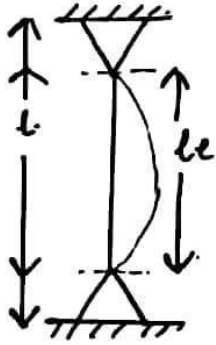
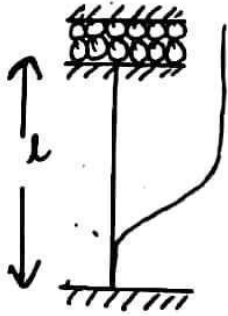
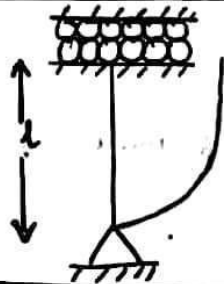
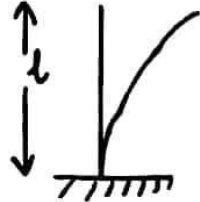
- The effective length of a compression member is the distance between the points of contraflexure.
- It depends on actual length (l) and support/end condition.

$$l_e = K l$$

l = actual length

K = effective length factor

(It depends on support condition)

End conditions	Theoretical (l_e)	IS code (l_e)
(i) 	$0.5 l$	$0.65 l$
(ii) 	$0.707 l$	$0.8 l$
(iii) 	l	l
(iv) 	l	$1.2 l$
(v) 	$2 l$	$2 l$
(vi) 	$2 l$	$2 l$

Note: → Effective length in IS code is slightly larger than the horizontal value to account for lack of 100% fixity at support.

2) for LACED column, above value are increased by 5%.

3) For BATTENED column, above values are increased by 10%.

SLENDERNESS RATIO (λ)

$$\lambda = \frac{l_e}{r_{\min}} = \frac{Kl}{r_{\min}}$$

l_e = effective length

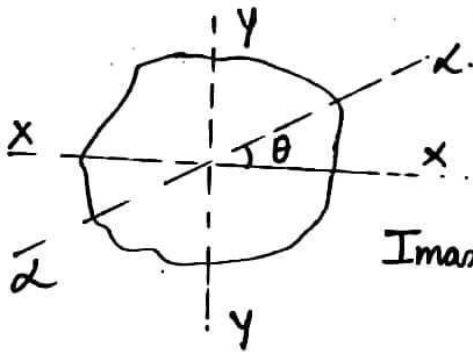
$r_{\min}$ = minimum radius of gyration

$$I_{\min} = A r_{\min}^2$$

$$r_{\min} = \sqrt{\frac{I_{\min}}{A}}$$

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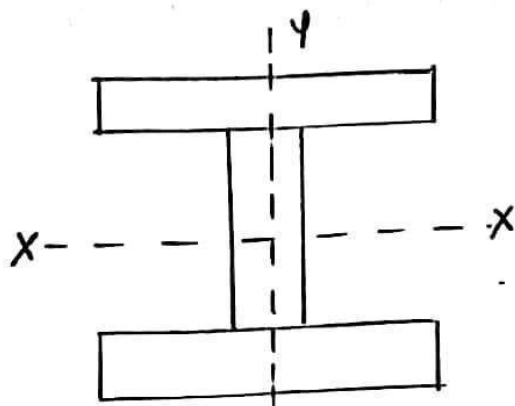
A = area of cross-section.



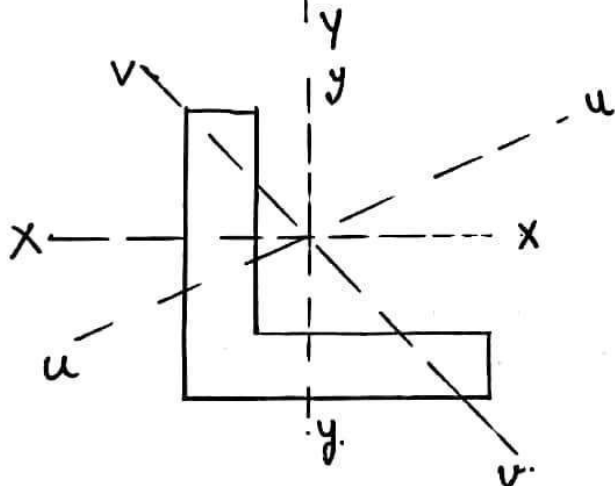
$$I_{\max/\min} = \frac{I_{xx} + I_{yy}}{2} \pm \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + I_{xy}^2}$$

$$I_{\alpha\alpha} = \frac{I_{xx} + I_{yy}}{2} + \frac{I_{xx} - I_{yy}}{2} \cos 2\theta - I_{xy} \sin 2\theta$$

$I_{\max/\min}$: Moment of Inertia about major & minor principal axis



Here $I_{xx} = I_{max}$ (area is away from xx).
 $I_{yy} = I_{min}$ (area is near to yy).



Here $I_{uu} = I_{max}$
 $I_{vv} = I_{min}$.

- An axis of symmetry and an axis perpendicular to this are principal axis.
- Axis from which area is concentrated at more distance is major principal axis and other is minor principal axis.
- The tendency of a member to buckle is usually measured by slenderness ratio.
- A section buckles about an axis having maximum slenderness ratio i.e. about minor principal axis. provided effective length is same about major and minor axis.

- Maximum value of slenderness ratio depends on type of member as follows.

Type of member	λ_{max}
a) A member carrying compressive load resulting from dead & live load.	180
b) A tension member in which reversal of direct stress is due to loads other than wind or earthquake	180
c) A member subjected to compressive load due to wind or earthquake	250
d) A member normally acting as a tension member but in which reversal of stresses is due to wind or earthquake.	350
e) Compression flange of beam.	300

- Slenderness ratio is limited for the following reasons
- a) To account for uncertainty of member being subjected to unexpected vibration
- b) To account for accidental or construction loads.

Classification of cross-section

- A plate may buckle locally due to compressive stresses, this local buckling can be avoided before the limit state is achieved by limiting the width to thickness ratio of each element of the cross-section subjected to axial force, moment or torsion (shear)
- On this basis four classes of section are being defined.

Lesson 8 Dec 22.

A) Plastic section (Class-I)

- section which can develop plastic hinge and has rotational capacity required for failure of structure by formation of plastic mechanism.

B) Compact section (Class-II)

- cross-section which can develop plastic hinge but have inadequate plastic hinge, rotational capacity for formation of plastic mechanism due to local buckling.

C) Semi-compact section (Class-III)

- cross-section in which extreme fibres can reach yield stress (f_y) but cannot develop plastic moment of resistance due to local buckling.

d) Slender section (Class-IV)

- cross section in which local buckling can occur before yield stress is reached.
- In this case, effective section for design shall be taken by deducting the width of the compression ~~plate~~ plane element excess of semi-compact section limit.
- With different elements of the cross-section following under different class, the overall section shall be classified as governed by lowest class.

Effective Area (Sectional) (A_e)

- For compression members built up of plastic, compact or semi compact sections, the gross sectional area is taken as effective sectional area, however the holes which are not filled by rivets, bolts or pins will be deducted from gross area, to calculate effective area " A_e ".

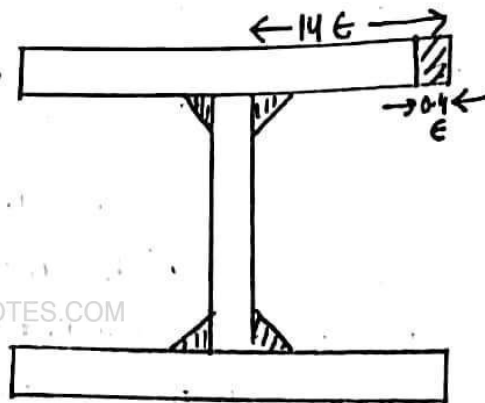
- But for slender section, the effective area is reduced to account for the effect of local buckling & is given by.

$$A_e = \sum b_e \cdot t$$

b_e = effective width of slender section (from code)
eg. $\rightarrow b_e = 15.7 \epsilon t_f$ for flange overhang of rolled I-section.

$b_e = 13.6 \epsilon t_f$ for flange overhang of welded I-section.

Here $\epsilon = \sqrt{\frac{250}{f_y}}$



Column Strength Curve

- For column with both ends hinged, the Euler critical load is given load

$$P_{cr} = \frac{\pi^2 E I_{min}}{l^2}$$

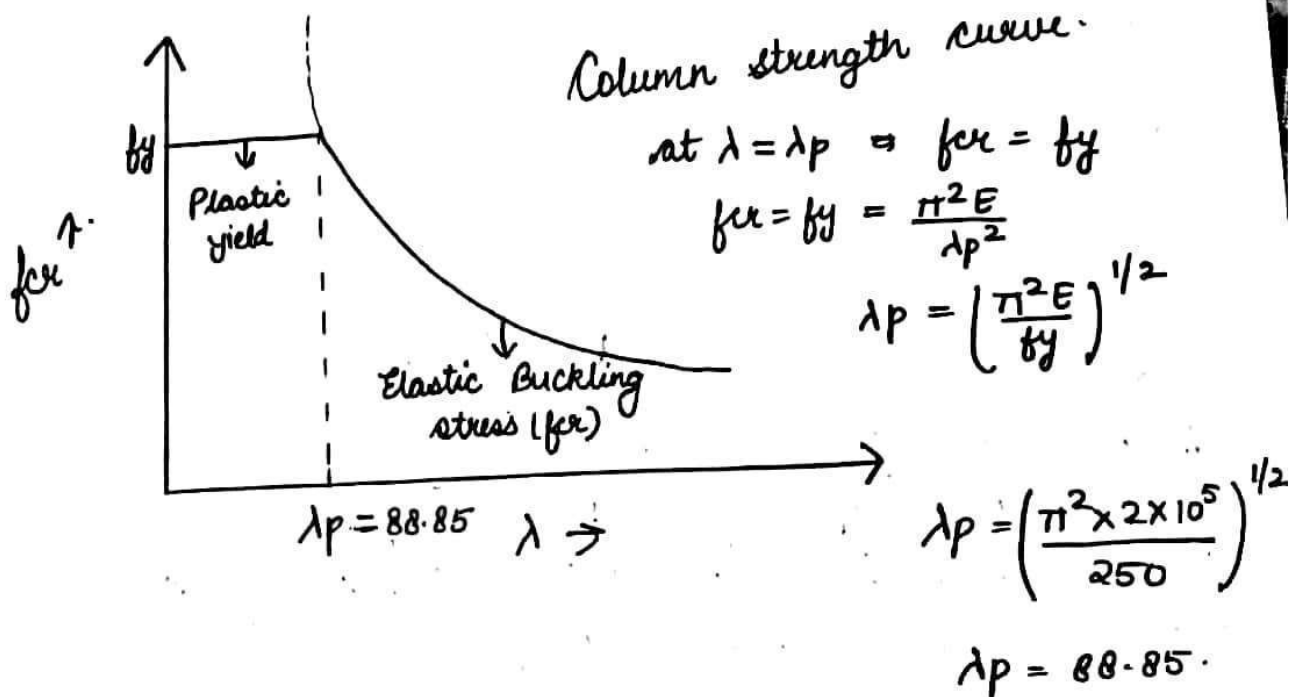
$$f_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E I_{min}}{l^2 \cdot A} = \frac{\pi^2 E A r_{min}^2}{A l^2}$$

$$= \frac{\pi^2 E}{(l/r_{min})^2} = \frac{\pi^2 E}{\lambda^2}$$

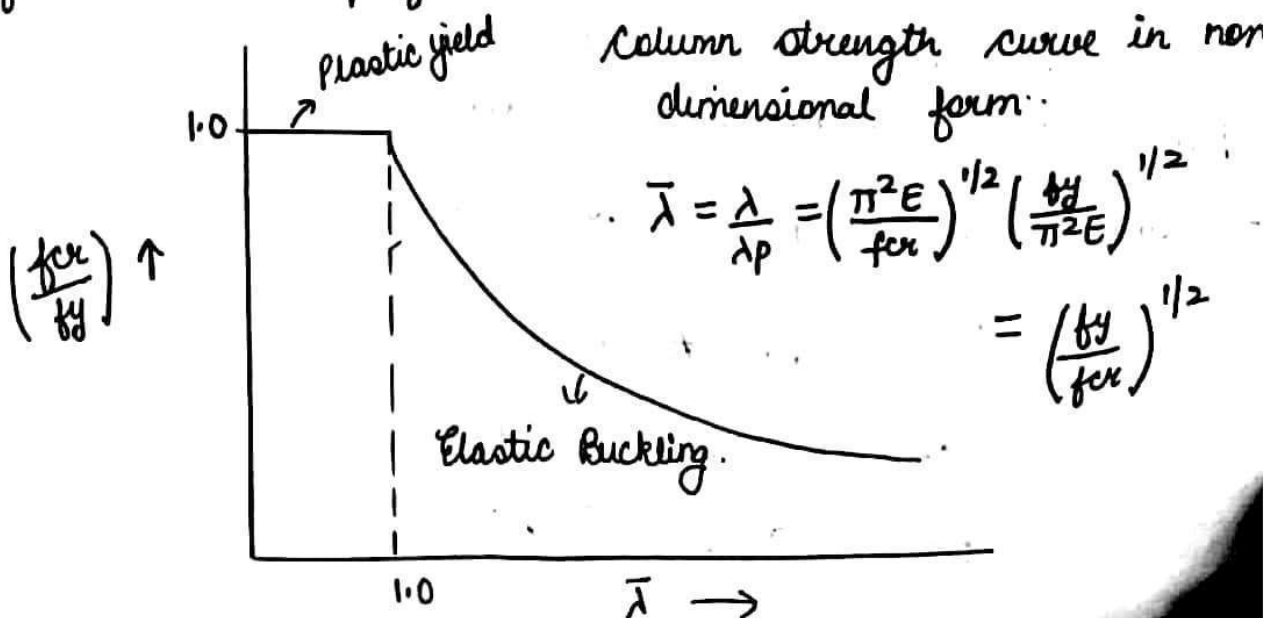
f_{cr} = Elastic buckling stress

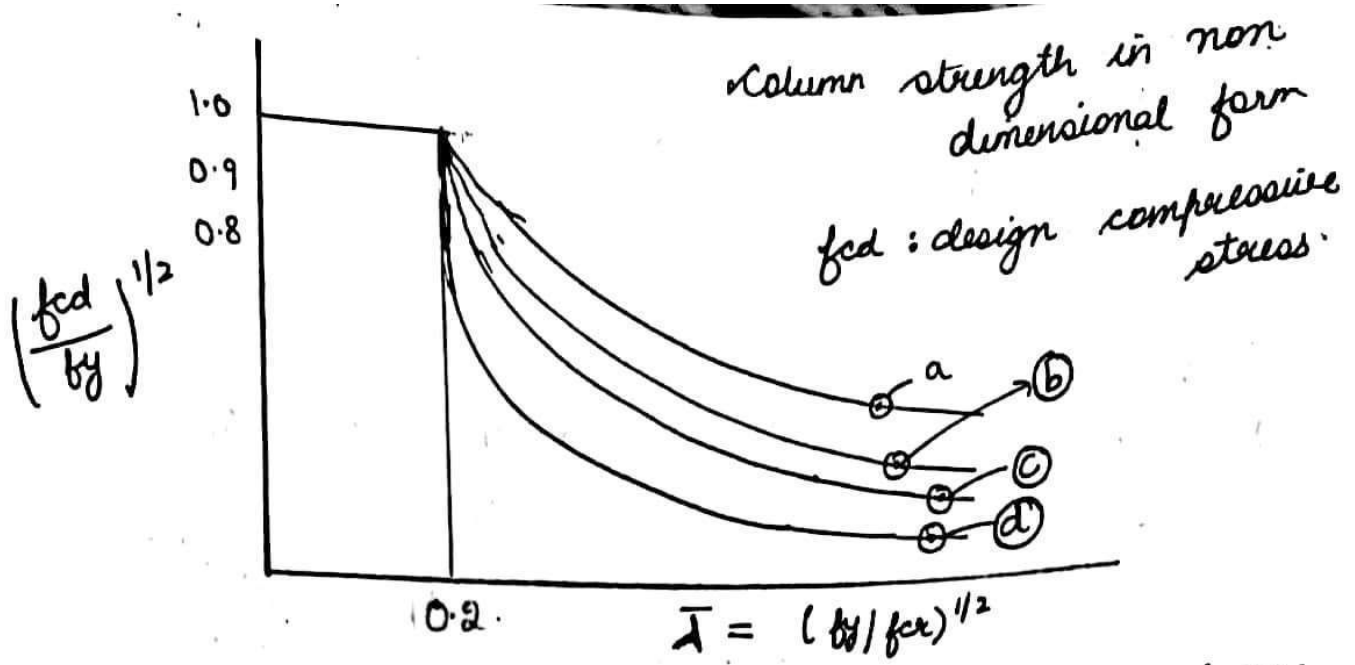
$\lambda = \frac{l}{r_{min}}$ = slenderness ratio

$f_{cr} \neq f_y$
and $f_{cr} \propto \frac{1}{\lambda^2}$



- Here ' λ_p ' is the slenderness ratio, defining the change over from yielding to buckling failure.
- IS 800-2007 proposes multiple column strength curve i.e. curve a, b, c, d in non-dimensional form based on PERRY ROBERTSON approach, where these curves are for different cross-section of columns and account for initial imperfections.





- Buckling strength of column is influenced by various factors like shape of cross-section, residual stress, initial imperfection, accidental eccentricity of load, end restraints & local buckling.
- Thus the behaviour of a real column is considerably different from that predicted by theoretical Euler's Eqⁿ.
- Here: curve (a) : represents perfect shape with least imperfection.
 curve (b) : represents shape with medium imperfection
 curve (c) : represents shape with lot of imperfection
 curve (d) : represents shape with maximum imperfection

Note : $\rightarrow$ for $\bar{\lambda} \leq 0.2$, the column can achieve the full plastic resistance i.e. Buckling effects may be ignored.

- The design compressive strength of column is given by

$$P_d = A_e \cdot f_{cd}$$

A_e = effective sectional area.

f_{cd} = design comp. stress.

' f_{cd} ' can be read from column strength curve given in code or can be found as

$$f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}$$

$$= \chi \frac{f_y}{\gamma_{m0}} \leq \frac{f_y}{\gamma_{m0}}$$

Here $\gamma_{m0} = 1.1$.

$$\text{Here } \chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}$$

$$\bar{\lambda} = \sqrt{\frac{f_y}{f_{cr}}}, \quad \text{for } \bar{\lambda} \geq 0.2, \quad \text{for } \bar{\lambda} > 0.2$$

$$\text{for } \bar{\lambda} > 0.2, \quad \text{for } \bar{\lambda} > 0.2$$

If $\bar{\lambda} < 0.2$, take $\bar{\lambda} = 0.2$.

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$$\phi = 0.5 (1 + \alpha (\bar{\lambda} - 0.2) + \bar{\lambda}^2).$$

α : Imperfection factor.

$\bar{\lambda}$ = Non dimensional slenderness ratio.

Buckling curve :	a	b	c	d
α :	0.21	0.34	0.49	0.76

- Values of α increases with imperfection.
- for larger values of α , smaller value of ' f_{cd} ' is obtained.

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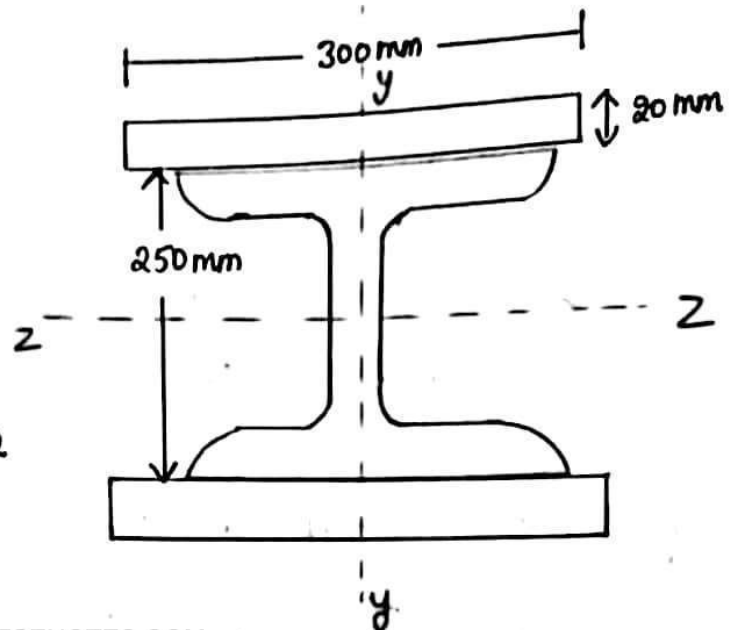
Q Calculate the value of least radius of gyration for a compound column consisting of ISHB 250 with cover plate $300\text{mm} \times 20\text{mm}$ on each flange. Properties of ISHB: 250 are as follows.
 $A = 6971\text{mm}^2$, $t_f = 99\text{mm}$, $t_w = 8.8\text{mm}$, $I_{zz} = 7983.9 \times 10^4\text{mm}^4$
 $I_{yy} = 2011.7 \times 10^4\text{mm}^4$.

$$r_{\min} = \sqrt{\frac{I_{\min}}{A}}$$

$$I_{\min} = I_{yy}$$

$$I_{yy\text{plates}} = \frac{20 \times 300^3 \times 2}{12}$$

$$= 9 \times 10^7\text{mm}^4$$



$$I_{yy} = 9 \times 10^7 + 2011.7 \times 10^4 = 11.0117 \times 10^7\text{mm}^4$$

$$A = 6971 + 2(300 \times 20) = 18971\text{mm}^2$$

$$r_{\min} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{11.0117 \times 10^7}{18971}} = 76.18\text{mm}$$

Q Calculate the design compressive load for a section ISHB 350, 3.5m height. The column is restrained in direction & in position at both ends. It is to be used as uncased column in a single storey building. use steel of grade Fe410.

Properties of ISHB 350 are

$$A = 9221\text{mm}^2, h = 350\text{mm}, b_f = 250\text{mm}$$

$$t_f = 11.6\text{mm}, t_w = 10.1\text{mm}, r_{zz} = 146.5\text{mm}$$

$$r_{yy} = 52.2\text{mm}$$

Use Perry Robertson Approach.

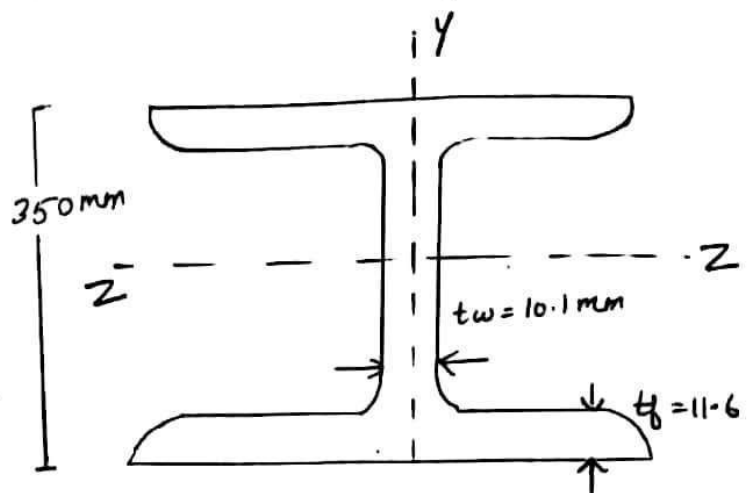
Since column is fixed
at ~~one~~ end
both

$$l_e = kl$$

$$l_e = 0.65 \times 3.5 \\ = 2.275 \text{ m}$$

$$r_{\min} = r_{yy} = 52.2 \text{ mm}$$

$$\lambda = \frac{l_e}{r_{\min}} = \frac{2.275 \times 10^3}{52.2} = 43.58$$



$$\frac{h}{b_f} = \frac{350}{250} = 1.4 > 1.2 \text{ and } t_f = 11.6 \text{ mm} < 40 \text{ mm}$$

and buckling about YY.
- Use buckling curve - b

Now using Perry Robertson Approach.

$$f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \leq \frac{f_y}{\gamma_{m0}}$$

$$\phi = 0.5 (1 + \alpha (\bar{\lambda} - 0.2) + \bar{\lambda}^2)$$

$$\alpha = 0.34 \text{ for buckling curve (b)}$$

$$\bar{\lambda} = \sqrt{\frac{f_y}{f_{cr}}} \text{ , } f_{cr} = \frac{\pi^2 E}{\lambda^2}$$

$$\bar{\lambda} = \sqrt{\frac{250}{2 \times 10^5}} \times \frac{43.58}{\pi} = 0.49 > 0.2$$

$$\phi = 0.5 (1 + 0.34 (0.49 - 0.2) + 0.49^2)$$

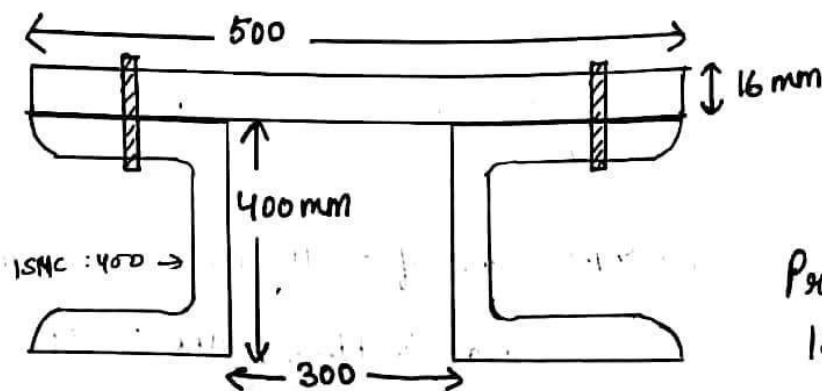
$$\phi = 0.669$$

$$f_{cd} = \frac{250 / 1.1}{0.669 + \sqrt{0.669^2 - 0.49^2}} = 202.113 \text{ N/mm}^2 < \frac{250}{1.1} \\ \leq 227 \text{ N/mm}^2$$

OK

Design compressive strength $P_d = A_e f_{cd}$
 $= 9221 \times 202.113 \times 10^{-3}$
 $= 1863.69 \text{ kN}$

Q Calculate the design compressive load which the member as given can support, if the member is of 5.5m effective length. Use steel of grade 410.



Properties of
ISMC : 400

$$A = 6293 \text{ mm}^2$$

$$b_f = 100 \text{ mm}$$

$$t_f = 15.3 \text{ mm}$$

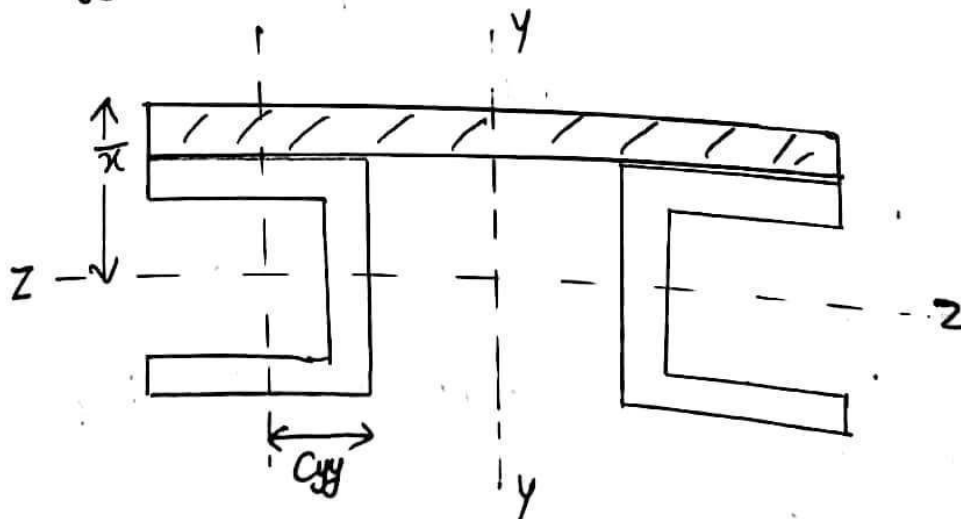
$$t_w = 8.6 \text{ mm}$$

$$I_{zz} = 15082.8 \times 10^4 \text{ mm}^4$$

Ky x	30	40	50	60	70
fcd	211	198	183	168	152

$$I_{yy} = 504.8 \times 10^4 \text{ mm}^4$$

$$C_{yy} = 24.2 \text{ mm}$$



$$I_{yy} = \frac{16 \times 500^3}{12} + 2 [504.8 \times 10^4 + 6293 (150 + 24 \cdot 2)^2]$$

$$= 5.587 \times 10^8 \text{ mm}^4$$

$$\bar{x} = \frac{A_1 \bar{x}_1 + A_2 \bar{x}_2}{A_1 + A_2} = \frac{500 \times 16 \times 8 + 2 \times 6293 (200 + 16)}{500 \times 16 + (2 \times 6293)}$$

$$= 135.168 \text{ mm}$$

$$I_{33} = \frac{500 \times 16^3}{12} + 500 \times 16 (135.168 - 8)^2 + 2 \times 6293 [15682.8 \times 10^4 + 6293 (216 - 135.168)^2]$$

$$I_{33} = 5.134 \times 10^8 \text{ mm}^4$$

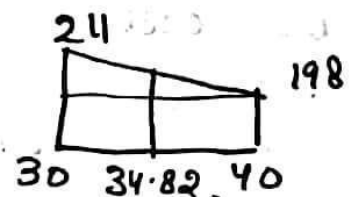
$$I_{33} < I_{yy} \Rightarrow I_{\min} = I_{33} = 5.134 \times 10^8 \text{ mm}^4$$

$$\lambda = \frac{kl}{r_{\min}} = \frac{l_e}{r_{\min}} = \frac{5.5 \times 10^3}{157.9} = 34.82$$

$$r_{\min} = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{5.134 \times 10^8}{500 \times 16 + 2 \times 6293}} = 157.9 \text{ mm}$$

for $\lambda = 34.82$ from given table.

$$f_{cd} = 204.734 \text{ N/mm}^2$$



Design compressive strength

$$P_d = A_e f_{cd}$$

$$= (500 \times 16 + 2 \times 6293) \times 204.734 \times 10^{-3}$$

$$\Rightarrow 4214.65 \text{ kN}$$

- Recommendations for Angle struts
- In case of bolted, riveted or welded truss & braced frames, effective length of the compression member shall be taken as (0.7-1.0) times the distance between the centre of connections, depending on the degree of end restraints provided.

a) Single Angle struts

- The compression in single angle may be transformed either concentrically to its C.G. through end gusset or eccentrically by connecting one of its leg to the gusset plate.

(i) Concentric loading.

- In this case, calculation of design compressive stress will be done by the formulation already discussed considering class (c) buckling curve
- Effective length for single angle in this case will be taken as "l" unless mentioned otherwise.

ii) Eccentric loading (loading through one leg.)

- limit state consider flexural torsional buckling in this case.
- The flexural torsional buckling strength of single angle loaded in compression through one of its leg may be computed using equivalent slenderness ratio. (λ_e)

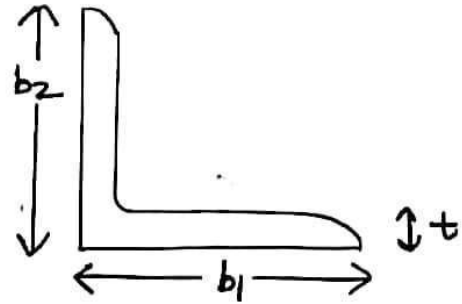
$$\lambda_e = \sqrt{K_1 + K_2 \lambda_{vv}^2 + K_3 \lambda \phi^2}$$

here K_1, K_2, K_3 are constants depending on end restraints.

$$\lambda_{vv} = \frac{l / r_{vv}}{\epsilon \sqrt{\frac{\pi^2 E}{250}}} \quad , \quad \epsilon = \sqrt{\frac{250}{f_y}}$$

$r_{uv} \Rightarrow$ minimum radius of gyration

$$\lambda \phi = \frac{(b_1 + b_2) / 2t}{\epsilon \sqrt{\frac{\pi^2 E}{250}}}$$



No. of bolts at each end condition	Gusset plate or connecting member fixity	K_1	K_2	K_3
(i) ≥ 2 bolts	Hinged	0.7	0.6	5
	Fixed	0.2	0.35	20
(ii) 1 bolt	Hinged	1.25	0.5	60
	Fixed	0.15	0.35	20

Note: For welded struts, joint may be treated as fixed condition with ≥ 2 bolts.

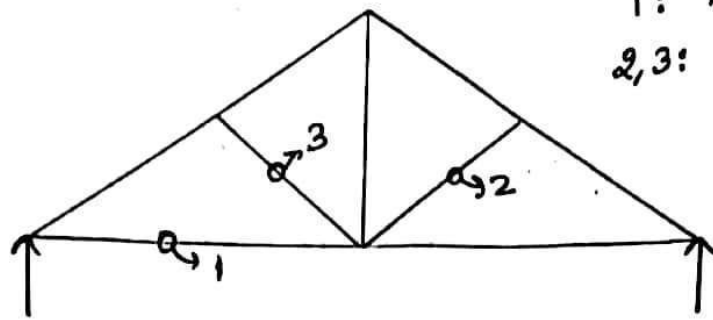
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b) Double Angle strut

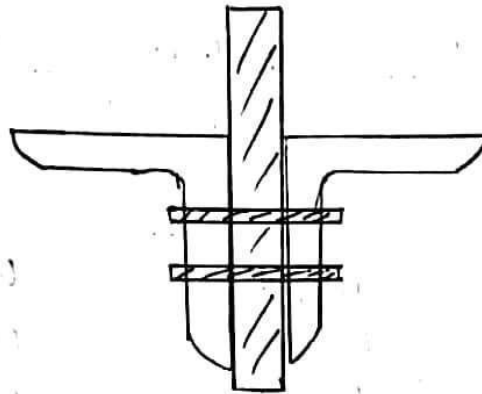
Case (i): Double angle discontinuous strut, connected back to back on opposite side of gusset plate or a section by not less than 2 bolts or weld in line along the angle at each end can be treated as axially loaded & k will be calculated as discussed

$l_{eff} = (0.7 - 0.85)l$: for in plane buckling.

$l_{eff} = l$: for out of plane buckling.



1: continuous strut
2,3: discontinuous strut



Case (ii) Double angle discontinuous truss connected back to back on same side of gusset plate by more than one belt or rivet or by weld, should be designed as single angle.

Case (iii) Continuous double angle strut are designed as axially loaded compression member.

$$l_{eff} = (0.7-1.0)L : \text{for in plane buckling.}$$

$$l_{eff} = L : \text{for out of plane buckling.}$$

Design steps of Compression members.

(i) Find the factored load

(ii) Choose f_{cd} between (0.4-0.6) f_y
or

Instead of choosing f_{cd} value, choose initial trial slenderness ratio $\lambda = \frac{l_{eff}}{r_{min}}$ from the given range.

Type of member	$\lambda = l_{eff} / r_{min}$
- single angle	100 - 150
- double angle	80 - 120
- single channel	90 - 150
- double channel	40 - 80
- single I-section	80 - 150
- double I-section	30 - 60
- Rolled section	90 - 150 (120)
- Built up section	50 - 60 (60)

(iii) after choosing the slenderness ratio we choose the buckling class & find 'fcd'.

(iv) Calculate A_{gross} required.

$$A_{gross} = \frac{\gamma_f P}{f_{cd}} \quad P: \text{working load}$$

(v) Choose trial section

(vi) for the chosen section calculate slenderness ratio

(vii) Find f_{cd} for the above slenderness ratio

(ix) Check $f_{cd} \text{ provided} \times A_g \geq \gamma_f \cdot P$, if not $\rightarrow$ Redesign.

Note: $\Rightarrow$ 1) we also check for max. slenderness ratio

2) If the design compressive strength of column exceed, the factored load over the column by more than 5% the section needs a redesign (as it is oversafe & uneconomical)

Q An ISA : $100 \times 100 \times 6$ mm (Fe 410) is used as a strut in a truss. The length of the strut b/w interconnection at each end is 3m. Compute the strength of eccentrically loaded strut if,

a) It is connected by 2 bolts at each end

b) " " " 1 bolt " " "

c) It is welded at each end.

Properties of ISA $100 \times 100 \times 6$ mm are $A = 1167 \text{ mm}^2$

$$r_{zz} = 30.9 \text{ mm}, \quad r_{yy} = 30.9 \text{ mm}, \quad r_{uu} = 39.1 \text{ mm}, \quad r_{vv} = 19.5 \text{ mm}$$

Assume gusset plate to be fixed.

$$r_{\min} = r_{vv} = 19.5 \text{ mm}.$$

for eccentric loading, check is applied flexural torsional buckling using equivalent slenderness ratio.

$$\lambda_e = \sqrt{K_1 + K_2 \lambda_{vv}^2 + K_3 \lambda_\phi^2}$$

$$\lambda_{vv} = \frac{L / r_{vv}}{\epsilon \sqrt{\frac{\pi^2 E}{250}}}, \quad \epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$\lambda_{vv} = \frac{3 \times 10^3 / 19.5}{\sqrt{\frac{\pi^2 \times 2 \times 10^5}{250}}} = 1.73.$$

$$\lambda_\phi = \frac{(b_1 + b_2) / 2t}{\epsilon \sqrt{\frac{\pi^2 E}{250}}} = \frac{(100 + 100) / 2 \times 6}{\sqrt{\frac{\pi^2 \times 2 \times 10^5}{250}}} = 0.188.$$

a) for 2 bolts and fixed connection

$$K_1 = 0.2, \quad K_2 = 0.35, \quad K_3 = 20$$

$$\lambda_e = \sqrt{0.2 + 0.35 \times 1.73^2 + 20 \times 0.188^2} = 1.398702$$

Using Perry Robertson Approach, $f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + \sqrt{\phi^2 - \lambda_e^2}}$

$$\phi = 0.5 [1 + \alpha (\lambda_e - 0.2) + \lambda_e^2]$$

for buckling class (c), $\alpha = 0.49$

$$\phi = 0.5 [1 + 0.49(1.398 - 0.2) + (1.398)^2] \\ = 1.77$$

$$f_{cd} = \frac{250}{1.1(1.77 + \sqrt{1.77^2 - 1.398^2})} = 80 \text{ N/mm}^2 < \frac{250}{1.1} \\ < 227.27 \frac{\text{N}}{\text{mm}^2} \\ \underline{\underline{\text{OK}}}$$

Design compressive strength, $P_d = f_{cd} \times A$
 $= 80 \times 1167 \times 10^{-3} = 93.36 \text{ kN}$

b) for 1 bolt and fixed connection

$$K_1 = 0.75, \quad K_2 = 0.35 \quad K_3 = 20$$

$$\lambda_e = \sqrt{0.75 + 0.35 \times 1.73^2 + 20 \times 0.188^2} = 1.58 > 0.2$$

$$\phi = 0.5 [1 + 0.49(1.58 - 0.2) + 1.58^2] = 2.09$$

$$f_{cd} = \frac{250/1.1}{2.09 + \sqrt{2.09^2 - 1.58^2}} = 65.7 < \frac{250}{1.1} \\ < 227.27 \text{ N/mm}^2 \\ \underline{\underline{\text{OK}}}$$

Design compressive strength $P_d = f_{cd} \times A$
 $= 65.7 \times 1167 \times 10^{-3} = 76.7 \text{ kN}$

c) Welded strut joints are treated as fixed joints with ≥ 2 bolts.

$$P_d = 93.36 \text{ kN}$$

Q Design a single angle discontinuous strut to carry a factored load, of 65 kN compressive in nature. The length of strut is 3 m between intersections. It is connected to a 12 mm thick gusset plate by 20 mm dia bolt, 4.6 grade bolts, use steel of grade Fe 410.

sections available are: ISA: 70 x 70 x 6 mm

$$A = 806 \text{ mm}^2, r_{uu} = 26 \text{ mm}, r_{vv} = 13.1$$

ISA: 70 x 70 x 8 mm.

$$A = 1058 \text{ mm}^2, r_{uu} = 26.7 \text{ mm}, r_{vv} = 13.5 \text{ mm}.$$

Sol 1) Factored load = 65 kN

(ii) Assume $\lambda = \frac{l_{eff}}{r_{min}} = 120$, buckling curve of class C

$$\text{and } f_y = 250 \text{ N/mm}^2$$

$$\chi = 0.368 \text{ from Table 8(c)}$$

$$f_{cd} = \chi \frac{f_y}{\gamma_{mo}} = 0.368 \times \frac{250}{1.1} = 83.6 \text{ N/mm}^2$$

$$(A_{gross})_{reqd.} = \frac{F}{f_{cd}} = \frac{65 \times 10^3}{83.6} = 777.51 \text{ mm}^2$$

Adopt ISA 70 x 70 x 6 mm $A_g = 806 \text{ mm}^2$

Design strength of bolt (V_{db}).

$$V_{db} = \frac{f_{ub}}{\sqrt{3} \times 1.25} \times 0.78 \times \frac{\pi d^2}{4}$$

$$= \frac{400}{\sqrt{3} \times 1.25} \times 0.78 \times \frac{\pi (20)^2}{4} \times 10^{-3}$$

$$= 45.272 \text{ kN}$$

$$V_{dpb} = 2.5 K_b d t \frac{f_u}{1.25}$$

$$t_{\min} = \min \{12, 6\} = 6 \text{ mm}$$

$$K_b = \min \left\{ \frac{e}{3d_0}, \frac{p}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1 \right\}$$

$$\text{assume } e = e_{\min} = 1.5d_0 = 1.5(20+2) = 33 \text{ mm}$$

$$p = p_{\min} = 2.5d_0 = 2.5 \times 20 = 50 \text{ mm}$$

$$K_b = 0.5$$

$$V_{dpb} = \frac{2.5 \times 0.5 \times 20 \times 6 \times \frac{410}{1.25}}{\times 10^3}$$

$$= 49.2 \text{ kN}$$

$$V_{db} = \min \{V_{deb}, V_{dpb}\} = 45.272 \text{ kN}$$

$$\text{No. of bolts at each end} = \frac{F}{V_{db}} = \frac{65}{45.272} = 1.43 \approx 2$$

$$\text{for 2 bolts \& fixed end, } K_1 = 0.2, K_2 = 0.35,$$

$$K_3 = 20$$

$$\lambda e = \sqrt{K_1 + K_2 \lambda_{vv}^2 + K_3 \lambda_{\phi}^2}$$

$$\lambda_{vv} = \frac{V_{vv}}{E \sqrt{\pi^2 E / 250}}, \quad e = \sqrt{\frac{250}{f_y}}$$

$$\lambda_{vv} = \frac{3 \times 10^3 / 13.6}{\sqrt{\frac{\pi^2 \times 2 \times 10^5}{250}}} = 2.48$$

$$\lambda_{\phi} = \frac{(b_1 + b_2) / 2t}{E \sqrt{\frac{\pi^2 E}{250}}} = \frac{(70 + 70) / 2 \times 6}{1 + \left(\frac{\pi^2 \times 2 \times 10^5}{250} \right)^{1/2}} = 0.13$$

$$\lambda e = \sqrt{0.2 + 0.35(2.48)^2 + 20(0.13)^2} = 1.64 > 0.2$$

$$\alpha = 0.49$$

$$\phi = 0.5 [1 + \alpha(\lambda e - 0.2) + \lambda e^2]$$

$$= 0.5 [1 + 0.49(1.64 - 0.2) + 1.64^2]$$

$$\phi = 2.19$$

$$f_{cd} = \frac{f_y / \gamma_{mo}}{\phi + \sqrt{\phi^2 - \lambda e^2}} = \frac{250/1.1}{2.19 + \sqrt{2.19^2 - 1.64^2}} = 62.43 \text{ N/mm}^2 < 227.27 \text{ OK.}$$

$$P_d = f_{cd} \times A_g = 62.43 \times 806 \times 10^{-3} = 50.3 \text{ kN} < 65 \text{ kN}$$

Not Safe.

Hence adopt ISA: 70x70x8 mm, $A_g = 1058 \text{ mm}^2$
 Design strength of bolt calculation is same,
 hence no. of bolts = 2.

$$\lambda_{vv} = \frac{1/\lambda_{vv}}{e \sqrt{\frac{\pi^2 E}{250}}} = \frac{3 \times 10^3 / 13.5}{\left(\frac{\pi^2 \times 2 \times 10^5}{250} \right)^{1/2}} = 2.5$$

$$\lambda_{\phi} = \frac{(70+70)/(2 \times 8)}{1 \times \left(\frac{\pi^2 \times 2 \times 10^5}{250} \right)^{1/2}} = 0.098$$

$$\lambda e = \sqrt{0.2 + 0.35 \times 2.5^2 + 20 \times (0.098)^2} = 1.60$$

$$\phi = 0.5 [1 + 0.49(1.60 - 0.2) + 1.60^2] = 2.12$$

$$f_{cd} = \frac{250/1.1}{2.12 + \sqrt{2.12^2 - 1.6^2}} = 64.6 \text{ MPa} < 227.27 \text{ MPa}$$

OK

$$P_d = f_{cd} \times A_g = 64.6 \times 1058 \times 10^{-3} = 68.35 \text{ kN} > 65 \text{ kN}$$

Hence Safe.

Adopt angle ISA: 70x70x8 mm, $A_g = 1058 \text{ mm}^2$ with 2 bolts.

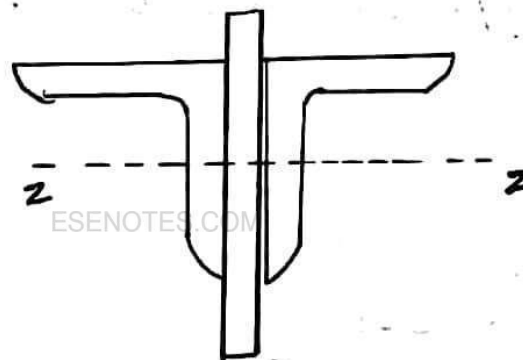
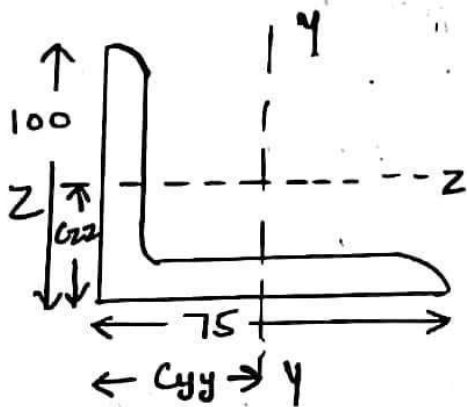
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Q Calculate the strength of the discontinuous strut of length 3.2 m. The strut consists of 2 unequal angles 100 mm x 75 mm x 8 mm with long legs connected and placed on the opposite sides of gusset plate 10 mm thick. Use steel of grade 410. Properties of ISA 100 x 75 x 8 mm are as follows.

$$A = 1336 \text{ mm}^2, I_{yy} = 63.3 \times 10^4 \text{ mm}^4, I_{zz} = 131.6 \times 10^4 \text{ mm}^4.$$

$$C_{yy} = 18.7 \text{ mm}, C_{zz} = 31 \text{ mm}.$$

Since angles are connected on opposite sides of gusset plate therefore it is analysed as concentrically loaded column.



$$I_{yy} = 2 (63.3 \times 10^4 + 1336 (18.7 + 5)^2)$$

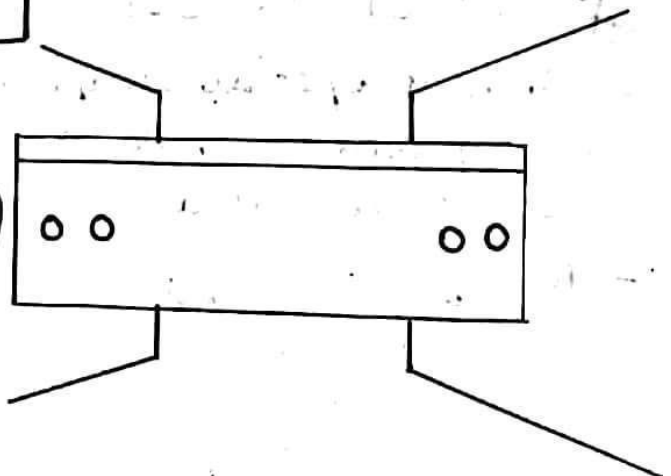
$$= 2.76 \times 10^6 \text{ mm}^4.$$

$$I_{zz} = 2 (131.6 \times 10^4)$$

$$= 2.63 \times 10^6 \text{ mm}^4.$$

for in plane buckling, $l_e = 0.85 l$
 $= 0.85 \times 3.2 = 2.72 \text{ m}.$

$$\lambda_1 = \frac{l_e}{r} = \frac{l_e}{\sqrt{\frac{I_{zz}}{A}}} = \frac{2.72 \times 10^3}{\sqrt{\frac{2.63 \times 10^6}{2 \times 1336}}} = 86.7$$



for out of plane buckling, $l_e = l = 3.2 \text{ m}$.

$$\lambda_2 = \frac{l_e}{r} = \frac{l_e}{r_{yy}} = \frac{l_e}{\sqrt{\frac{I_{yy}}{A}}} = \frac{3.2 \times 10^3}{\sqrt{\frac{2.76 \times 10^6}{2 \times 1336}}} = 99.6$$

$$\lambda = \max(\lambda_1, \lambda_2) = 99.6$$

for Buckling class C & $\lambda = 99.6$ & $f_y = 250 \text{ MPa} \Rightarrow \chi = 0.473$
(from table B(C) IS 800)

$$f_{cd} = \chi \frac{f_y}{\gamma_{m0}} = 0.473 \times \frac{250}{1.1} = 107.5 \text{ N/mm}^2$$

Design compressive strength, $P_d = f_{cd} \times A$
 $= 107.5 \times 2 \times 1336 \times 10^{-3}$
 $= 287.2 \text{ kN}$

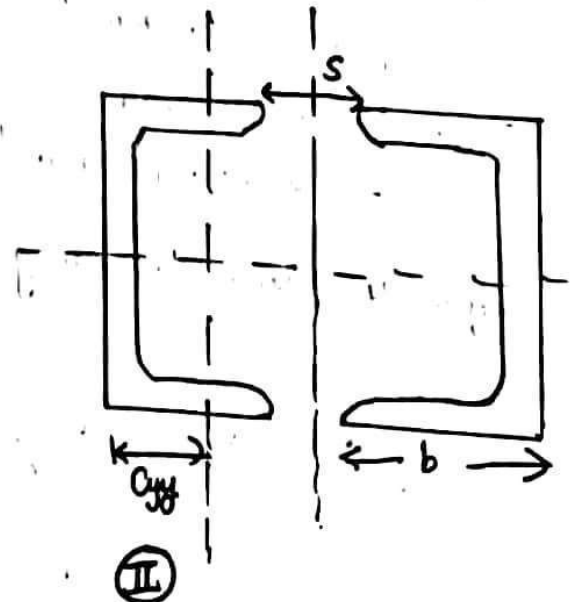
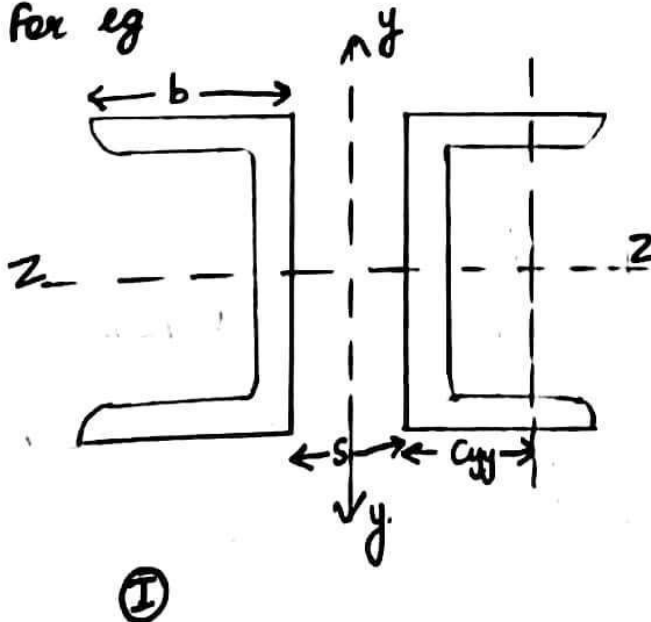
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Lesson 12 Dec 26

BUILT-UP COLUMNS (Latticed columns).

- When a special shape or large radius of gyration is required in two directions, a built up section is used / fabricated.

- For eg



- For economical design of heavily loaded long columns the least radius of gyration of column section is increased to maximum.

$$(r_{yy} > r_{zz}) \text{ [slightly]} \Rightarrow r_{\min} = r_{zz}$$

- This is achieved by increasing the spacing "s" where y-y is perpendicular to the plane of lattice system and z-z is in plane of lattice system.

for case (I)

$$I_{yy} = 2 \left[I_{yy}^{\text{one}} + A \left(C_{yy} + \frac{s}{2} \right)^2 \right]$$

$$I_{zz} = 2 (I_{zz}^{\text{one}})$$

for case (II)

$$I_{yy} = 2 \left[I_{yy}^{\text{one}} + A \left(\frac{s}{2} + (b - C_{yy}) \right)^2 \right]$$

$$I_{zz} = 2 (I_{zz}^{\text{one}})$$

- Different components of the built up sections are connected by some connecting system known as "LATTICE SYSTEM".

- The commonly used lattice are

(a) Lacing Bars

(b) Battered Plate

- Lattice system ensures unified behaviour of built up sections.

Note: Lacing bars is generally preferred in case of eccentric loads and battering is preferred in case of axial loads.

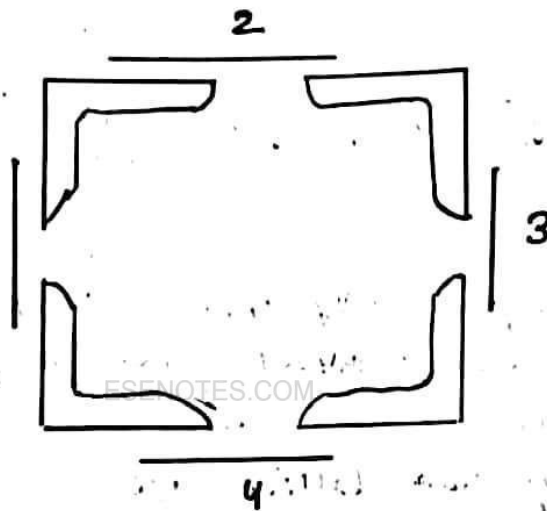
(A) LACING

- Single lacing non intersecting arrangement is better than single lacing intersecting arrangement as failure of bolt connecting the lacing bar in later arrangement will double the length "c" & increases the chances of buckling of individual section.

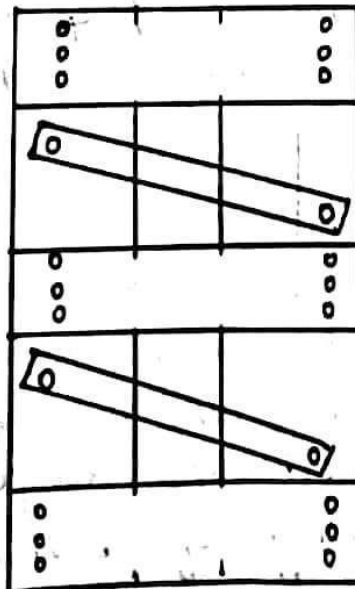
- Double lacing is provided when even increasing the inclination angle of lacing with vertical, the spacing "c" is not reduced significantly to safeguard against local buckling.
- Lacing should not be varied throughout the length of strut, i.e. the cross-section area and inclination with longitudinal axis (θ), should not change along the length.
- Single lacing on opposite side of main component should preferably in same direction, so that one is mirror image of the other; however on adjacent faces it must be staggered.

Face

1 & 3, 2 & 4 mirror image
1 & 2 & 3 & 4 staggered



- At top & bottom of laced column, end battens are provided.
- cross members are not desirable i.e. lacing & battering is not clubbed / combined.



(X)

11 Recommendations for Lacing.

- Slenderness ratio of laced column is 1.05(λ_0) (increased by 5%)

λ_0 : Maximum actual slenderness ratio of column.

- $r_{yy} > r_{zz}$ (slightly) such that $r_{\min} = r_{zz}$.
- Angle of inclination of lacing (θ) with longitudinal axis of column should be in range of 40° - 70° .
- The max. spacing of lacing bar (C) should be such that slenderness ratio of the component member

$$\left. \begin{array}{l} \frac{C}{r_{\min}} \neq 50 \\ \neq 0.7 \lambda_{\text{whole}} \end{array} \right\} \text{Same in Batten also}$$

Here λ_{whole} : slenderness ratio of whole section ($1.05\lambda_0$).

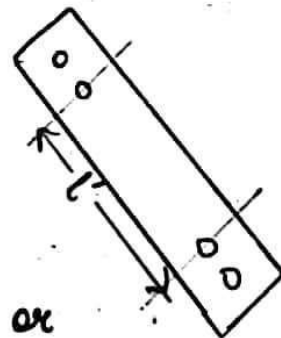
- Effective length of lacing (l_e)

$l_e = l'$: single lacing

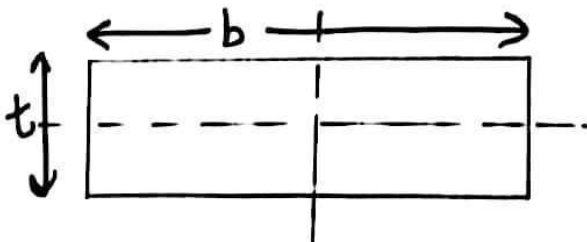
$l_e = 0.7l'$: double lacing

$l_e = 0.7l'$: welded lacing

l : distance b/w inner ends of bolts or welds.



- Slenderness ratio of lacing bar : $\frac{l_e}{r_{\min}} \neq 145$



$$r_{\min} = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{\frac{bt^3}{12}}{bt}}$$

$$r_{\min} = \frac{t}{2\sqrt{3}}$$

$$\frac{l_e}{\frac{t}{2\sqrt{3}}} \neq 145$$

- Minimum width of lacing bar depends on nominal dia of bolt as follows.

Nominal dia of bolt (mm)	Minimum width. (b) mm
22	65
20	60
18	55
16	50

Note: $\rightarrow$ "b" is 3 times bolt dia & nearest multiple of 5.

- Minimum thickness of lacing bar (t)

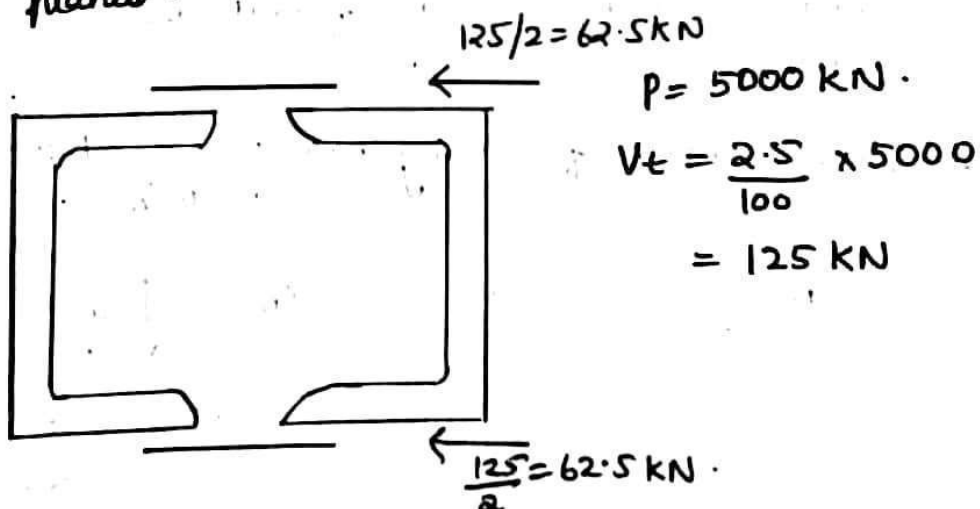
$t \geq 1/40$: for single lacing

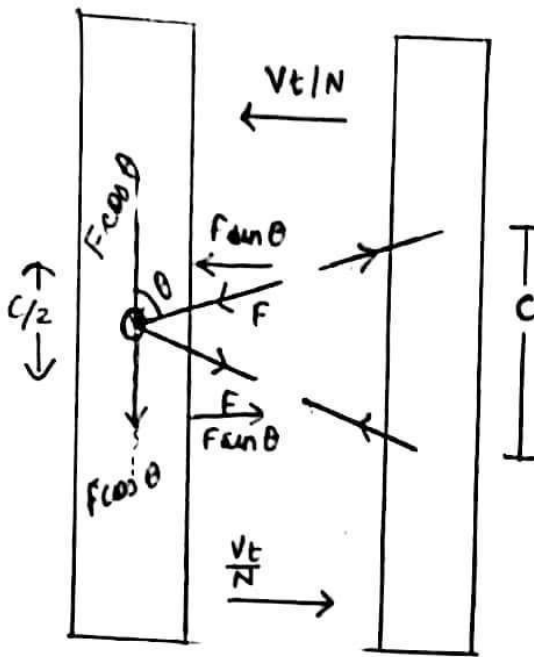
$t \geq 1/60$: for double lacing.

- Force for which lacing bar should be designed is shear force in column arising due to bending, lateral loading or accidental eccentricity.

- The transverse shear (V_t) is taken as 2.5% of axial loading & this shear is divided equally in all parallel planes.

For eg.





$$\frac{V_t}{N} = F \sin \theta$$

$$F = \frac{V_t}{N} \operatorname{cosec} \theta \quad (\text{force generated in one lacing})$$

N = no. of parallel plates

$N = 2$: for single lacing

$N = 4$: for double lacing.

Note: - At one joint if two lacing bars are meeting, force in one will be tensile & in other will be compressive & equal magnitude.

- Check for tension

$$F \geq \frac{0.9 f_u (b - d_o) t}{1.25}$$

$$\geq \frac{f_y (b \times t)}{1.1}$$

- Check for compression

$$F \geq f_{cd} \times b t$$

f_{cd} is function of (λ_{lacing} , α)

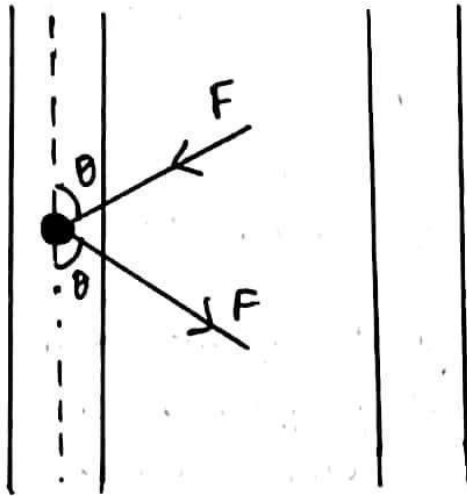
$$\text{where } \lambda_{lacing} = \frac{l_e}{r_{min}} = \frac{l_e}{t/2\sqrt{3}}$$

α : Imperfection factor corresponding to buckling class ©

Lesson 14 Dec 28

- Design for connection

a)

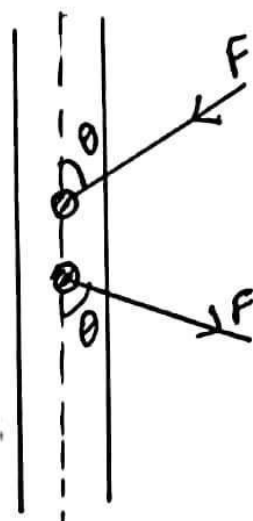


$$n = \frac{2F \cos \theta}{V_{db}} = \frac{2 \frac{V_t}{N} \operatorname{cosec} \theta \cos \theta}{V_{db}}$$

$$n = \frac{2 \frac{V_t}{N} \cot \theta}{V_{db}}$$

n = no. of bolts

b)



$$n = \frac{F}{V_{db}} = \frac{\frac{V_t}{N} \operatorname{cosec} \theta}{V_{db}}$$

$$n = \frac{\frac{V_t}{N} \operatorname{cosec} \theta}{V_{db}}$$

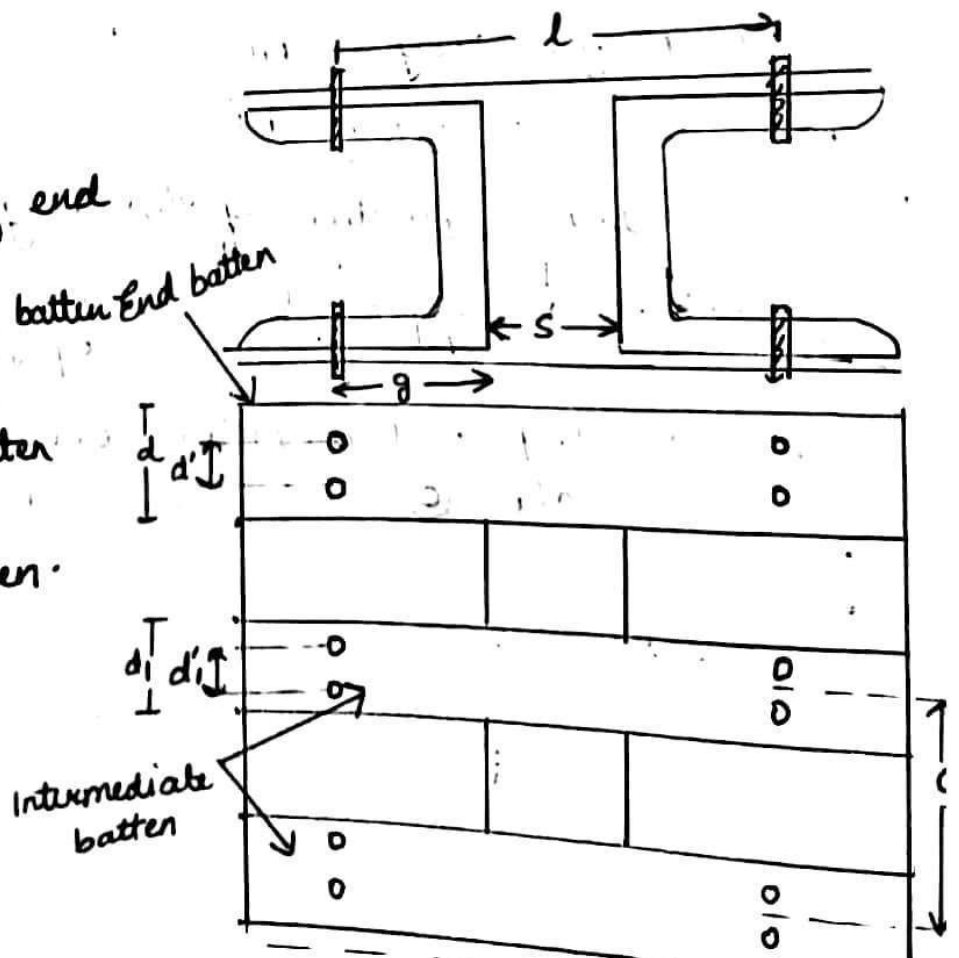
B) BATTENS

d = overall depth of end batten.

d' = eff. depth of end batten.

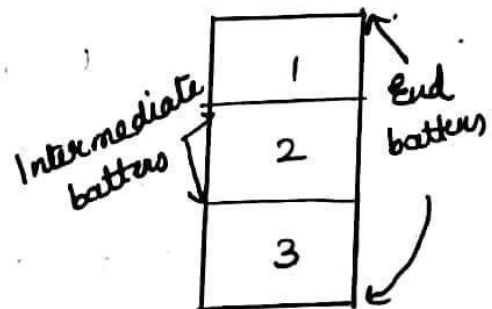
d_1 = overall depth of intermediate batten.

d'_1 = eff. depth of intermediate batten.



- Battened columns are uneconomical as compared to laced columns.
- Battens on opposite face should be mirror image.
- $r_{yy} > r_{zz}$ (slightly) spacing change.
- No. of battens should be such that, column is divided into not less than 3 parts longitudinally i.e. minimum no. of intermediate battens should be 2.

- Effective slenderness ratio of battened column is $1.1\lambda_0$ (Increased by 10%).
where λ_0 : maximum slenderness ratio of column.

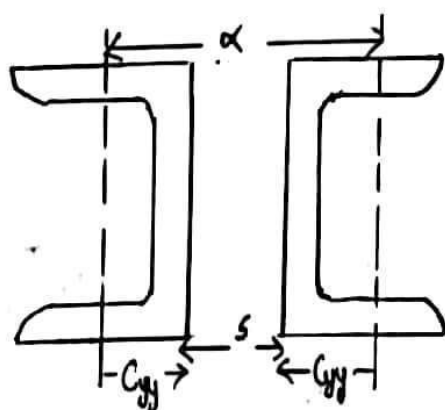


- Battens are not recommended for a column subjected to an eccentric load in plane of connecting system

$$\frac{C}{r_{\min}} \neq 50$$

$$\neq 0.7 \lambda_{\text{whole}}$$

- The effective depth of the end batten should not be less than the distance b/w centroid of component member & it should also be ^{not} less than twice the width of the component member in the plane of the batten (b).



for end battens

$$d' \neq \alpha$$

$$d' \neq 2b$$

$$d = d' + 2 \text{ edge d/s. (e)}$$

$$\alpha = S + 2C_{yy}$$

Similarly for intermediate battens.

$$d_1' \neq \frac{3}{4} \alpha$$

$$d_1' \neq 2b$$

$$d_1 = d_1' + 2 \text{ edge distance } (e)$$

where l = distance b/w innermost connecting line of bolts or welds.

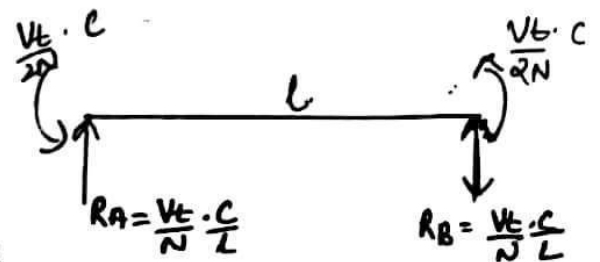
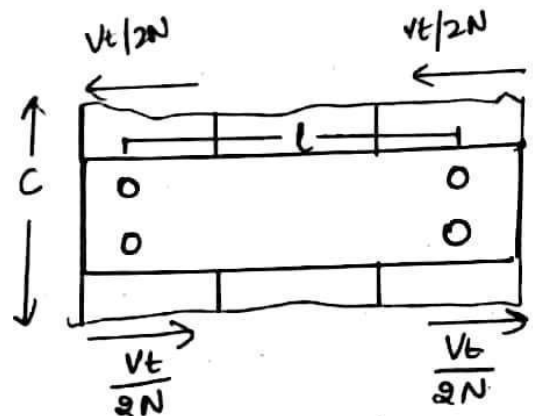
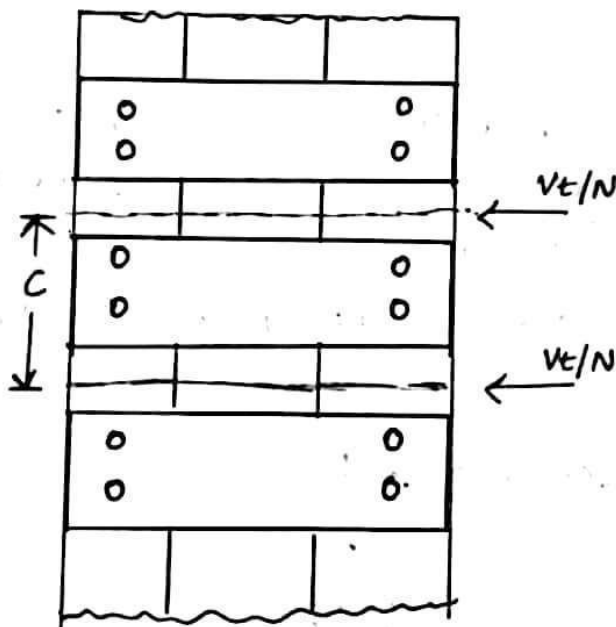
- Thickness of battens (t) $\neq \frac{l}{50}$

$$l = s + 2g$$

- Battens should be designed to carry the bending moment & shear arising out of transverse shear (V_t)

where $V_t = 2.5\%$ of axial load.

- The transverse shear will be equally divided b/w the parallel planes of battens.



$$\sum M_B = 0$$

$$R_A \cdot l = \frac{2V_t}{2N} \cdot C$$

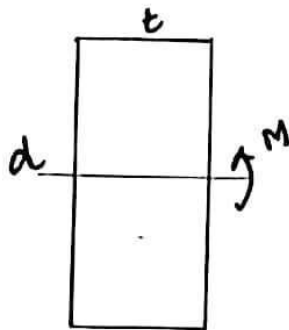
$$R_A = \frac{V_t}{N} \cdot \frac{C}{l} \Rightarrow R_A + R_B = 0 \quad R_A = -R_B$$

- Design BM for batten $(M) = \frac{V_t \cdot c}{2N}$

- Design shear force for batten $(V_b) = \frac{V_t \cdot c}{Nl}$

$N = \text{no. of parallel planes} = 2$

- Check for Bending



Max. Bending stress $\nless \frac{f_y}{\gamma_{mo}}$

$$\frac{M_y}{I} = \frac{\frac{V_t \cdot c}{2N} \cdot \frac{d}{2}}{\frac{t d^3}{12}} \nless \frac{f_y}{\gamma_{mo}}$$

- Check for shear.

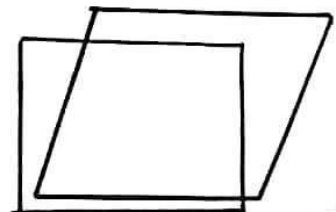
$$\tau_{avg} = \frac{V_b}{dt} \nless \frac{f_y}{\sqrt{3} \gamma_{mo}}$$

Note $\Rightarrow$ These checks should be done only for intermediate battens; as if intermediate battens are safe, end battens will always be safe as depth is more in end batten.

- End battens are designed as eccentric connections.

Note $\Rightarrow$ In design of built up column following failure modes are checked.

- Buckling of column
- Buckling of component member
- Buckling of
- Failure of lattice member.
- Distortion of the cross-section.



Q Design a built up column 10 m long carrying an axial load of 750 kN. Use 2 channels placed back to back with eff. length of 10 m. Provide single lacing system with bolted connection. Assume steel of grade 410 and M20 bolts of grade 4.6. Sections available are

ISMC : 300		ISMC : 350	
$A = 4564 \text{ mm}^2$	$b = 90 \text{ mm}$	$A = 5366 \text{ mm}^2$	$b = 100 \text{ mm}$
$r_{zz} = 118.1 \text{ mm}$	$g = 50 \text{ mm}$	$r_{zz} = 136.6 \text{ mm}$	$g = 60 \text{ mm}$
$r_{yy} = 26.18 \text{ mm}$		$r_{yy} = 28.3 \text{ mm}$	
$C_{yy} = 23.6 \text{ mm}$		$C_{yy} = 24.4 \text{ mm}$	

→ Assume $\lambda = 60$ (for built up section)

- For Buckling class (C) & $f_y = 250 \text{ N/mm}^2 \Rightarrow \chi = 0.74$

$$f_{cd} = \chi \frac{f_y}{\gamma_{mo}} = 0.74 \times \frac{250}{1.1} = 168.18 \text{ N/mm}^2$$

$$A_{gross} = \frac{P}{f_{cd}} = \frac{1.5 \times 750 \times 10^3}{168.18} = 6689.26 \text{ mm}^2$$

$$\text{area of one channel s/c reqd} = \frac{6689.26}{2} = 3344.63 \text{ mm}^2$$

adopt ISMC : 300 $A_g = 4564 \text{ mm}^2$

check for compression

$$l_e = 10 \text{ m}$$

$$\text{assume } r_{yy} > r_{zz} \Rightarrow r_{min} = r_{zz} = \sqrt{\frac{I_{zz}^{whole}}{A}}$$

$$\Rightarrow \sqrt{\frac{2 I_{zz}^{one}}{2 A^{one}}}$$

$$r_{min} = \sqrt{\frac{I_{zz}^{one}}{A^{one}}} \Rightarrow 118.1 \text{ mm}$$

$$\lambda = \frac{l_{eff}}{r_{min}} = \frac{10 \times 10^3}{118.1} = 84.67 \text{ mm}.$$

- Increase λ by 5%, $\lambda = 1.05 \times 84.67 = 88.84$

for Buckling class ©, $f_y = 250$, $\Rightarrow \chi = 0.542$

$$- f_{cd} = \chi \frac{f_y}{\gamma_{m0}} = 0.542 \times \frac{250}{1.1} = 123.18 \text{ N/mm}^2.$$

$$P_d = f_{cd} \times A_e.$$

$$= 123.18 \times 2 \times 4564 \times 10^{-3} = 1124.38 \text{ kN} < 1125 \text{ kN}$$

Not Safe.

- adopt ISMC : 350, $A_g = 5366 \text{ mm}^2$.

$$\lambda = \frac{l_e}{r_{min}} = \frac{10 \times 10^3}{136.6} \Rightarrow 73.2 \quad \Rightarrow \chi = 0.65.$$

$$\lambda = 1.05 \times 73.2 = 76.86$$

$$P_d f_{cd} = \chi \frac{f_y}{\gamma_{m0}} = 0.65 \times \frac{250}{1.1} = 147.72 \text{ N/mm}^2.$$

$$P_d = f_{cd} \times A_e = 147.72 \times 2 \times 5366 \times 10^{-3} = 1585.4 \text{ kN} > 1125 \text{ kN}$$

Hence Safe.

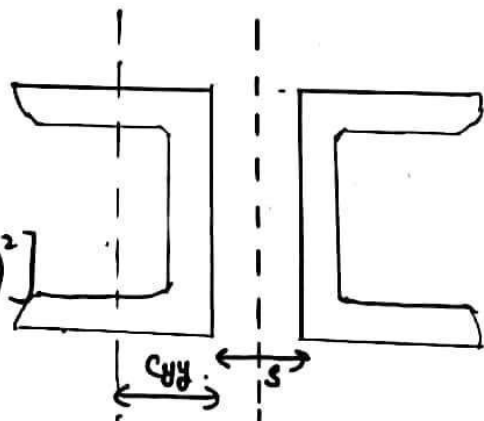
Lesson 15: Sec 29

- Design for lacing.

$\rightarrow r_{yy} > r_{zz}$ (slightly).

$$I_{yy}^{\text{combined}} = 2 \left[I_{yy}^{\text{one}} + A_{\text{one}} \left(C_{yy} + \frac{s}{2} \right)^2 \right]$$

$$I_{zz}^{\text{combined}} = 2 I_{zz}^{\text{one}}$$



$$\sqrt{\frac{2 \left[I_{yy}^{\text{one}} + A_{\text{one}} \left(C_{yy} + \frac{s}{2} \right)^2 \right]}{2 A_{\text{one}}}}$$

$$> \sqrt{\frac{2 I_{zz}^{\text{one}}}{2 A_{\text{one}}}}$$

$$\sqrt{r_{yy}^2 + (C_{yy} + s/2)^2} > \sqrt{r_{zz}^2}$$

$$28.3^2 + (24.4 + s/2)^2 > 136.6^2$$

$$s > 218.47 \text{ mm}$$

Adopt $s = 220 \text{ mm}$

Assume $\theta = 45^\circ \Rightarrow \frac{C}{2} = l$

$$\begin{aligned} C &= 2l \\ &= 2(s + 2g) \\ &= 680 \text{ mm} \end{aligned}$$

- Check for local buckling

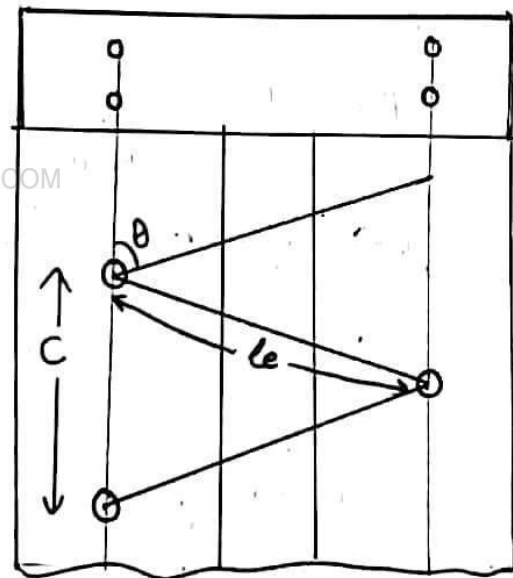
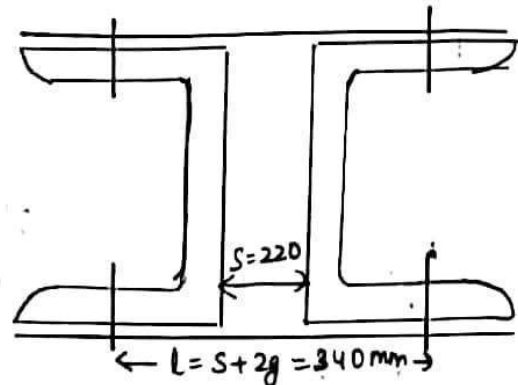
$$\frac{C}{r_{min}} \nless 50$$

$$\nless 0.7 \lambda_{whole}$$

$$\frac{680}{28.3} = 24 \nless 50$$

$$\begin{aligned} &\nless 0.7 \times 76.86 \\ &= 53.8 \\ &(\text{OK}) \end{aligned}$$

$\therefore$ Safe in local Buckling



- Thickness of lacing bar

$$t \nless \frac{l}{40}$$

$$t \nless \frac{340}{40}$$

$$\nless 8.5 \text{ mm}$$

adopt $t = 10 \text{ mm}$

- width of lacing bar (b) = $3d = 3 \times 20 = 60 \text{ mm}$.

adopt $b = 60 \text{ mm}$.

— slenderness ratio of lacing bar, $\lambda_{lacing} \neq 145$

$$\lambda_{lacing} = \frac{l_e}{r_{min}} = \frac{l'}{t/\sqrt{12}} = \frac{l\sqrt{2}}{t/\sqrt{12}} = 166.58 \neq 145$$

$$r_{min} = \frac{bt^3}{12}{b \times t} \Rightarrow \frac{t}{\sqrt{12}}$$

$$\text{Redesign } \lambda_{lacing} = 145$$

$$\frac{l\sqrt{2}}{t/\sqrt{12}} = 145 \Rightarrow t = 11.48 \text{ mm}$$

$$\text{adopt } t = 12 \text{ mm} \Rightarrow \lambda_{lacing} = 138.8 \neq 145$$

OK

Force in lacing bar, $F = \frac{V_t}{N} \csc \theta$

$N = 2$, for lacing bar (single)

$$V_t = \frac{2.5P}{100} = \frac{2.5 \times 1125}{100} = 28.125 \text{ kN}$$

$$F = \frac{28.125}{2} \csc 45^\circ = 19.88 \text{ kN}$$

— Check for tension $F \geq \frac{0.9 f_u (b-d_0) t}{1.25}$

$$\frac{0.9 \times 410 (60 - 22) \times 12}{1.25}$$

$$F \geq 134.61 \text{ kN} \quad \underline{\underline{OK}}$$

$$\text{also } F \geq \frac{f_y}{1.1} b t = \frac{250}{1.1} \times 60 \times 12$$

$$F \geq 163.63 \text{ kN} \quad \underline{\underline{OK}}$$

— Check for compression : $F \geq f_{cd} t$

for $\lambda_{lacing} = 138.8$, buckling class (C), $f_y = 250 \text{ MPa}$

$$\Rightarrow \chi = 0.295$$

$$f_{cd} = \chi \frac{f_y}{\gamma_{mo}} = 0.295 \times \frac{250}{1.1} = 67.139 \text{ MPa}$$

$$f_{cd} \times A_e = 67.139 \times 60 \times 12 \times 10^3 = 48.33 \text{ kN}$$

$$F = 19.88 \text{ kN} < 48.33 \text{ kN} \therefore \text{safe in compression}$$

- design of connection.

$$\text{design force for bolt } F_b = \frac{2Vt}{N} \cot \theta$$

$$= \frac{2 \times 2.5}{100} \times \frac{1125}{2} \cot 45^\circ$$

$$= 28.125 \text{ kN}$$

$$\text{design strength of bolt } V_{db} = V_{dsb} = \frac{f_{ub}}{\sqrt{3} \times 1.25} \times 0.78 \times \frac{\pi d^2}{4}$$

$$= 45.27 \text{ kN}$$

$$\therefore \text{no. of bolts } (n) = F_b / V_{db}$$

$$\Rightarrow \frac{28.125}{45.27} = 0.61; \text{ adopt 1 bolt}$$

Q design the same column with connecting system as battens with bolted connection.

$$\text{Sol}^n \quad S = 220 \text{ mm}$$

- check for local buckling

$$\frac{c}{r_{min}} > 50$$

$$> 0.7 \lambda_{whole}$$

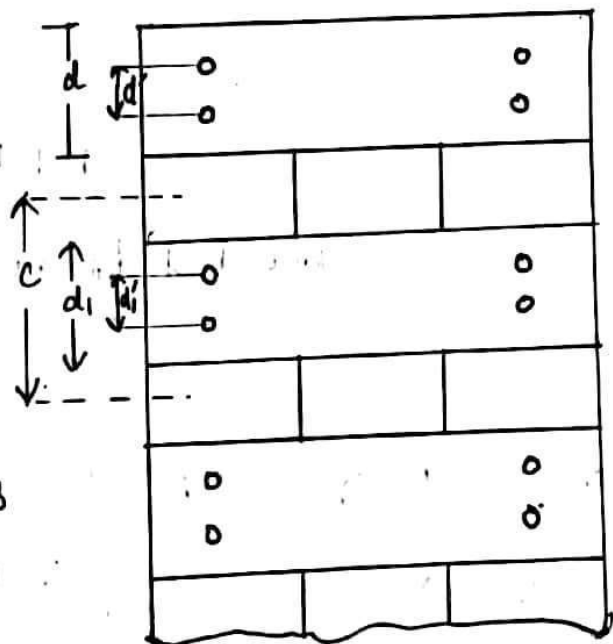
$$\lambda_{whole} = 1.1 \times 73.2 = 80.52$$

$$\frac{c}{28.3} > 50 \Rightarrow c > 50 \times 28.3$$

$$\Rightarrow 1415$$

$$\text{also } c > 0.7 \times 80.52 \times 28.3$$

$$\Rightarrow 1595 \text{ mm}$$



adopt $C = 1400 \text{ mm}$

- Depth of batten

$$\alpha = S + 2C_{yy}$$

$$= 220 + 2 \times 24.4 = 268.8 \text{ mm}$$

for end batten

$$d' \neq \alpha = 268.8 \text{ mm}$$

$$\neq 2b = 2 \times 100 = 200 \text{ mm}$$

adopt $d' = 270 \text{ mm}$

for intermediate batten

$$d_1' \leq \frac{3}{4} \alpha = \frac{3}{4} \times 268.8 = 201.6 \text{ mm}$$

$$< 2b = 200 \text{ mm}$$

adopt $d_1' = 210 \text{ mm}$

$$e_{min} = 1.5 d_0 = 1.5 \times 22 = 33 \text{ mm}$$

$$d = d' + 2e$$

$$d_1 = d_1' + 2e$$

$$d = 270 + 2(35)$$

$$= 210 + 2(35)$$

$$d = 340 \text{ mm}$$

$$d_1 = 280 \text{ mm}$$

- Thickness of batten ; $t \neq \frac{d}{50} = \frac{340}{50} = 6.8 \text{ mm}$

adopt $t = 8 \text{ mm}$

- Design BM for batten, $M = \frac{V_e \cdot C}{2N} = \frac{2.5 \times 1125 \times 1400 \times 10^{-3}}{100 \times 2 \times 2}$

$$= 9.84 \text{ kN-m}$$

- Design SF for bolts, $V_b = \frac{V_e \cdot C}{Nl} = \frac{2.5 \times 1125 \times 1400}{100 \times 340 \times 2}$

$$\Rightarrow 57.9 \text{ kN}$$

- Check for Bending & shear.

$$\frac{M_y}{I} = \frac{9.81 \times d_1/2}{\frac{\pi d_1^3}{32}} = \frac{9.81 \times 6 \times 10^6}{8 \times (280)^2} = 93.84 \text{ MPa} \times \frac{f_y}{\gamma_{mo}}$$

$$\frac{250}{1.1} = 227.2 \text{ MPa}$$

∴ safe in bending.

$$\frac{V_b}{d_1 t} = \frac{57.9 \times 10^3}{280 \times 8} = 25.84 \text{ N/mm}^2 \times \frac{f_y}{\sqrt{3} \times 1.1} = \frac{250}{\sqrt{3} \times 1.1}$$

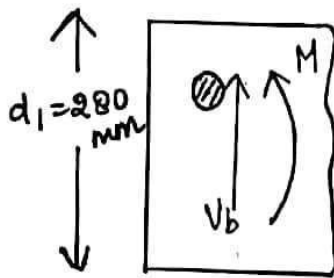
$$= 131.21 \text{ N/mm}^2$$

∴ safe in shear.

Design of Connection-

Bolt value $V_{db} = V_{dsb} = 45.27 \text{ kN}$.

$M = 9.81 \text{ kNm}$, $V_b = 57.9 \text{ kN}$, $m = 1$,



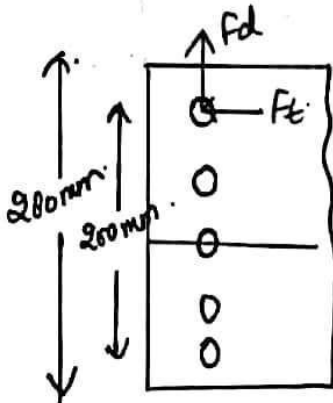
$$n = \sqrt{\frac{6M}{m p V_{db}}} = \sqrt{\frac{6 \times 9.81 \times 10^3}{1 \times 50 \times 45.27}} = 5$$

$$\phi = 2.5d = 2.5 \times 20 = 50 \text{ mm}$$

$$\text{end distance } e = \frac{280 - 4 \times 50}{2} = 40 \text{ mm}$$

$$F_d = \frac{V_b}{5} = \frac{57.9}{5} = 11.58 \text{ kN}$$

$$F_t = \frac{T \pi e^2}{\sum x_i^2} = \frac{9.81 \times 10^3 \times 100}{2 [100^2 + 50^2]} = 39.24 \text{ kN}$$

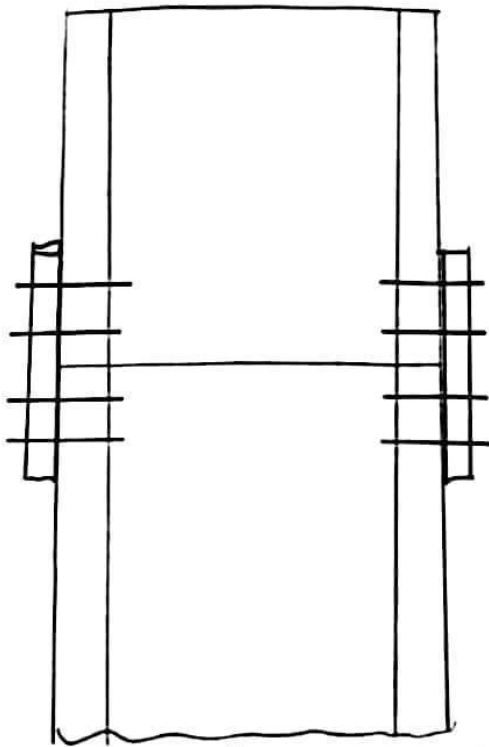


$$F_R = \sqrt{F_d^2 + F_t^2} = 40.9 \text{ kN} \times \frac{V_{db}}{45.27 \text{ kN}}$$

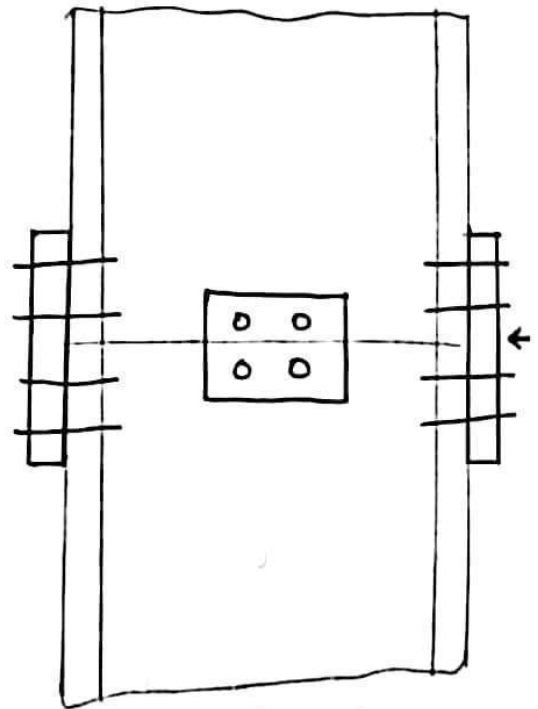
OK

Provide 5 Bolt on both side.

COLUMN SPLICES



Column flange having complete bearing



Web splice for shear

Case-I): When the top storey column & bottom storey column are of equal sizes, the top column storey flange rests completely on the bottom storey flanges.

- In this case two situation arises.

a) When column ends are MILLED for complete bearing.

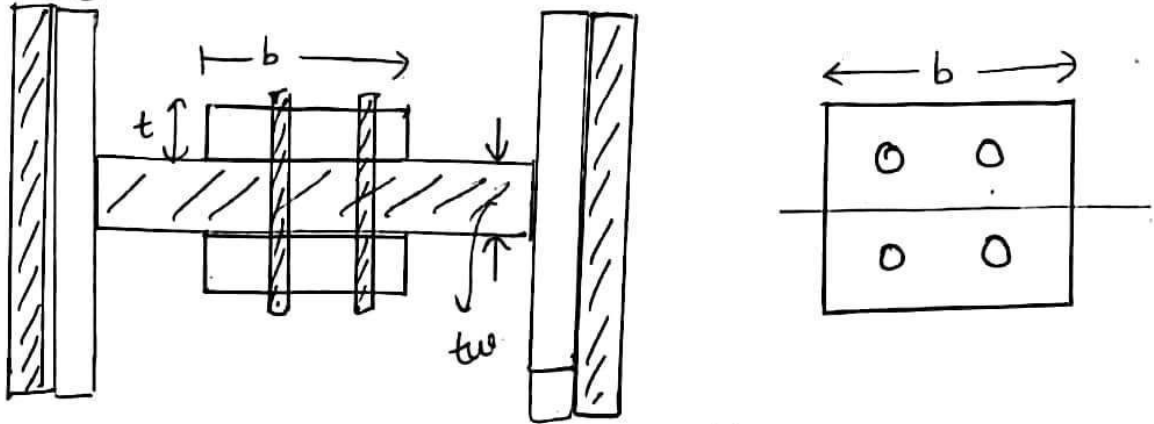
b) When column ends are not MILLED.

- When column ends are MILED for complete bearing all the direct load is assumed to have been transferred through direct bearing.

- In this case, if no moment is acting, purpose of the splice would be simply to hold the two members together.

- However for design purpose 50% of direct load will be assumed to have been transferred through direct bearing, remaining 50% through splice.
- The one splice plate will be designed for $\frac{P}{4}$ ($\frac{P/2}{2}$) load.
- If moments are also acting splice plate will be designed for $(\frac{M}{l} + \frac{P}{4})$ load.
 where M is factored bending moment. (P_e)
 l is c/c distance b/w 2 splice.
- When column ends are not MILLED, 100% of direct load will be assumed to have been transferred through splice plate.
- Thus one splice plate will be designed for $(\frac{M}{l} + \frac{P}{2})$ load in compression and same splice plate will be provided on other side also.
- However if $(\frac{P}{2} - \frac{M}{l}) < 0$, check for tension should be done in the splice plate.
- width of the splice plate should be taken as width of the column flange.
- splice plates are assumed to act as short column. (with zero slenderness ratio), hence its design compressive strength is given by $f_{cd} = \frac{f_y}{\gamma_{mo}}$.
- The required thickness is thus determined for the computed force.

- In case the column also carries shear we need to provide, shear splicing on the web.
- Two plates will be placed on the opposite sides of web.



- The bolt connecting shear splice with the web of column is in double shear.

- Here
$$n = \frac{V}{V_{db}}$$

V_{db} ⇒ design strength of bolt in double shear

$$V \nless \frac{f_y (bt)}{\sqrt{3} \gamma_{m0}}$$

$$V \nless \frac{0.9 f_u (b - nd_0)t}{\sqrt{3} \gamma_{m1}}$$

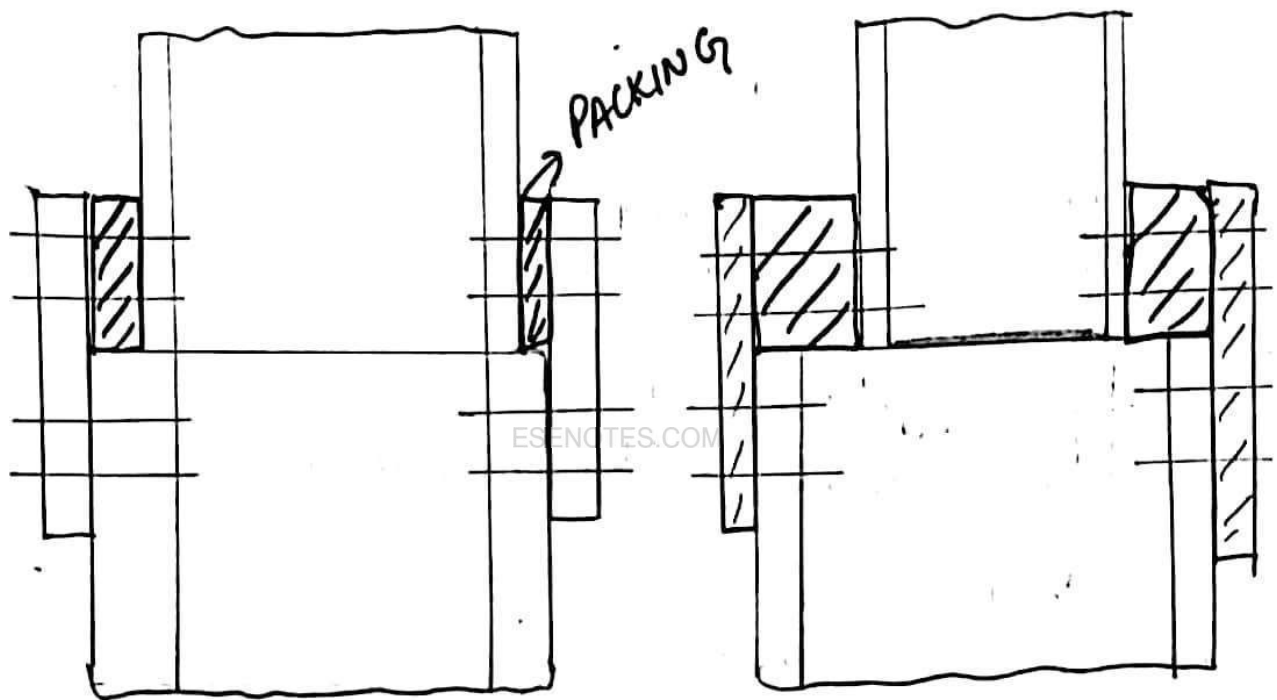
Case (II). When the top storey column size is smaller than the bottom storey column size.

- If the top storey column flange is resting entirely on the bottom storey column flange, the situation is similar to case (I), except that filler plate (packing plate) will be required i.e. packing will be provided.

Note 8 $\rightarrow$ If the $t_{pk} > 6\text{mm}$, the design strength of bolt is reduced as per specification.

b) If top storey column is resting partially on the bottom storey column, all the loads will be through splice.

- If there is difference b/w thickness of web, filler plates are also used in shear splicing

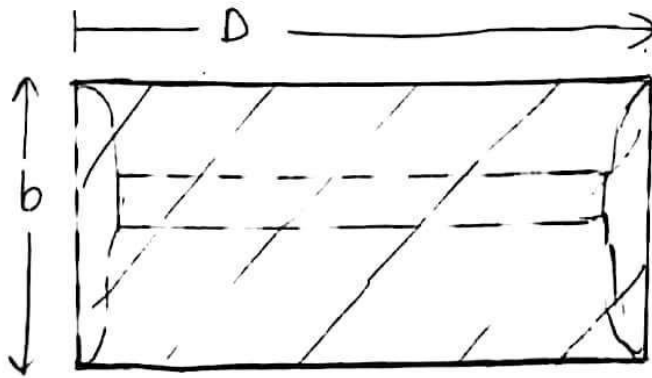


Column flange with partial Bearing.

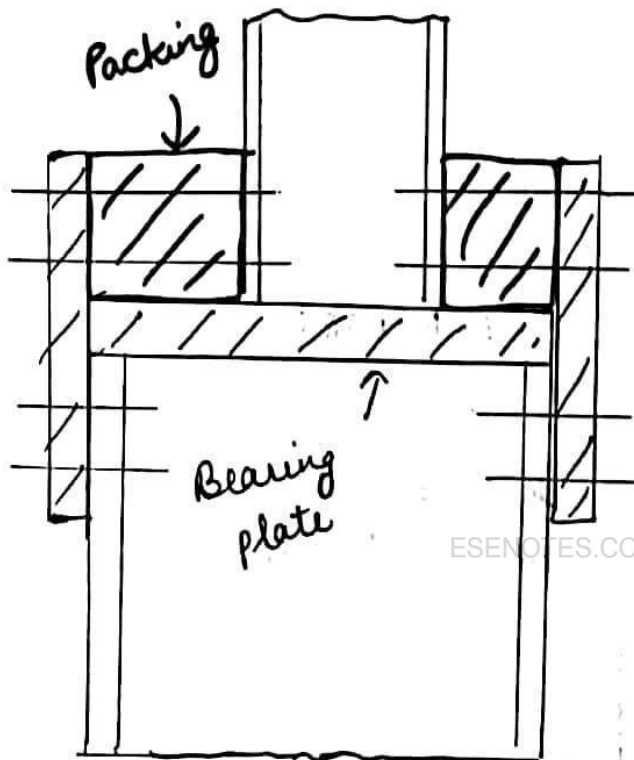
Case III

when the top storey column flange is not resting at all the bottom storey column flange.

- In this case a bearing plate will be used on the top of bottom storey column & the size of the plate will be equal to overall size of bottom storey column.

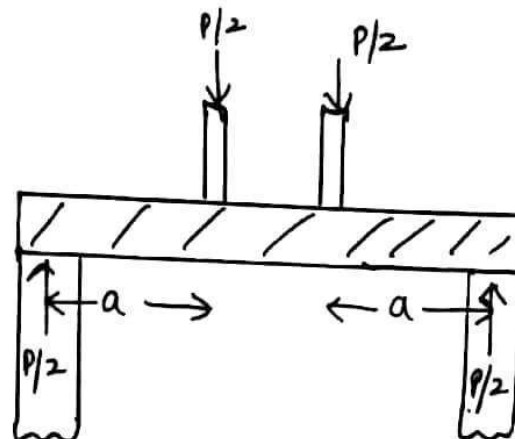


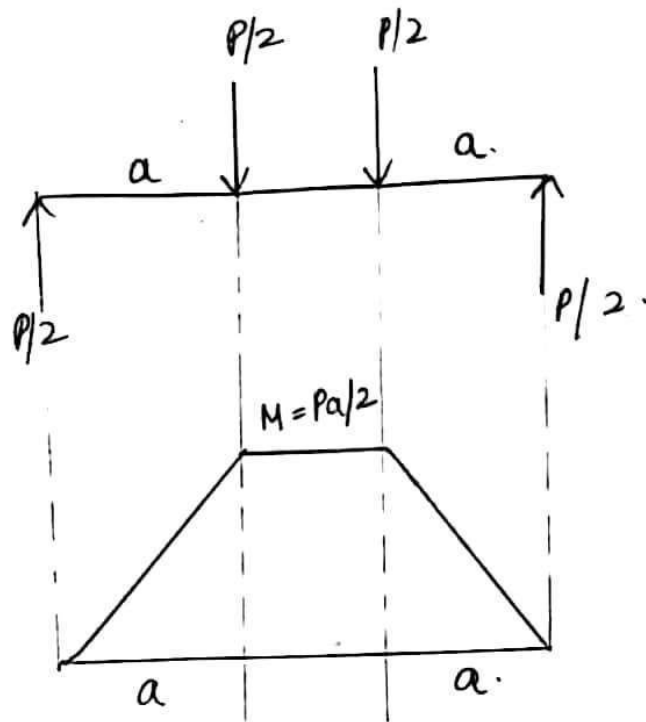
size of column $\Rightarrow b \times D$.



Column flange over the web of other column.

- The thickness of the plate will be calculated as follows.





$$\sigma = M/z = \frac{by}{\gamma_{mo}} = \cancel{M/z} \cdot M/bt_p^2$$

$$\Rightarrow \frac{M}{\frac{bt_p^2}{6}} = \frac{by}{\gamma_{mo}} \Rightarrow \frac{Pa/2}{\frac{bt_p^2}{6}} = \frac{by}{\gamma_{mo}}$$

$$t_p = \sqrt{\frac{6M}{b \frac{by}{\gamma_{mo}}}}$$

Ans: 17

Lesson 17

Q Design a column splice for applied load of 300 kN on 15HB 300 sections in compression, applied shear force & applied BM is 75 kN & 20 kNm respectively. Assume column end to be unlimited & Fe 410 grade steel, Use M18 Bolts of grade 4.6 properties of 15HB:300 are $t_w = 7.6 \text{ mm}$, $t_f = 10.8 \text{ mm}$, $b = 250 \text{ mm}$.

Since columns are unmilled, design force = $\frac{P}{2} + \frac{M}{L}$

$$P = 1.5 \times 300 = 450 \text{ kN}$$

$$M = 1.8 \times 20 = 36 \text{ kNm}$$

Since thickness of splice plate is not known, adopt $L = 300 \text{ mm}$
(Here $L < 300 + t \Rightarrow \text{Safe}$)

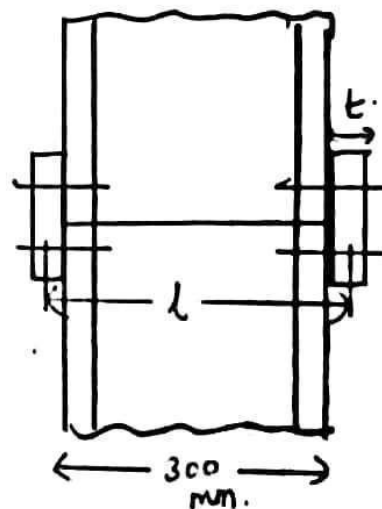
$$\text{Design force} = \frac{450}{2} + \frac{36}{300 \times 10^{-3}} = 325 \text{ kN}$$

- splice are designed as a short column

$$f_{cd} = \frac{f_y}{\gamma_{m0}}$$

$$\frac{325 \times 10^3}{250 \times t} = \frac{250}{1.1} \Rightarrow t = 5.72 \text{ mm}$$

adopt $t = 8 \text{ mm}$.



- Design of connection (Vdb)

$$\begin{aligned} \text{(i) } V_{dsb} &= \frac{f_{ub} \times 0.78 \times \frac{\pi d^2}{4}}{\sqrt{3} \times 1.25} = \frac{400 \times 0.78 \times \pi \times 18^2}{\sqrt{3} \times 1.25} \\ &= 36.67 \text{ kN} \end{aligned}$$

$$\text{(ii) } V_{dcb} = \frac{2.5 k_b d t f_u}{1.25}$$

$$t = \min \{ 8, 10.8 \} = 8$$

$$k_b = \min \left\{ \frac{e}{3d_0}, \frac{t}{3d_0} - 0.25, \frac{f_{ub}}{f_u}, 1 \right\}$$

$$e = e_{\min} = 1.5d_o = 1.5(18+2) = 30 \text{ mm.}$$

$$p = 2.5d = 2.5 \times 18 = 45 \text{ mm.}$$

$$k_b = 0.5$$

$$V_{dpb} = \frac{2.5 \times 0.5 \times 18 \times 8 \times 410 \times 10^{-3}}{1.25} = 59 \text{ kN.}$$

$$V_{db} = \min(V_{dsb}, V_{dpb}) = 36.67 \text{ kN}$$

$$\text{no. of bolts } (n) = \frac{325}{36.67} = 8.89$$

Adopt $n = 10$ bolts on each side.

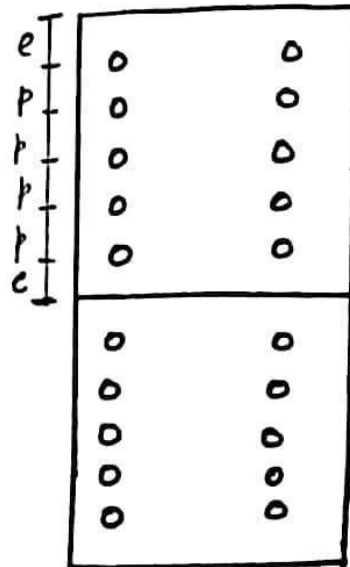
$$\text{Now length of splice plate} = 2(2e + 4p).$$

$$L = 2(2 \times 30 + 4 \times 45) = 480 \text{ mm.}$$

provide a splice of $L = 480 \text{ mm}$ on each

$$b = 250 \text{ mm side}$$

$$t = 8 \text{ mm}$$



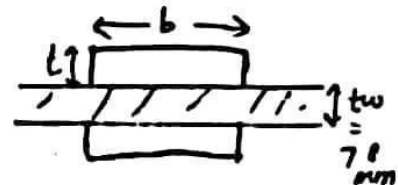
- Design of shear splice.

$$(i) V \neq \frac{b t f_y}{\sqrt{3} \gamma_{m0}}$$

$$b t \neq \frac{75 \times 1.5 \times 10^3 \times \sqrt{3} \times 1.1}{250} = 857.36 \text{ mm}^2$$

$$(ii) V \neq \frac{0.9 f_u A_n}{\sqrt{3} \times 1.25}$$

$$A_n \neq \frac{75 \times 1.5 \times 10^3 \times \sqrt{3} \times 1.25}{0.9 \times 410} = 660.08 \text{ mm}^2.$$



- Design of connection (V_{db})

$$(i) \quad V_{dsb} = \frac{400}{\sqrt{3} \times 1.25} \times 1.78 \times \pi \frac{d^2}{4} \times 10^{-3}$$

$$= 83.68 \text{ kN}$$

$$(ii) \quad V_{dpb} = \frac{2.5 k_b d t f_u}{1.25} = \frac{2.5 \times 0.5 \times 18 \times 7.6 \times 410 \times 10^{-3}}{1.25}$$

$$= 56.088 \text{ kN}$$

$$V_{db} = \min(V_{dsb}, V_{dpb}) = 56.088 \text{ kN}$$

$$\text{no. of bolts} = \frac{75 \times 1.5}{56.088} = 2$$

Provide 2 bolts on each side.

Adopt $t = 8 \text{ mm}$.

$$b = 2e + p = 2 \times 30 + 45$$

$$= 105 \text{ mm}$$

Now applying check $b \geq \frac{857.36}{t} = \frac{857.36}{8} = 107.17 \text{ mm}$

adopt $b = 110 \text{ mm}$.

$$A_n = (b - 2d_o)t = (110 - 2 \times (18 + 2)) \times 8 = 560 \text{ mm}^2$$

$$A_n \geq 660 \text{ mm}^2$$

Provide $b = 125 \text{ mm}$.

$$A_n = 680 \text{ mm}^2 > 660 \text{ mm}^2$$

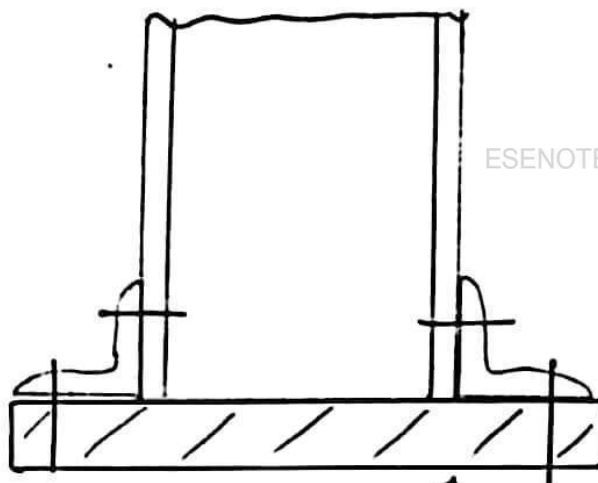
Conventional

COLUMN BASE

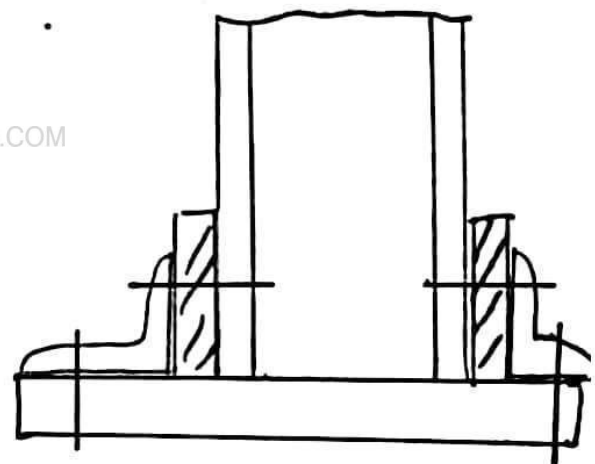
- If a steel column is directly placed on a concrete pedestal, the concrete will get crushed
- To safeguard against the crushing area of load transfer should be increased such that resulting stress on concrete is less than the permissible stress in direct compression in concrete.
- In order to increase the area, column bases are provided, which are of following types.

a) Slab Base

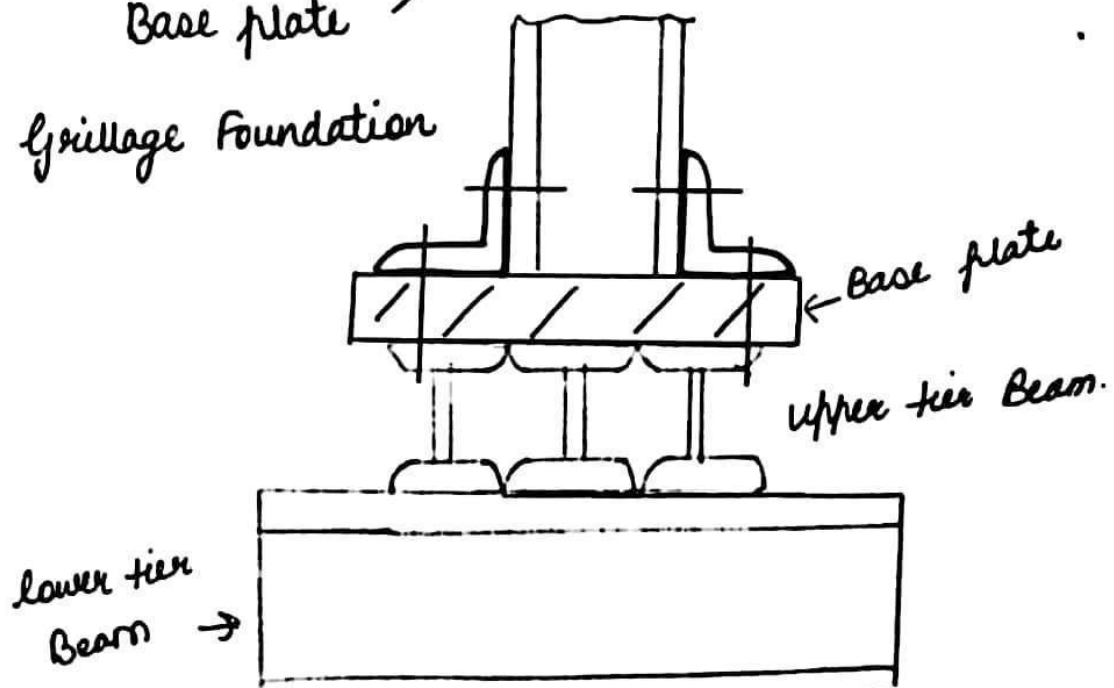
b) Gussat Plate



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c) Grillage Foundation

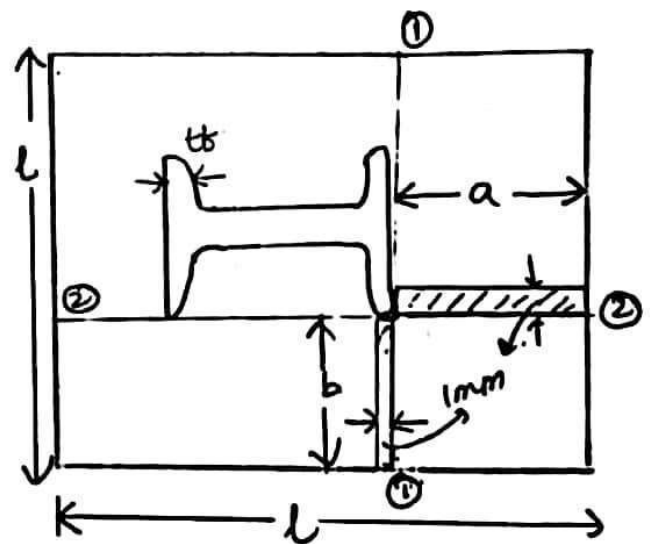
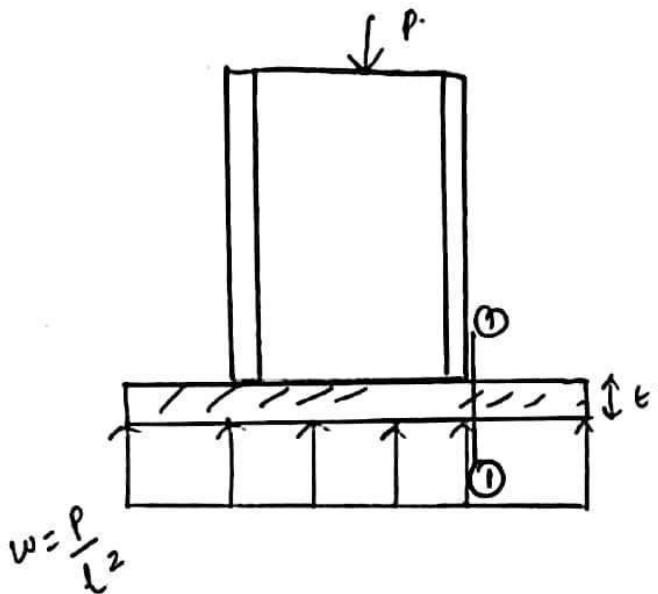


- Gusset plate is better for column carrying large moment or when the load is heavy.
- It provides more rigidity at the joint & thickness requirement of the base plate is reduced.
- When the load is very heavy and soil is weak or if foundation depth is shallow, we provide Grillage foundation, which provides larger area at the base.

Case-I: when the column is subjected to purely axial load.

- In this case, size of plate is computed so corresponding compressive stress in concrete $= 0.45 f_{ck}$.
- for pure axial loading base plate is assumed to square plate

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a = larger overhang

b = smaller overhang.

- The slab base tends to bend simultaneously about two axis ①-① & ②-②, thus the stress caused by Bending about one axis is influenced by the stress, due to bending about the other axis.
- Poisson's ratio ($\mu = 0.3$) is used to account for this effect

$$M_{11} = (w \times l) \times a \cdot \frac{a}{2} = \frac{wa^2}{2}$$

$$w = \frac{P}{l^2}$$

$$M_{22} = (w \times l) \cdot b \cdot \frac{b}{2} = \frac{wb^2}{2}$$

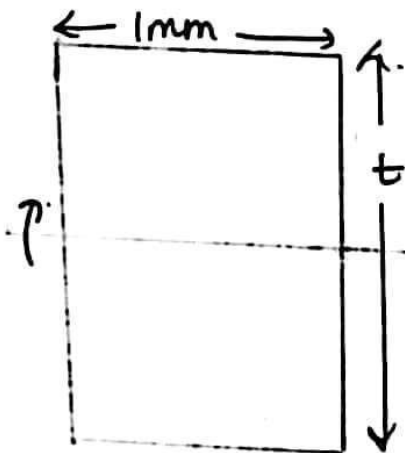
$$M_{net} = M_{11} - \mu M_{22}$$

Now, moment capacity of section (M_d).

$$M_d = \frac{1.2 f_y \cdot z_e}{\gamma_{mo}}$$

M_d = design moment

z_e = section modulus of plate



$$z_e = \frac{I}{y_{max}} = \frac{1 \times t^3}{12 \cdot \frac{t}{2}} = \frac{t^2}{6}$$

$$M_d = \frac{1.2 f_y}{\gamma_{mo}} \cdot \frac{t^2}{6}$$

for safety. $M_{net} \leq M_d$.

$$M_{11} - \mu M_{22} \leq M_d$$

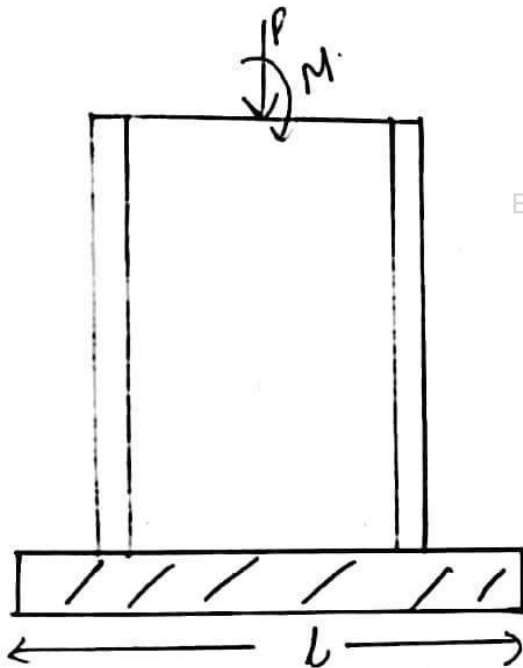
$$\frac{wa^2}{2} - \mu \frac{wb^2}{2} \leq \frac{1.2 f_y}{\gamma_{mo}} \cdot \frac{t^2}{6}$$

$$\left\{ t \geq \sqrt{\frac{2.5 w (a^2 - 0.3 b^2)}{f_y / \gamma_{mo}}} \right\}$$

also $t > t_f$ (thickness of flange of column)

Note: As per IS 456-2000 bearing strength of concrete is $0.45 f_{ck}$ and as per 800-2007, bearing strength of concrete is $0.6 f_{ck}$. But we will consider $0.45 f_{ck}$.

Case (II) When slab base is subjected to axial load as well as Bending.



Here $M = Pe \Rightarrow e = \frac{M}{P}$

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& also for no tension $e \leq \frac{l}{6}$

$$\sigma_T = \frac{P}{A} \pm \frac{M}{I} y$$

$$= \frac{P}{A} \pm \frac{Pe}{Z}$$

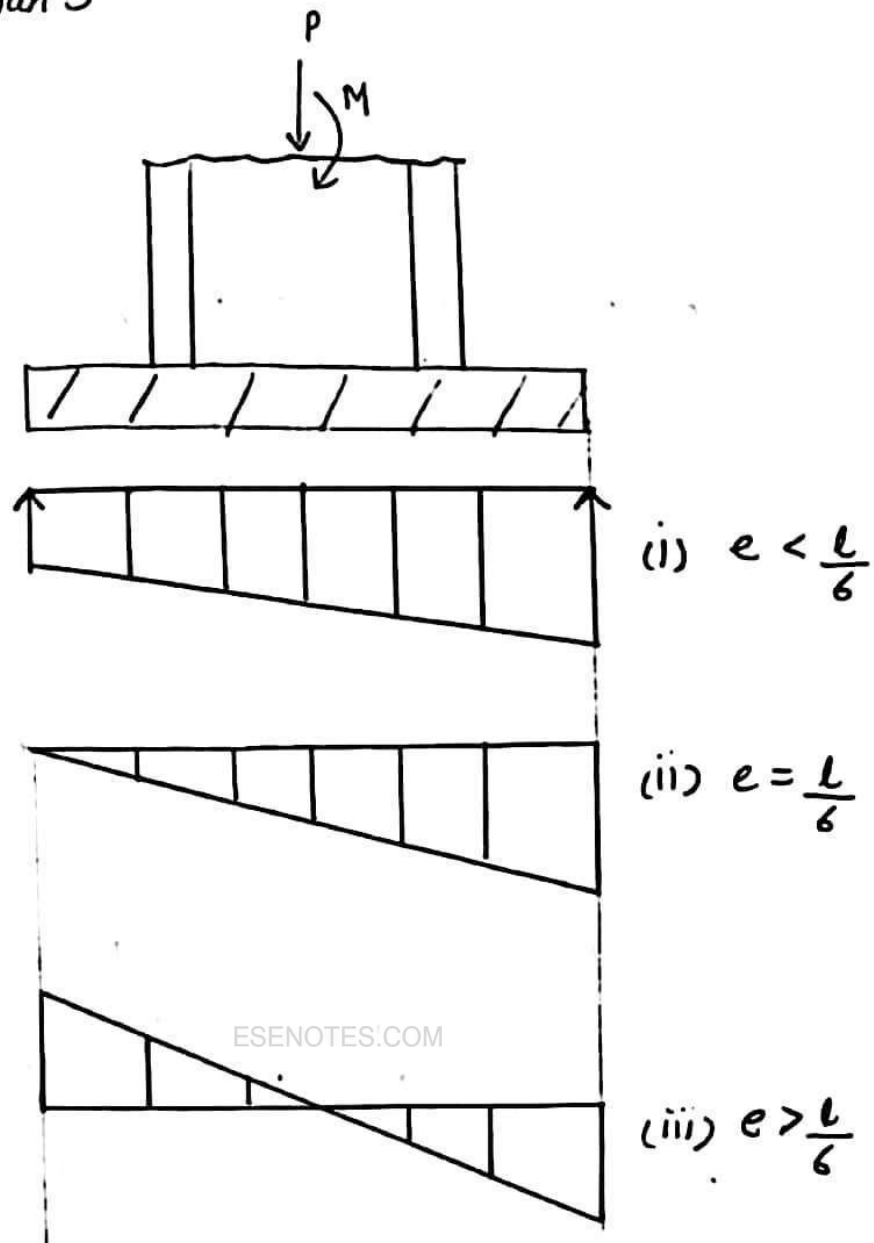
$$= \frac{P}{bl} \pm \frac{Pe}{\frac{bl^2}{6}}$$

$$\sigma_T = \frac{P}{bl} \left[1 \pm \frac{6e}{l} \right]$$

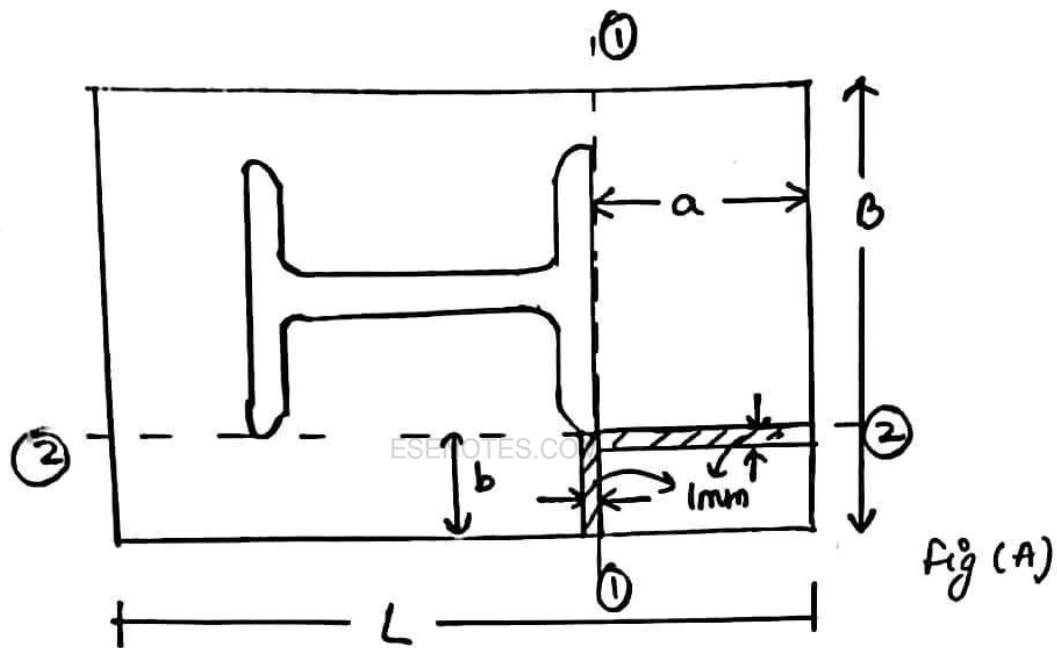
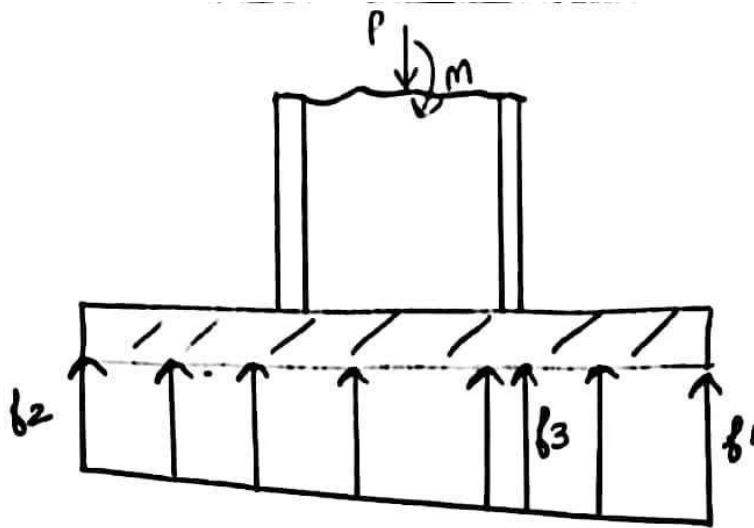
for no tension $\sigma_T \geq 0$

$$\frac{P}{A} \left[1 \pm \frac{6e}{l} \right] \geq 0 \Rightarrow e \leq \frac{l}{6}$$

Lesson 21 Jan 3



- if we have option for selecting " l " then we consider $l > 6e$.
- since " l " is now known, now " B " is selected such that max. pressure on concrete do not exceed the bearing strength ($0.45 f_{ck}$)
- a) When eccentricity $e \leq \frac{l}{6}$.



Now . $f_1 = \frac{P}{Bl} \left(1 + \frac{6e}{l} \right)$

$f_2 = \frac{P}{Bl} \left(1 - \frac{6e}{l} \right)$

for safety $f_1 \neq 0.45 f_{ck}$

$\frac{P}{Bl} \left(1 + \frac{6e}{l} \right) \neq 0.45 f_{ck} \quad - (i)$

from above eq (i) "B" can be obtained.

$$\text{Now, } f_3 = f_2 + \frac{(f_1 - f_2)}{l} (l - a)$$

$$M_{11} = \frac{1}{2} (f_1 + f_2) (a \times 1) \cdot \frac{a}{3} \left(\frac{2f_1 + f_3}{f_1 + f_3} \right)$$

$$M_{22} = f_3 \times (b \times 1) \cdot \frac{b}{2} = \frac{f_3 b^2}{2}$$

$$M_{\text{net}} = M_{11} - \mu M_{22}$$

$$\text{Design moment } M_d = \frac{1.2 f_y z_e}{\gamma_{mo}}$$

$$z_e = \frac{t^2}{6}$$

$$M_d = \frac{1.2 f_y \frac{t^2}{6}}{\gamma_{mo}}$$

$$\text{for safety } M_{\text{net}} \leq M_d$$

$$M_{11} - \mu M_{22} \leq \frac{1.2 f_y \frac{t^2}{6}}{\gamma_{mo}} \quad \text{--- (ii)}$$

from above eq (ii), "t" can be computed.

also $t > t_f$ (thickness of column flange).

$$b). \text{ when } e > \frac{l}{6}$$

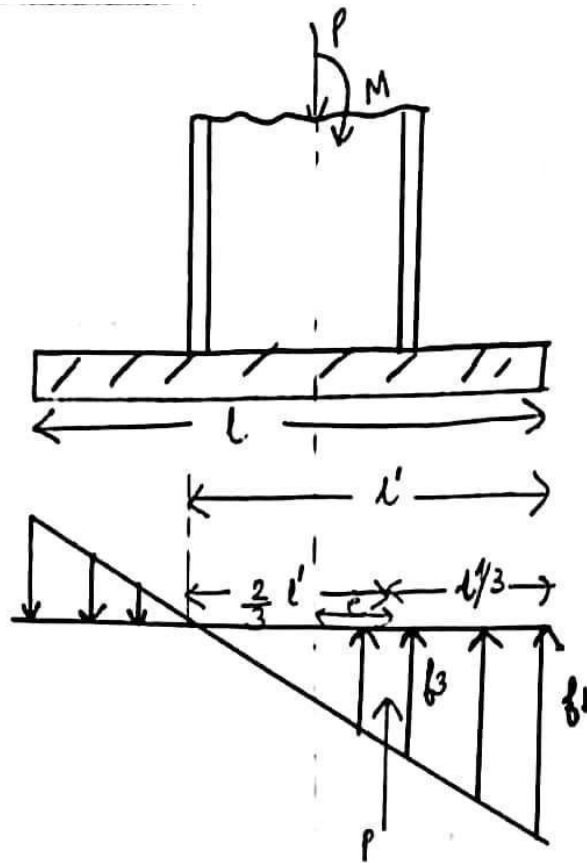
l' = length of contact.

$$\frac{l'}{3} + e = \frac{l}{2}$$

$$l' = 3 \left(\frac{l}{2} - e \right) \quad \text{--- (i)}$$

$$\text{also } \frac{1}{2} l' \cdot f_1 \cdot B = P$$

$$f_1 = \frac{2P}{B l'} \quad \text{--- (ii)}$$



$$f_3 = \frac{f_1}{l'} \times (l' - a) \quad \text{--- (iii)}$$

Plan of this case is as same fig (A)

Now, for safety. $f_1 \neq 0.45 f_{ck}$.

$$\frac{2P}{Bl'} \neq 0.45 f_{ck} \quad \text{--- (iv)}$$

from eq (i) + (iv), "B" can be computed.

$$\text{also, } M_{11} = \frac{1}{2} (f_1 + f_3) (ax_1) \times \frac{a}{3} \left(\frac{2f_1 + f_3}{f_1 + f_3} \right)$$

$$M_{22} = \frac{f_3 \cdot b^2}{2}$$

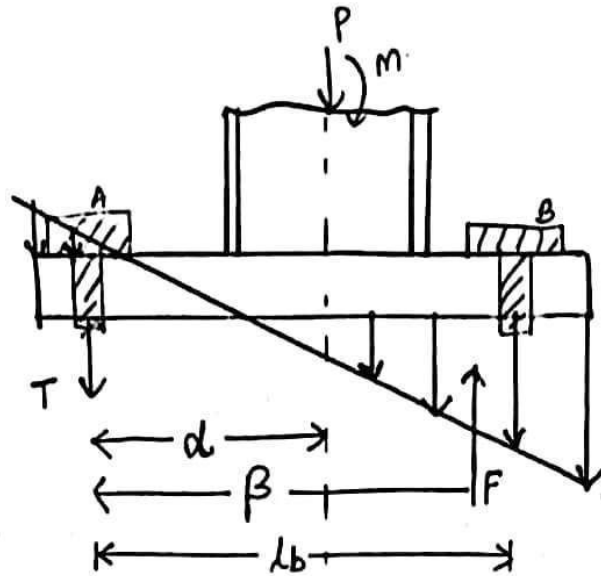
$$M_{net} = M_{11} - \mu M_{22}$$

for safety $M_{net} \leq M_d$.

$$M_{11} - \mu M_{22} \leq \frac{1.2 f_d}{\gamma_{mo}} Z_e \quad \text{--- (v)}$$

$$(z_c = \frac{t^2}{6})$$

from eq (v), "t" can be computed.



- For design of Bolt.

$$\sum F_y = 0 \quad T + P = F \quad \text{--- (i)}$$

$$\sum M_A = 0 \quad M + P\alpha = F\beta \quad \text{--- (ii)}$$

from eq (i) & (ii), "T" can be computed.

also For design purpose $Tlb = M$ (for simplicity)

$$T = \frac{M}{lb}$$

Now, tension force in one belt $T_b = \frac{T}{n}$.

Here n is no. of belts on one side.

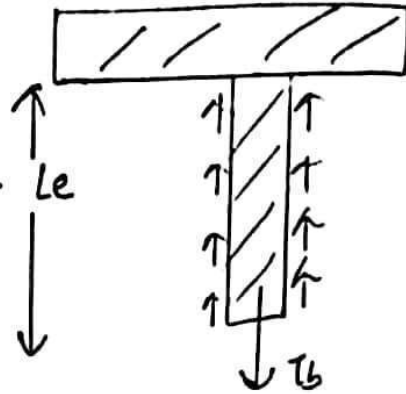
$$\text{For safety, } T_b \geq \frac{0.9 f_{ub}}{1.25} \times 0.78 \times \frac{\pi d^2}{4}$$

$$\text{also } T_b \geq \frac{f_{yb}}{1.1} \times \frac{\pi d^2}{4}$$

for embedment length of bolt (l_e)

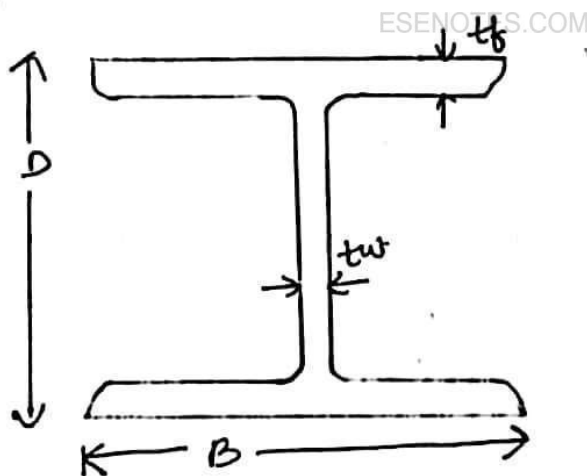
$$\frac{T_b}{\pi d l_e} \neq \tau_{bd}$$

τ_{bd} : bond strength of concrete $\cdot l_e$



Q A column of section 200 is to support an axial compressive load of 700 kN & moment of 55 kN-m, take $\mu = 0.3$. Design column slab base assuming bearing ends to be milled. Grade of concrete is M25 & of steel Fe410.

Properties of section 200, $B = D = 200$ mm, $t_w = 9$ mm, $t_f = 15$ mm.



Solⁿ

$$P = 1.5 \times 700 = 1050 \text{ kN}$$

$$M = 1.5 \times 55 = 82.5 \text{ kN-m}$$

$$e = \frac{M}{P} = \frac{82.5 \times 10^3}{1050} = 78.57 \text{ mm}$$

$$\text{adopt } l \geq 6e = 6 \times 78.57 = 471.42, \text{ consider } l = 600 \text{ mm}$$

for safety $f_1 \neq 0.45 f_{ck}$.

$$\frac{P}{B L} \left(1 + \frac{6e}{L} \right) \neq 0.45 f_{ck}.$$

$$\frac{1050}{600 \times B} \left(1 + \frac{6 \times 78.57}{600} \right) \neq 0.45 \times 25 \times 10^3$$

$$B \neq 277.7 \text{ mm}$$

adopt $B = 300 \text{ mm}$.

$$a = \frac{600 - 200}{2} = 200 \text{ mm}$$

$$b = \frac{300 - 200}{2} = 50 \text{ mm}$$

$$f_1 = \frac{P}{B L} \left(1 + \frac{6e}{L} \right)$$

$$\frac{1050 \times 10^3}{300 \times 600} \left(1 + \frac{6 \times 78.57}{600} \right)$$

$$f_1 = 10.42 \text{ MPa}$$

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$$f_2 = \frac{P}{B L} \left(1 - \frac{6e}{L} \right) = 1.28 \text{ MPa}.$$

$$f_3 = f_2 + (f_1 - f_2) \frac{(L - a)}{L} = 1.28 + (10.42 - 1.28) \frac{(600 - 200)}{600}$$

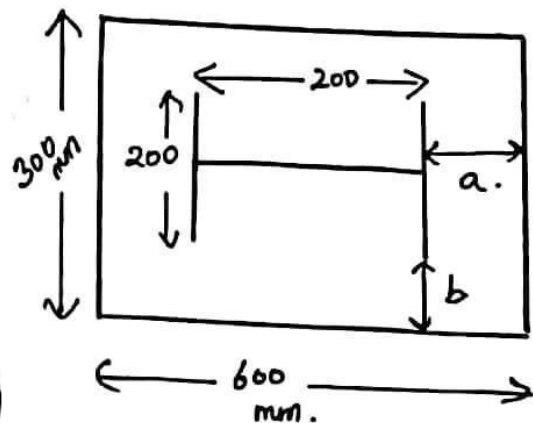
$$f_3 = 7.37 \text{ N/mm}^2$$

$$M_{11} = \frac{1}{2} (f_1 + f_3) (a \times 1) \cdot \frac{a}{3} \left(\frac{2f_1 + f_3}{f_1 + f_3} \right) \times 10^{-6}$$

$$M_{11} = 6.188 \text{ kNm}.$$

$$M_{22} = f_3 \frac{b^2}{6} \times 10^{-6}$$

$$= 9.21 \times 10^{-3} \text{ kN-m}$$



$$\text{Design moment, } M_d = \frac{1.2 f_y z_e}{\gamma_{mo}} = \frac{1.2 f_y}{\gamma_{mo}} \cdot \frac{t^2}{6}$$

for safety, $M_{net} \leq M_d$.

$$M_{11} - \mu M_{22} \leq M_d$$

$$(0.188 - 0.3 \times 9.21 \times 10^3) \leq \frac{1.2 \times 250 \times t^2}{1.1 \times 6} \times 10^{-6}$$

$$t \geq 63.83 \text{ mm.}$$

adopt $t = 65 \text{ mm.}$

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BEAMS

- Beam is a structural member subjected to transverse load (loads perpendicular to the longitudinal axis of beam) which give rise to shear force and bending moment in the beam.

- Beams are classified as follows.

a) JOIST: These are closely spaced beams to support the roof.

b) GIRDER: A large beam supporting a no. of joist.

c) PURLIN: These are beams provided over the trusses to support roofing b/w the adjacent truss.

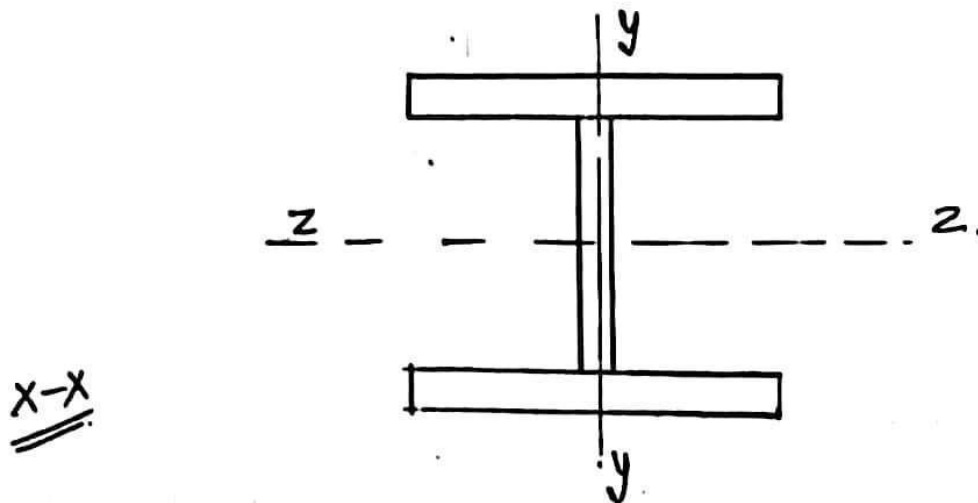
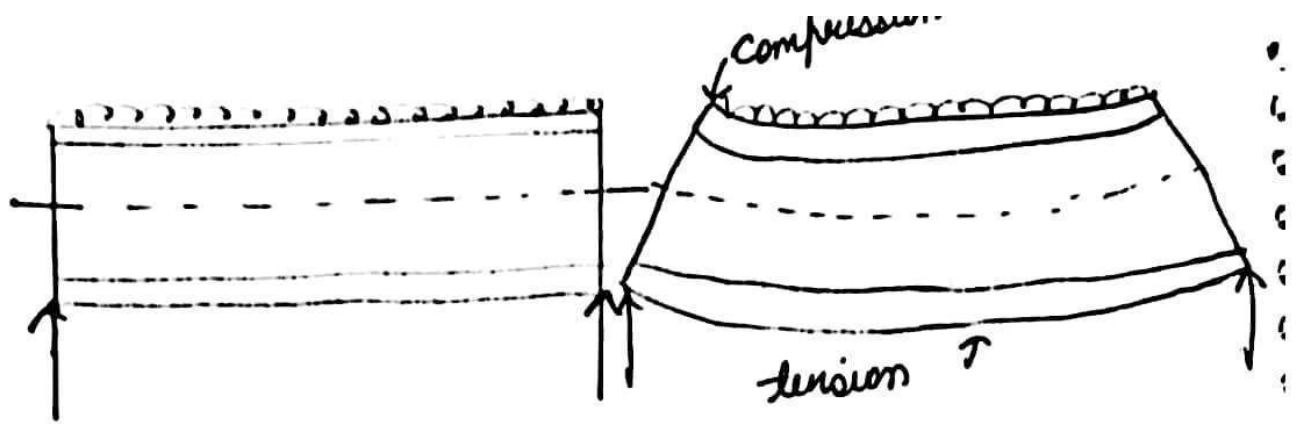
d) LINTELS: These are the beams supporting the walls over doors or window openings.

Note: $\Rightarrow$ our scope is limited to beams with symmetrical cross section & subject to only transverse loading in plane of symmetry, thus no torsion or axial force is considered.

Lateral Stability of Beams

- For a simply supported beam one of the flange is subjected to compression & other is subjected to tension.

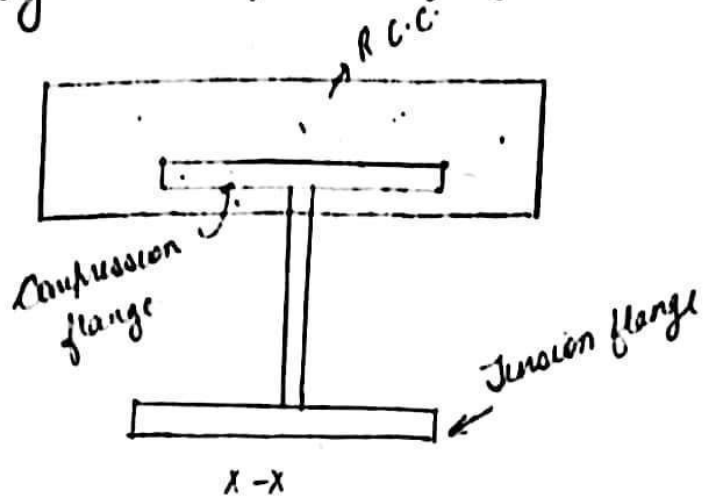
- Hence to achieve the economy in design of beam, the sections are proportioned such that moment of inertia about one axis (Z-Z axis) is considerably larger than the other moment of inertia about other axis (Y-Y axis).



- Since such sections are weak in bending resistance about $y-y$ axis, the compression flange tends to buckle laterally and the bottom flange being in tension tends to remain straight results in twisting of the entire cross-section.
- Hence, lateral buckling of compression flange is also accompanied by twisting & this type of instability is termed as "lateral torsion buckling".
- Thus, it is desirable to restrain the compression flange of beam against lateral buckling, so that beam can develop its bending strength.
- Based on the degree of lateral restraint, the beams are classified as
 - a) Laterally restrained Beam (our scope is limited to these beams)
 - b) Laterally unrestrained Beam

- In laterally restrained beam, compression flange is restrained by

- (i) Providing cross bracing
- (ii) Making compression flange heavier
- (iii) Inserting the compression flange in RCC slab.



Classification of cross-section

- The local buckling may affect the bending behaviour of steel section, thus different classes of cross-sections are defined on the basis of point at which local buckling occur during bending.

- This classification of each element of a cross-section subjected to compression is based on its width to thickness ratio.

a) Class-I / Plastic cross-section

- These sections can develop plastic hinge and have the rotational capacity required for failure of structure by formation of plastic mechanism.

$$M_d = Z_p \frac{f_y}{\gamma_{mo}} \quad Z_p - \text{plastic section modulus.}$$

b) Class-II / Compact cross-section.

- Such sections can develop plastic moment of resistance but have inadequate rotational capacity for formation of plastic mechanism due to local buckling.

Here $\boxed{M_d = Z_p \frac{f_y}{\gamma_{mo}}}$

c) Class - III / Semi compact cross-section.

- These are the section in which extreme fibres in compression can reach the yield stress but cannot develop plastic moment of resistance due to local buckling.

Here $\boxed{M_d = Z_e \frac{f_y}{\gamma_{mo}}}$ $Z_e = \text{elastic section modulus.}$

d) Class - IV / Slender cross-section.

- These sections have elements which buckle locally even before reaching yield stress.

Here $\boxed{M_d < Z_e \frac{f_y}{\gamma_{mo}}}$

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Bending strength of Laterally supported Beams.

- In simple bending, with beams adequately supported against lateral torsional buckling, the design bending strength (M_d) of the section should not be less than factored design moment (M_u) at that section, due to external loading.

i.e. $\boxed{M_d \geq M_u}$

Note: $\rightarrow$ This relationship is based on assumption that web of beam will not buckle due to shear

i.e. $\frac{d}{t_w} \leq 67 \epsilon$

- For beam with plastic, compact, semi compact flanges and slender web i.e. $\frac{d}{t_w} > 67 \epsilon$, web is susceptible to shear buckling.

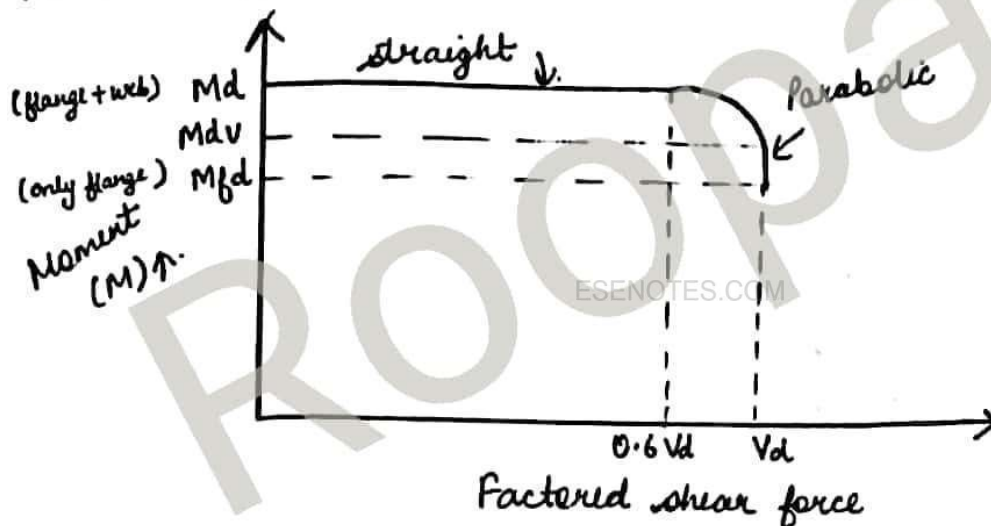
- According to IS 800:2007, the design bending strength of a section, which is not susceptible to web buckling under shear before yielding (i.e. $\frac{d}{t_w} \leq 67\epsilon$), depends on the design shear strength & factored design shear force (V_d).

a) Low shear case ($V \leq 0.6 V_d$)

b) High shear case ($V > 0.6 V_d$)

Lesson 27 Jan 17

- The following Interaction diagram blue moment & shear is adopted by IS code.



- For factored shear force (V) $\leq 0.6 V_d$, moment capacity of section is constant & independent of shear & for $V > 0.6 V_d$, a moment capacity of beam is affected by shear & the reduced moment capacity is denoted by M_{dV} .

- According to IS code following formulation for the moment capacity in case of low & high shear is used.

Case I: When $V \leq 0.6 V_d$ (low shear case).

$$M_d = \beta_b Z_p \frac{f_y}{\gamma_{m0}}$$

also. $M_d \leq 1.2 Z_e \frac{f_y}{\gamma_{m0}}$ (for SS)

$$\leq 1.5 Z_e \frac{f_y}{\gamma_{m0}} \text{ (for cantilever)}$$

here $\beta_b = 1.0$ for plastic & compact section

$$= \frac{Z_e}{Z_p} \text{ for semi compact section}$$

More Note: The additional check of $M_d \leq 1.2 Z_e \frac{f_y}{\gamma_{mo}}$ (for

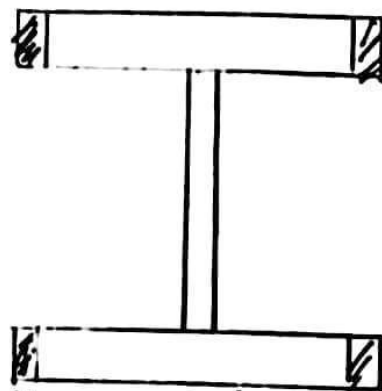
simply supported and $M_d \leq 1.5 Z_e \frac{f_y}{\gamma_{mo}}$ (for cantilever Beam)

is provided to prevent the onset of plasticity under unfactored load (DL, LL, WL) i.e. excessive deformations are prevented at working loads.

- Now, for slender section, $M_d = Z_e \frac{f_y}{\gamma_{mo}}$

Z_e = effective elastic section modulus.

- It is calculated by limiting the effective width of beam flanges upto the specified limit for semi-compact section.



Effect of Bolt Holes.

- The effect of bolt holes in tension flange need not to be taken if.

$$\frac{0.9 f_u A_{nf}}{\gamma_{m1}} \geq \frac{f_y A_{gf}}{\gamma_{mo}}$$

$$\text{or } \frac{A_{nf}}{A_{gf}} = \frac{f_y}{f_u} \cdot \frac{\gamma_{m1}}{\gamma_{mo}} \times \frac{1}{0.9}$$

However, if the above condition is not met, then the area of tension flange should be taken as

$$A_{nf} = A_{gf} \frac{f_y}{f_u} \times \frac{\gamma_{m1}}{\gamma_{mo}} \times \frac{1}{0.9}$$

- If bolt holes in the web of beams are generally considered to be insignificant as they have ^{almost} no effect on section modulus calculation

Case - II when $V > 0.6 V_d$ (high shear case)

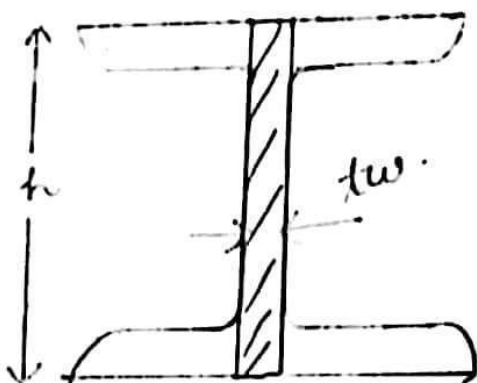
- Design bending strength (M_{dv}) in this case is less than the moment capacity of section (M_d) due to interaction b/w moment & shear. ($M_{dv} < M_d$)
- The reduced moment capacity M_{dv} can be determined as follows.
- for plastic & compact section

$$M_{dv} = M_d - \beta (M_d - M_{fd})$$

$$\text{check} \leq 1.2 Z_e \frac{f_y}{\gamma_{m0}}$$

M_d = plastic design moment capacity of section disregarding high shear force i.e. M_d is taken corresponding to $V \leq 0.6 V_d$.

M_{fd} = plastic design strength of area of cross-section excluding shear area & considering the partial factor of safety.



$$M_{fd} = \frac{f_y}{\gamma_{m0}} \left(Z_p - \frac{t_w h^2}{4} \right)$$

Note: $\rightarrow$ plastic section modulus of rectangle: $\frac{bd^2}{4}$.

$$\text{and } \beta = \left(\frac{2V}{V_d} - 1 \right)^2$$

V = factored S.F.

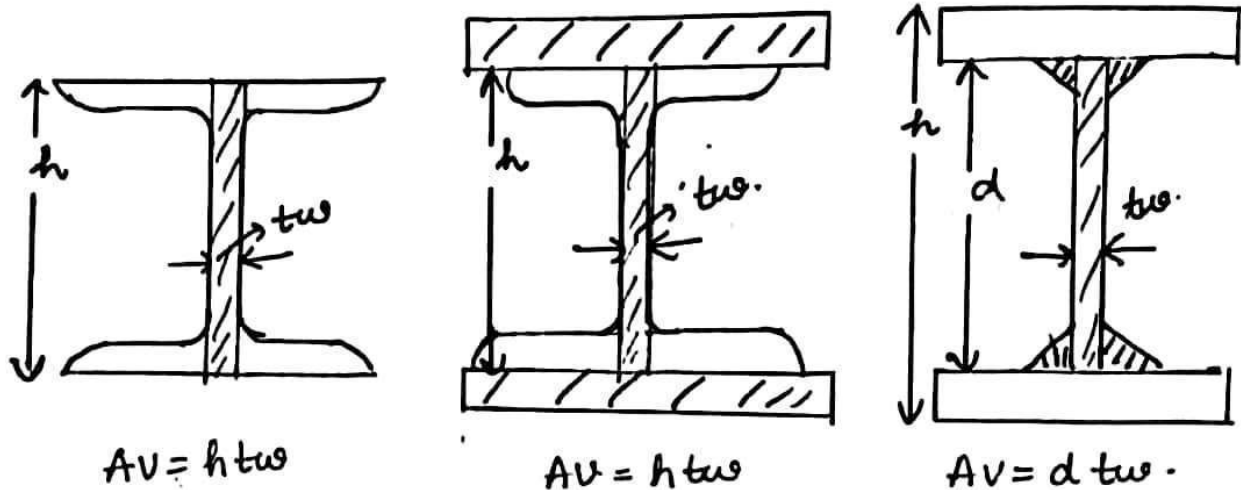
V_d = Design shear str. as governed by web yielding or web buckling

- for semi compact section, $M_d = Z_e \frac{f_y}{\gamma_{m0}}$

- The design shear strength, where there is no buckling of the web is given by

$$V_d = \frac{f_y A_v}{\sqrt{3} \gamma_{m0}}$$

Here A_v is shear area.



- for safety in shear, factored shear force (V) $\leq V_d$

Check for deflection:

- Calculation for max. deflection should be done using service load as deflection is serviceability criteria.
- The max. permissible deflection is given in IS code.
- In general case, max. deflection should be less than permissible deflection $\left(\frac{\text{span}}{300} \right)$

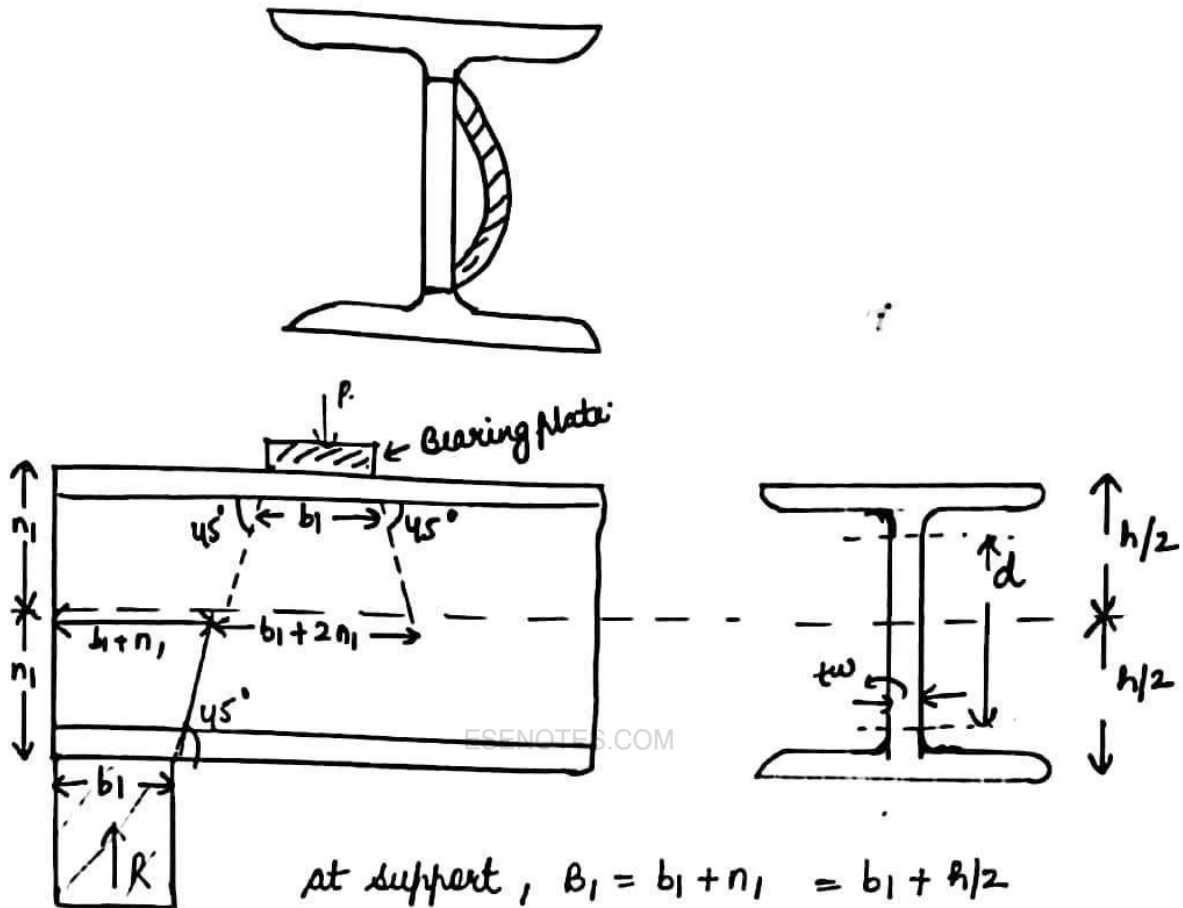
Check for web buckling.

- If $\frac{d}{t_w} \leq 67 \epsilon$, web is considered safe in web buckling (for low shear case)
- However in case of high shear, even if $\frac{d}{t_w} \leq 67 \epsilon$, then also web should be checked for web buckling.

- Hence, web is checked for buckling when:

a) $\frac{d}{t_w} > 67 \epsilon$

b) $V > 0.6 V_d$



at support, $B_1 = b_1 + n_1 = b_1 + h/2$

at point load, $B_1 = b_1 + 2n_1 = b_1 + h$

Here b_1 : stiffness bearing length.

- The web in a rolled steel section, behaves as a column when placed under concentrated load.
- The web buckling strength at support & below concentrated load (P) is given by.

$F_{wb1} = B_1 t_w f_{cd}$ (at support)

$F_{wb} = B_1 t_w f_{cd}$ (at point load)

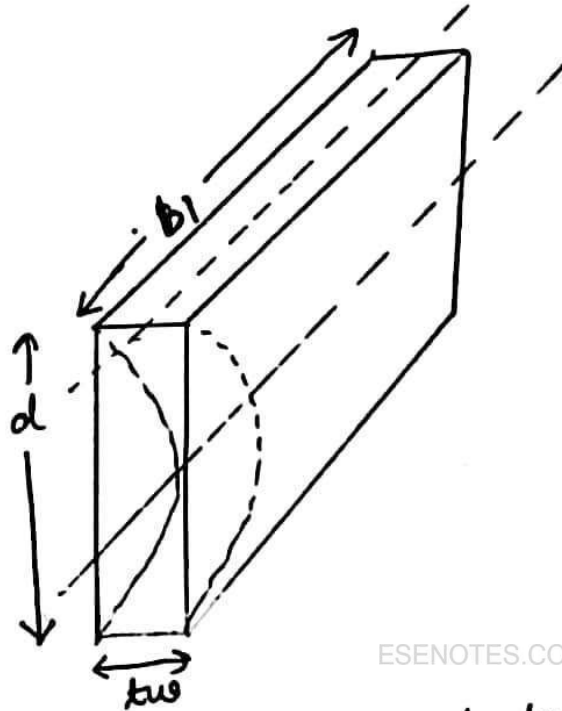
f_{cd} = design compressive stress corresponding to assumed web column

$$\lambda = \frac{l_{eff}}{r_{min}} = \frac{0.7d}{t_w/\sqrt{12}} = 0.7 \times \sqrt{12} \frac{d}{t_w}$$

$$\lambda = 2.42 \frac{d}{t_w}$$

$$\lambda \approx 2.5 \frac{d}{t_w}$$

$$\left\{ \begin{aligned} r_{min} &= \sqrt{\frac{I_{min}}{A}} \\ &= \sqrt{\frac{b t_w^3}{12 (b t_w)}} \\ &= \frac{t_w}{\sqrt{12}} \end{aligned} \right.$$



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- corresponding to this slenderness ratio (λ) & buckling class © find f_{cd} .

for safety $P \neq F_{wb}$

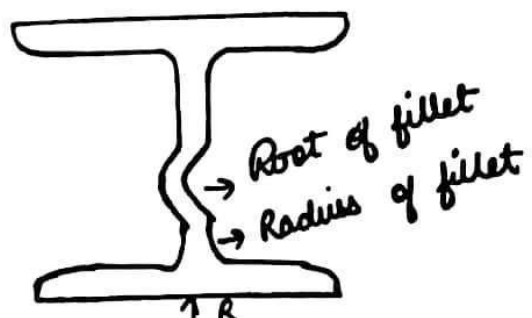
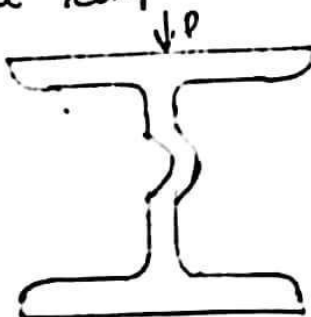
$R \neq F_{wb}$

Hence P & R are factored load.

Lesson 28 Jan 18.

Check for web crippling

- web crippling is local buckling of the web caused by the compressive stress delivered through the flange



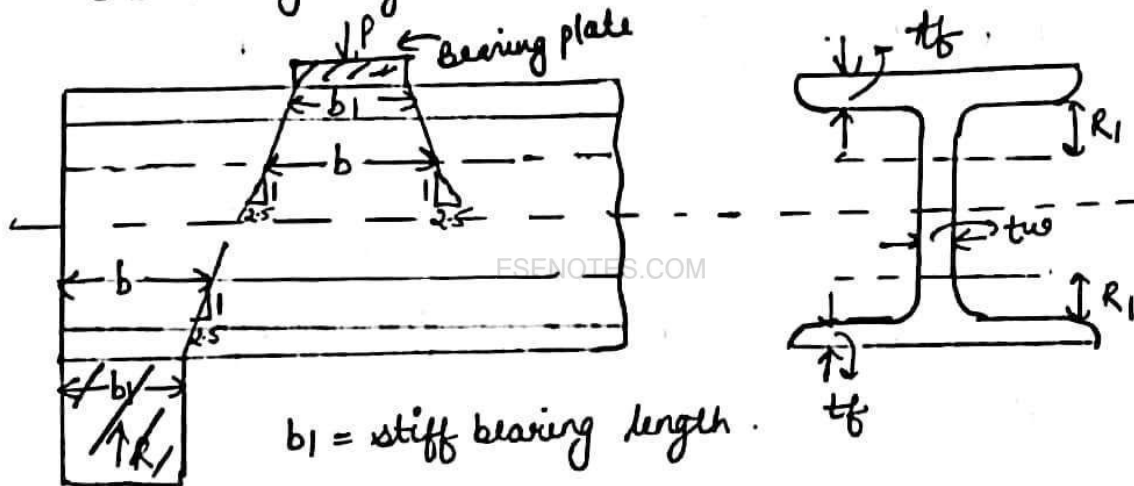
- The web of steel beam is subjected to large bearing compressive stresses under the concentrated loads & above the reactions from the support.
- The root of the fillet is the most critical location for failure as the resisting area is least at this section.
- The crippling strength or bearing strength of the web is given by

$$F_w = \frac{A_e f_{yw}}{\gamma_{mo}}$$

f_{yw} = yield stress of web.

A_e = effective area in bearing = $b t_w$.

b = bearing length as calculated below.



at support

$$b = b_1 + 2.5 (t_b + R_1)$$

$$F_{w1} = [b_1 + 2.5 (t_b + R_1)] t_w \frac{f_{yw}}{\gamma_{mo}}$$

for safety $P \nless F_{w1}$

$R \nless F_{w1}$

under concentrated load.

$$b = b_1 + 2 \times 2.5 (t_b + R_1)$$

$$F_w = [b_1 + 5 (t_b + R_1)] t_w \times \frac{f_{yw}}{\gamma_{mo}}$$

Note: If the beam is safe in crippling, normally it comes out to be safe in web buckling also.

Design procedure for Rolled Beams.

- Compute factored load & max BM & max SF.
- Choose a trial section for plastic section modulus

$$Z_p = \frac{M}{f_y / \gamma_{mo}}$$

- Classify the section on the basis $\frac{b}{t_f}$ & $\frac{d}{t_w}$ ratio as.

plastic, compact, semi-compact or slender.

- Trial section is chosen is checked for shear i.e.

$$V \leq V_d$$

Here
$$V_d = \frac{f_y h t_w}{\sqrt{3} \gamma_{mo}}$$

h = overall depth of beam
 t_w = thickness of web.

- Check either case is high shear or low shear.
- Check the ~~entire~~ trial section for design bending strength.

a) for low shear case ($V \leq 0.6 V_d$)

$$\begin{aligned} M_d &= \beta_b Z_p f_y / \gamma_{mo} \\ &\leq 1.2 Z_e f_y / \gamma_{mo} \\ &\leq 1.5 Z_e f_y / \gamma_{mo}. \end{aligned}$$

$\beta_b = 1$ (for plastic & compact section).

$$= \frac{Z_e}{Z_p} \text{ (for semi compact section)}$$

b) for high shear case ($V > 0.6 V_d$)

$$M = M_{dv} \text{ (for plastic & compact s/c)}$$

$$M = Z_e f_y / \gamma_{mo} \text{ (for semi compact s/c)}$$

$$M_{dv} = M_d - \beta (M_d - M_{fd}).$$

$$\beta = \left(\frac{2V}{V_d} - 1 \right)^2$$

$$\leq 1.2 z_e f_y / \gamma_{mo}.$$

If $M < M_d$, trial section is safe in bending.

- Check for web buckling, if a) $\frac{d}{t_w} > 67\epsilon$ b) $V > 0.6 V_d$.

- Check for web crippling.

- Check for deflection.

Q Determine the design bearing strength of ISLB: 350, considering the beam to be laterally supported. Assume low shear case. & steel of grade Fe 410.

Properties of ISLB: 350 are as follows.

$$h = 350 \text{ mm.}$$

$$b_f = 165 \text{ mm.}$$

$$t_f = 11.4 \text{ mm.}$$

$$t_w = 7.4 \text{ mm}$$

$$R_1 = 16 \text{ mm.}$$

$$I_{zz} = 1315.8 \times 10^4 \text{ mm}^4$$

$$I_{yy} = 631.9 \times 10^4 \text{ mm}^4.$$

$$Z_{pz} = 851.11 \times 10^3 \text{ mm}^3$$

$$Z_{zz} = 751.9 \times 10^3 \text{ mm}^3$$

$$x_y = 31.7 \text{ mm}$$

$$\epsilon = \sqrt{\frac{250}{f_y}}$$

	b/t_f	d/t_w
Plastic	9.4ϵ	84ϵ
Compact	10.5ϵ	105ϵ
Semi-compact	15.7ϵ	126ϵ

Solⁿ section classification

a) flange: $\frac{b}{t_f} = \frac{165/2}{11.4} = 7.23$

$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$\frac{b}{t_f} = 7.23 < 9.4 \epsilon = 9.4$$

⇒ flange is plastic.

b) web: $\frac{d}{t_w} = \frac{h - 2(t_f + R_1)}{t_w} = \frac{350 - 2(11.4 + 16)}{7.4} = 39.89$

$$< 84.6 \\ = 84$$

⇒ web is plastic.

∴ section is plastic.

for low shear case in plastic section.

$$M_d = \beta_b Z_p f_y / \gamma_{mo}$$

$$\leq 1.2 Z_e f_y / \gamma_{mo}$$

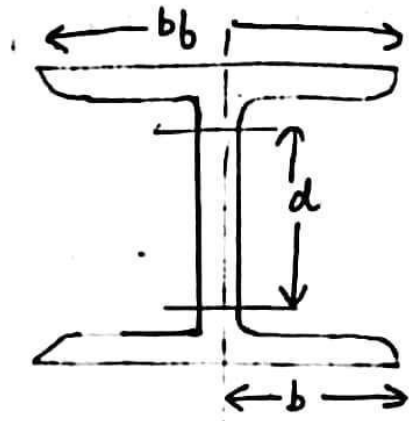
$\beta_b = 1$ for plastic section

$$M_d = 1 \times 851.11 \times 10^3 \times \frac{250}{1.1} \times 10^{-6} = 193.43 \text{ kN-m.}$$

$$\text{also } 1.2 Z_e f_y / \gamma_{mo} = 1.2 \times 751.9 \times 10^3 \times \frac{250}{1.1} \times 10^{-6} \\ = 205.06 \text{ kN-m.}$$

$$M_d = 193.43 \text{ kN-m} < 1.2 Z_e f_y / \gamma_{mo} \text{ Ok.}$$

$$\text{Hence } M_d = 193.43 \text{ kN-m.}$$



Q A simply supported steel beam of effective span 4m is laterally supported throughout, it carries a total UDL of 40 kN (inclusive of self wt.) Design an appropriate section using steel of grade Fe 410

Available section are

ISLB : 200	ISMB : 200
$h = 200 \text{ mm}$	$h = 200 \text{ mm}$
$Z_{pz} = 184.34 \times 10^3 \text{ mm}^3$	$Z_{pz} = 253.8 \times 10^3 \text{ mm}^3$
$b_f = 100 \text{ mm}$	$b_f = 100 \text{ mm}$
$t_f = 7.3 \text{ mm}$	$z_{pz} = 223.5 \times 10^3 \text{ mm}^3$
$t_w = 5.4 \text{ mm}$	$t_f = 10.8 \text{ mm}$
$R_1 = 9.5 \text{ mm}$	$t_w = 5.7 \text{ mm}$
	$R_1 = 11.0 \text{ mm}$

Total load = 40 kN

$$wL = 40$$

$$w = \frac{40}{L} = \frac{40}{4} = 10 \text{ kN/m}$$

- Factored load = $1.5w = 1.5 \times 10 = 15 \text{ kN/m}$

$$\text{Maximum BM (M)} = \frac{1.5wL^2}{8} = \frac{15 \times 4^2}{8} = 30 \text{ kN-m}$$

$$\text{Maximum SF (V)} = \frac{1.5wL}{2} = \frac{15 \times 4}{2} = 30 \text{ kN}$$

$$Z_{p \text{ req}} = \frac{M}{f_y / \gamma_{m0}} = \frac{30 \times 10^6}{250 / 1.1} = 132 \times 10^3 \text{ mm}^3$$

Adopt ISLB : 200, $Z_p = 184.34 \times 10^3 \text{ mm}^3$

- section classification

$$t = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

for flange : $\frac{b}{t_f} = \frac{100/2}{7.3} = 6.48 < 9.46 = 9.4 \Rightarrow$ flange is plastic.

for web : $\frac{d}{t_w} = \frac{h - 2(t_f + R_1)}{t_w} = \frac{200 - 2(7.3 + 9.5)}{5.4} = \frac{30.8}{5.4} = 84 < 84$

$\Rightarrow$ web is plastic.

Hence, overall section is plastic.

- Check for shear

$$V_d = \frac{f_y A_{tw}}{\sqrt{3} \gamma_{mo}} = \frac{250 \times 200 \times 5.4 \times 10^{-3}}{\sqrt{3} \times 1.1} = 141.71 \text{ kN}$$

$$V = 30 \text{ kN} < V_d = 141.71 \text{ kN} \Rightarrow \text{safe in shear}$$

- Check for shear case,

$$V < 0.6 V_d = 0.6 \times 141.71 = 85.02 \text{ kN} \Rightarrow \text{low shear case.}$$

- Check for Bending.

$$M_d = \beta_b Z_p f_y / \gamma_{mo}$$

$$= 1 \times 184.34 \times 10^3 \times \frac{250}{1.1} \times 10^{-6} = 41.89 \text{ kN-m}$$

$$\text{also } 1.2 Z_e f_y / \gamma_{mo} = 1.2 \times 169.7 \times 10^3 \times \frac{250}{1.1} \times 10^{-6}$$

$$= 46.28 \text{ kNm.}$$

$$M_d < 1.2 Z_e f_y / \gamma_{mo} \Rightarrow \text{OK.}$$

$$\text{Now } M < M_d = 41.89 \text{ kNm} \Rightarrow \text{safe in Bending}$$

- Check for Buckling

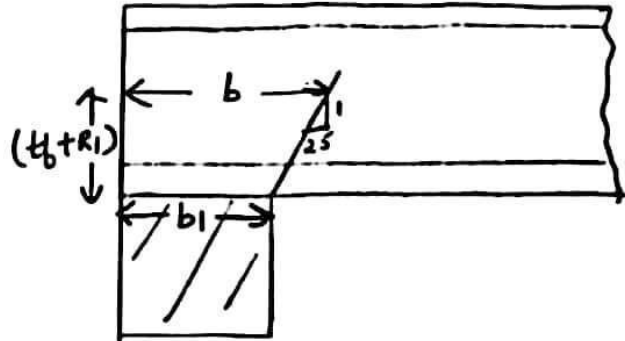
$$\frac{d}{t_w} = 30.8 < 67.6 = 67 \Rightarrow \text{safe in web buckling.}$$

→ check for web crippling.

assume, stiff bearing length $b_1 = 75 \text{ mm}$.

bearing length $b = b_1 + 2.5(t_f + R_1)$.

$$b = 75 + 2.5(7.3 + 9.5) \\ = 117 \text{ mm}.$$



$$F_w = \frac{f_y b t_w}{\gamma_{m0}}$$

$$= \frac{250 \times 117 \times 5.4 \times 10^{-3}}{1.1}$$

$$= 143.59 \text{ kN}$$

$R = V = 30 \text{ kN} < F_w \Rightarrow$ safe in crippling.

⇒ check for deflection.

$$\delta_{\max} = \frac{5}{384} \frac{W l^4}{E I_{zz}}$$

$$= \frac{5}{384} \times \frac{(10 \times 4 \times 10^3)^4}{2 \times 10^5 \times 169.7 \times 10^5}$$

$$= 9.82 \text{ mm}$$

$$I_{zz} = Z_{ezz} y_{\max}$$

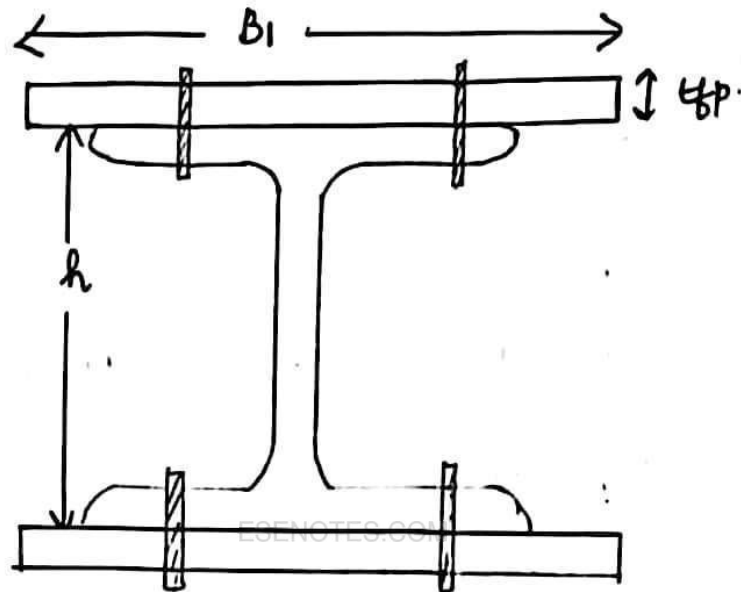
$$= 169.7 \times 10^3 \times \frac{200}{2}$$

$$= 169.7 \times 10^5 \text{ mm}^4$$

$$\delta_{\max} < \frac{l}{300} = \frac{4 \times 10^3}{300} = 13.33 \text{ mm} \Rightarrow \text{safe.}$$

Built-up Beams (Plated Beams)

- For long spans & heavy loads large BM are generated in such case is available rolled beam sections do not have sufficient strength to resist the external BM, then they may be reinforced with cover plates along the entire length or part of it.
- This is termed as Built up beam or plated beam.



$$I_{p \text{ reqd}} = I_p + 2A_p \left(\frac{h}{2} \right)^2 \quad \text{if } t_f \text{ is neglected.}$$

$I_{p \text{ reqd}}$: plastic moment of inertia
reqd. to resist the factored moment.

$$I_p + 2A_p \left(\frac{h}{2} + \frac{t_f}{2} \right)^2 \quad \text{neglected.}$$

I_p : plastic moment of inertia of rolled section.

A_p : area of the flange cover plate = $B_1 t_{fp}$.

B_1 ⇒ width of flange cover plate.

t_{fp} ⇒ thickness of flange cover plate.

$$I_{p \text{ reqd}} = I_p + 2A_p \left(\frac{h}{2} \right)^2.$$

$$\frac{I_{p \text{ reqd}}}{h/2} = \frac{I_p}{h/2} + A_p h.$$

$$Z_{p\text{ reqd}} = Z_{p\text{ rolled}} + A_p h$$

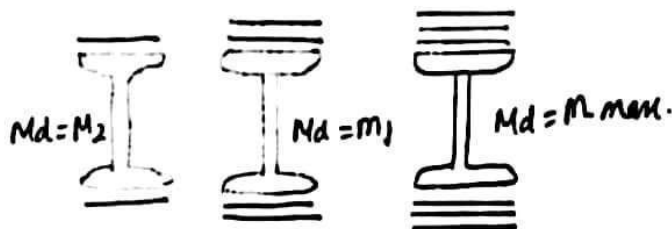
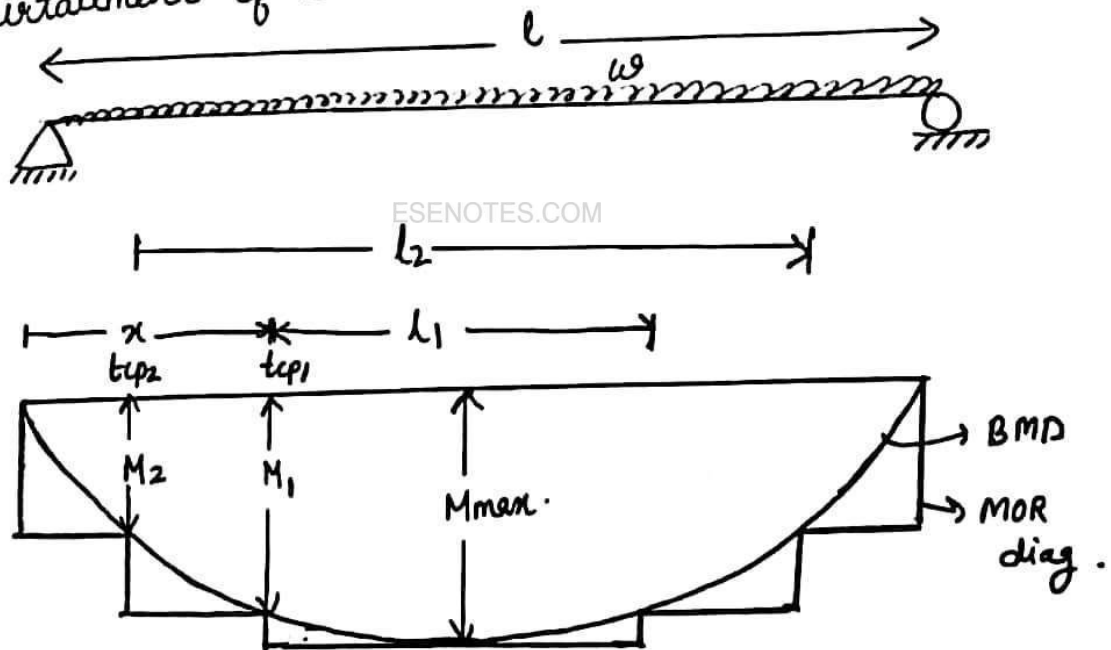
$$A_p = \frac{Z_{p\text{ reqd}} - Z_{p\text{ rolled}}}{h}$$

$$Z_{p\text{ reqd}} = \frac{M}{f_y / \gamma_{m0}}$$

⇒ If a bolted connection is used to account for loss in area due to holes, the area of the plate is increased by 10%.

Lesson 29. Jan 19.

Curtailment of the Plates.



- MOR diagram should envelope BMD.
- At least one plate must continue throughout the span.
- Theoretical point of cutoff is the point at which MOR of the continuing beam is equal to applied BM.

for eg: To compute t_{cp}

$$BM_x = \frac{wL}{2}x - \frac{wx^2}{2} = M_1$$

from above eq, " x " can be computed

$$x = \frac{L - l_1}{2} \Rightarrow \text{from here "l}_1\text{" can be computed}$$

- Normally curtailment will be done beyond theoretical point of cutoff (t_{cp}) so that connection on the extension can develop the load in the plate at the cutoff point & will also help in developing the BM.

Q Design a laterally supported beam of effective span 6m for the following data, grade of steel is Fe410, Max BM = 150 kN-m, max SF = 210 kN. Check for deflection is not required. Assume that the given moment & shear to be factored moment & shear. Available sections are:->

ISLB : 325	ISLB : 350
$h = 325 \text{ mm}$	$h = 350 \text{ mm}$
$b_f = 165 \text{ mm}$	$b_f = 165 \text{ mm}$
$t_f = 9.8 \text{ mm}$	$t_f = 11.4 \text{ mm}$
$t_w = 7 \text{ mm}$	$t_w = 7.4 \text{ mm}$
$R_1 = 10 \text{ mm}$	$R_1 = 16 \text{ mm}$
$Z_{pz} = 687.76 \times 10^3 \text{ mm}^3$	$Z_{pz} = 851.11 \times 10^3 \text{ mm}^3$
$Z_{ez} = 607.7 \times 10^3 \text{ mm}^3$	$Z_{ez} = 751.9 \times 10^3 \text{ mm}^3$

- $BM = 150 \text{ kN-m}$, $V = 210 \text{ kN}$.

- $Z_{p reqd} = \frac{M}{f_y / \gamma_{mo}} = \frac{150 \times 10^6}{250 / 1.1} = 660 \times 10^3 \text{ mm}^3$.

Provide ISLB 325, $Z_p = 687.76 \times 10^3 \text{ mm}^3$.

- Classify the section

$$\epsilon = \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

for flange: $\frac{b}{t_f} = \frac{165/2}{9.8} = 8.141 < 9.4 \epsilon = 9.4 \Rightarrow \text{plastic}$

for web: $\frac{d}{t_w} = \frac{325 - 2(9.8 + 10)}{7} = 40.77 < 84 \epsilon = 84 \Rightarrow \text{plastic}$

Hence, section is plastic.

$\Rightarrow$ check for shear.

$$V_d = \frac{f_y A_{tw}}{\sqrt{3} \gamma_{mo}} = \frac{250 \times 325 \times 7 \times 10^{-3}}{\sqrt{3} \times 1.1} = 298.5 \text{ kN}.$$

$V = 210 \text{ kN} < V_d = 298.5 \text{ kN} \Rightarrow \text{safe in shear.}$

$V > 0.6 V_d \Rightarrow 0.6 \times 298.5 = 179.1 \text{ kN} \Rightarrow \text{high shear case}$

$\Rightarrow$ check for Bending.

For plastic section, $M_{dv} = M_d - \beta (M_d - M_{fd})$

$$M_d = \beta_b Z_p \frac{f_y}{\gamma_{mo}} = 1 \times 687.76 \times 10^3 \times \frac{250}{1.1} \times 10^{-6} = 156.3 \text{ kN}.$$

$$1.2 Z_e \frac{f_y}{\gamma_{mo}} = 1.2 \times 607.7 \times 10^3 \times \frac{250}{1.1} \times 10^{-6} = 165.73 \text{ kNm}$$

$$M_d < 1.2 Z_e \frac{f_y}{\gamma_{mo}} \Rightarrow \underline{\underline{OK}}$$

$$M_{fd} = \frac{b_f}{\gamma_{mo}} \left(z_p - \frac{t_w h^2}{4} \right)$$

$$= \frac{250}{1.1} \left(687.76 \times 10^3 - 7 \times \frac{325^2}{4} \right) \times 10^6$$

$$= 114.3 \text{ kN-m}$$

$$\beta = \left(\frac{2V}{V_d} - 1 \right)^2 = \left(\frac{2 \times 210}{298.5} - 1 \right)^2 = 0.165$$

$$M_{dv} = 156.3 - 0.165 (156.3 - 114.3) = 149.37 \text{ kN-m}$$

$$M_{dv} = 149.37 \text{ kN-m} < M = 150 \text{ kN} \Rightarrow \text{Not safe in bending.}$$

Hence Adopt ISLB: 350, $z_p = 851.11 \times 10^3 \text{ mm}^3$

- classify the section
 $\epsilon = 1$

$$\text{for flange: } \frac{b}{t_f} = \frac{165/2}{11.4} = 7.23 < 9.4 \epsilon = 9.4 \Rightarrow \text{flange is plastic.}$$

$$\text{for web: } \frac{d}{t_w} = \frac{350 - 2(114 + 16)}{7.4} = 39.89 < 84 \epsilon = 84 \Rightarrow \text{plastic}$$

Hence, section is plastic

- Check for shear,

$$V_d = \frac{b_f h t_w}{\sqrt{3} \gamma_{mo}} = \frac{250 \times 350 \times 7.4 \times 10^{-3}}{\sqrt{3} \times 1.1} = 339.85 \text{ kN}$$

$$V = 210 \text{ kN} < V_d = 339.85 \text{ kN} \Rightarrow \text{safe in shear}$$

$$- V > 0.6 V_d = 0.6 \times 339.85 = 203.9 \text{ kN} \Rightarrow \text{high shear case}$$

- check for Bending

$$M_{dv} = M_d - \beta (M_d - M_{fd})$$

$$M_d = \beta_b z_p \frac{b_f}{\gamma_{mo}} = 1 \times 851.11 \times 10^3 \times \frac{250}{1.1} \times 10^{-6} = 193.43 \text{ kN-m}$$

$$1.2 Z_e \frac{f_y}{\gamma_{m0}} = 12 \times 751.9 \times 10^3 \times \frac{250}{1.1} = 250.06 > M_d = \underline{OK}$$

$$\beta = \left(\frac{2V}{V_d} - 1 \right)^2 = \left(\frac{2 \times 210}{339.85} - 1 \right)^2 = 0.055$$

$$M_{fd} = \frac{b_d}{\gamma_{m0}} \left(Z_p - \frac{t_w h^2}{4} \right) = \frac{250}{1.1} \left(851.11 \times 10^3 - \frac{7.4 \times 350^2}{4} \right) \times 10^{-6}$$

$$= 141.92 \text{ kN-m}$$

$$M_{dv} = 193.43 - 0.055 (193.43 - 141.92) = 190 \text{ kN-m} > M = 150$$

⇒ Safe in Bending.

- Check for web buckling,

Assume stiff bearing length $b_1 = 100 \text{ mm}$.

$$B_1 = b_1 + \frac{h}{2} = 100 + \frac{350}{2} = 275 \text{ mm}$$

$$F_{web} = B_1 t_w f_{cd}$$

$$\lambda = 2.5 \frac{d}{t_w} = 2.5 \times \frac{295.2}{7.4} = 99.72$$

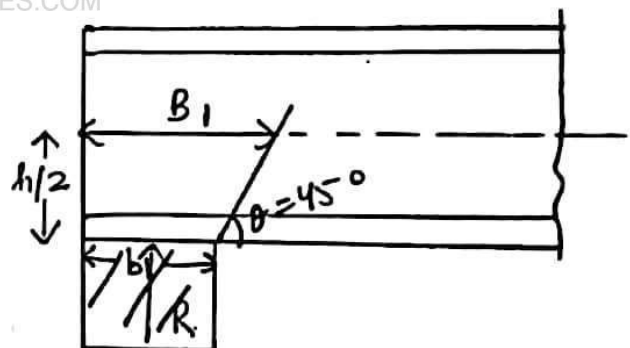
for buckling class (C)

$$\eta = 0.4727$$

$$f_{cd} = \frac{f_y}{\gamma_{m0}} \eta = \frac{250}{1.1} \times 0.4727 = 107.44 \text{ N/mm}^2$$

$$F_{web} = 275 \times 7.4 \times 107.44 \times 10^{-3} = 218.64 \text{ kN} > R = V = 210 \text{ kN}$$

⇒ Safe in web buckling



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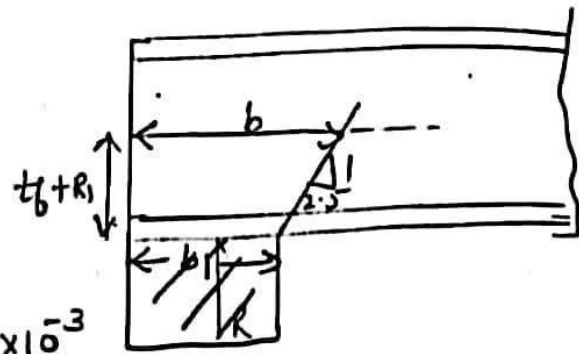
- Check for web crippling

$$b = 100 \text{ mm}$$

$$b = b_1 + 2.5(t_f + R_1)$$

$$= 100 + 2.5(11.4 + 16)$$

$$= 168.5 \text{ mm}$$



$$F_w = \frac{b_f}{\gamma_{mo}} b_{tw} = \frac{250}{1.1} \times 168.5 \times 7.4 \times 10^{-3}$$

$$= 283.38 \text{ kN} > R = V = 210 \text{ kN}$$

$\Rightarrow$ safe in web crippling

Q Design a steel beam section for supporting the roof of a hall for the following data: Assume Fe 410 grade steel. clear span = 6.5 m, end bearing = 150 mm, C/C spacing of beam = 3 m. Imposed load on the beam = 10 kN/m^2 , $DL = 4 \text{ kN/m}^2$.

Restriction on beam depth = 375 mm. Assume the compression flange of the beam to be laterally supported throughout section available as IS WB 350 with following properties

$$h = 350 \text{ mm}$$

$$t_w = 8 \text{ mm}$$

$$b_f = 200 \text{ mm}$$

$$R_1 = 12 \text{ mm}$$

$$t_f = 11.4 \text{ mm}$$

$$I_{zz} = 15521.7 \times 10^4 \text{ mm}^4$$

$$Z_{pz} = 995.49 \times 10^3 \text{ mm}^3$$

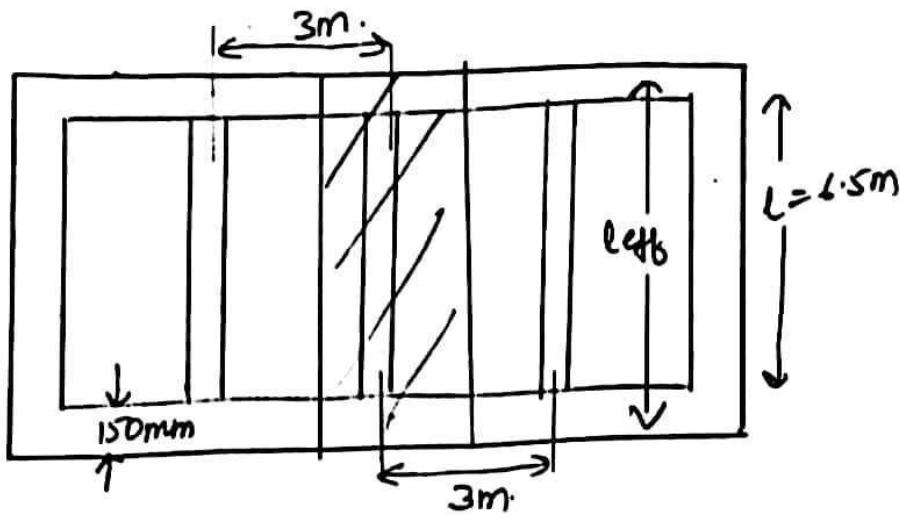
$$Z_{ez} = 887 \times 10^3 \text{ mm}^3$$

Lesson 30 Jan 20

$$\text{Total load } W = 1.5 (U + DL)$$

$$= 1.5 (10 + 4) \times 3 = 63 \text{ kN/m}$$

$$l_{eff} = l + \frac{2(150)}{2} \times 10^{-3} = 6.5 + 0.15 = 6.65 \text{ m}$$



Design moment $M = \frac{w l_{eff}^2}{8} = \frac{63 \times 6.65^2}{8} = 348.25 \text{ kN-m}$

Design shear force $V = \frac{w l_{eff}}{2} = \frac{63 \times 6.65}{2} = 209.475 \text{ kN}$

$Z_{p req} = \frac{M}{f_y / \gamma_{mo}} = \frac{348.25 \times 10^6}{250/1.1} = 1.5323 \times 10^6 \text{ mm}^3$

$Z_{p rolled} = 995.49 \times 10^3 \text{ mm}^3$

Since $Z_{p reqd} >> Z_{p rolled} \Rightarrow$ Hence flange cover plate is provided such that area of plate on one side is given by

$$A_p = \frac{Z_{p reqd} - Z_{p rolled}}{h}$$

$$A_p = \frac{1.523 \times 10^6 - 995.49 \times 10^3}{350} = 1533.74 \text{ mm}^2$$

adopt the width of flange cover plate $b_{fp} = 200 \text{ mm}$.

Thickness of flange cover plate $t_{fp} = \frac{A_p}{b_{fp}} = \frac{1533.74}{200} = 7.67 \text{ mm}$

Adopt $t_{fp} = 8 \text{ mm}$.

$$Z_{p \text{ provided}} = 995.49 \times 10^3 + 200 \times 8 \times 2 \left(\frac{350+8}{2} \right) \\ = 1.56 \times 10^6 \text{ mm}^3.$$

$$Z_{p \text{ provided}} > Z_{p \text{ reqd}} \Rightarrow \underline{\text{OK}}$$

$$\text{Total depth of beam} = 350 + 2 \times 8 \\ = 366 \text{ mm} \\ < 375 \text{ mm} \\ \Rightarrow \underline{\text{OK}}$$

$\Rightarrow$ classify the ~~star~~ section

$$C = \sqrt{\frac{250}{250}} = 1$$

$$\text{for flange: } \frac{b}{t_f} = \frac{200/2}{11.4} = 8.77 < 9.4 \epsilon \Rightarrow 9.4 \times 1 = 9.4 \Rightarrow \text{plastic}$$

$$\text{for web: } \frac{d}{t_w} = \frac{350 - 2(11.4 + 12)}{8} = 37.9 < 84 \epsilon = 84 \Rightarrow \text{plastic}$$

Hence; section is plastic

$\Rightarrow$ check for shear.

$$V_d = \frac{b_f h t_w}{\sqrt{3} \times 1.1} = \frac{250 \times 350 \times 8 \times 10^{-3}}{\sqrt{3} \times 1.1} = 367.4 \text{ kN}$$

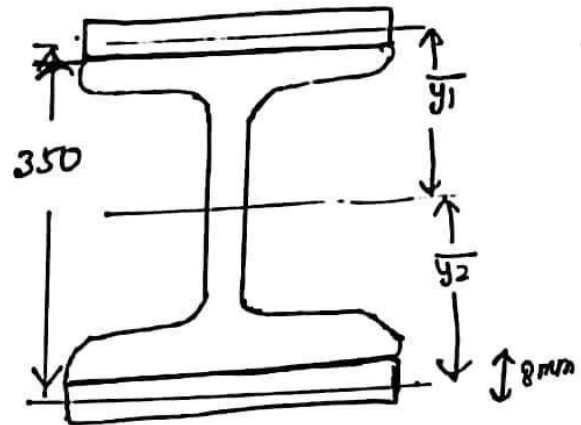
$$V = 209.475 \text{ kN} < V_d = 367.4 \text{ kN} \Rightarrow \text{safe.}$$

$$- V < 0.6 V_d = 0.6 \times 367.4 = 220.44 \text{ kN} \Rightarrow \text{low shear case.}$$

check for Bending.

$$M_d = \beta_b Z_p \frac{f_y}{\gamma_{m0}} \leq 1.2 Z_e \frac{f_y}{\gamma_{m0}}$$

$$= 1 \times 1.56 \times 10^6 \times \frac{250}{1.1} \times 10^{-6} = 354.54 \text{ kN-m}$$



$$\text{Now, } 1.2 Z_e \frac{f_y}{\gamma_{m0}} = 1.2 \left[15521.7 \times 10^4 + 2 \left(\frac{200 \times 8^3}{12} + 200 \times 8 (4+175)^2 \right) \right] \times \frac{250}{175+8. \quad \frac{250}{1.1}}$$

$$= 384.15 \times 10^6 \text{ N-mm.}$$

$$\Rightarrow 384.15 \text{ kN-m} > M_d \Rightarrow \text{OK.}$$

$M_d > M \Rightarrow$ safe in Bending.

$\Rightarrow$ check for web buckling.

$$\frac{d}{t_w} = 37.9 < 67.6 = 67 \Rightarrow \text{safe in web buckling.}$$

$\Rightarrow$ check for web crippling.

$$b = b_1 + 2.5 (t_f + t_w + R_1)$$

$$150 + 2.5 (8 + 11.4 + 12) = 228.5 \text{ mm}$$

$$F_w = \frac{f_y b t_w}{\gamma_{m0}} = \frac{250 \times 228.5 \times 8 \times 10^{-3}}{1.1} = 415.4 \text{ kN.}$$

$$R = V = 209.475 \text{ kN} < F_w.$$

$\Rightarrow$ safe in web crippling.

$\Rightarrow$ Check for deflection

$$\delta_{\max} = \frac{5}{384} \frac{w l^4}{E I_{zz}}$$

$$= \frac{5}{384} \times \frac{63}{1.5} \times \frac{(6.65 \times 10^3)^4}{2 \times 10^5 \times 257.765 \times 10^6}$$

$$\delta_{\max} = 20.74 \text{ mm}$$

$$\delta_{\text{permitted}} = \frac{l_{\text{eff}}}{300} = \frac{6.65 \times 10^3}{300} = 22.16 \text{ mm.}$$

$\delta_{\max} < \delta_{\text{permitted}} \Rightarrow$ Safe in deflection.

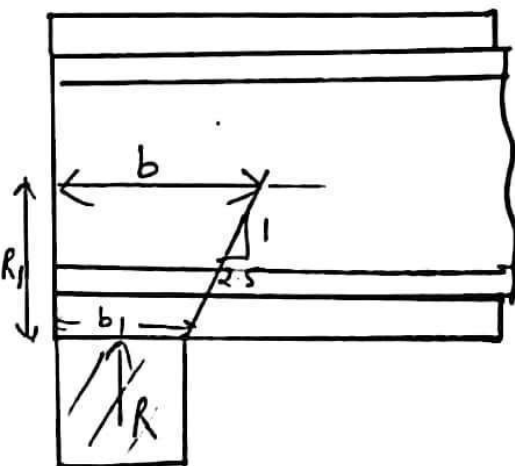
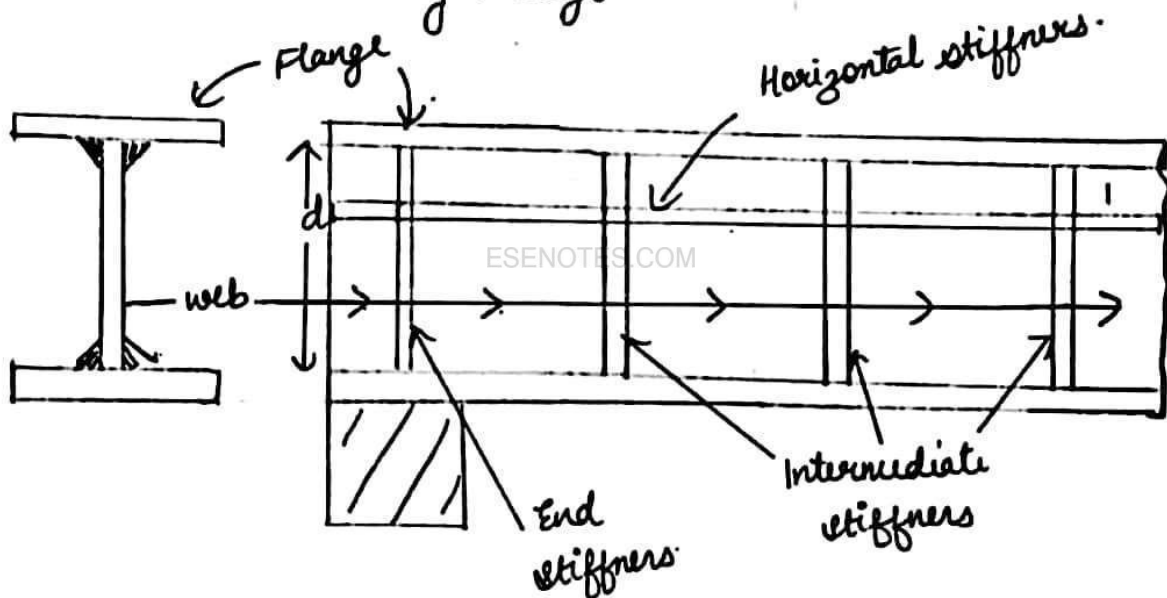


Plate Girders

Design of Plate Girders

- When span & load increases, the available rolled section may not be sufficient even after strengthening with cover plates.
- Such situations are common in the following cases.
 - a) Large column free halls in the lower floor of a multi storey building.
 - b) In a workshop where girders are required to carry crane beams.
 - c) In road or railway bridge.



Note :- In today's time only welded plate girders are built as bolted / riveted plate girders are heavy, costly, unaesthetic.

Ques 32 Jan 22

a) Web: Web of required depth & thickness are provided to keep the flange plates at required distance & to resist the shear in the plate girder.

b) Flange: Flange of required width & thickness are provided to resist bending moment acting on the beam.

c) Stiffeners: These are provided to safeguard the web against local buckling failure

- These are of two types

(i) Transverse / Vertical stiffeners

(ii) Longitudinal / Horizontal stiffeners

(i) Transverse stiffeners: These are further of two types

1) Bearing / End stiffeners: These are provided to transfer the load from the beam to the support.

- At the end section, portion of the web of beam acts as a compression member & hence there is possibility of crushing of web, there by web needs stiffener to transfer the load to the support.

2) Intermediate stiffeners: If concentrated load acting on a plate girder (due to cross beam), Intermediate stiffeners are required.

- To resist average shear stress, the thickness of web required is very less, but if it is provided it may result in to buckling due to shear.

- Hence when thin webs are used, intermediate transverse stiffeners are provided to improve the buckling strength of web.

(ii) Longitudinal stiffeners: These are also provided to increase the buckling strength of the web.

- If only one longitudinal stiffener is provided, it

is at the depth of 0.2d from the compression flange & if another longitudinal stiffener is to be provided it is at mid depth of the web.

Self wt. of plate girder.

- self wt. of the plate girder is found empirically as follows.

$$w = \frac{W}{200}$$

w = factored self wt.

W = Total factored load on girder

Economical depth

- The depth of web (d) which would require least cost for given gross cross-sectional area of the girder (A) is termed as "economical depth" of girder.

- Assuming that the moment " M " is carried out by flanges only.

$$M = \frac{f_y}{\gamma_{mo}} A_f d$$

$$A_f = \frac{M \gamma_{mo}}{f_y \cdot d}$$

f_y = yield stress in flange plate.

Now, total gross sectional area of girder (A) = $2A_f + dtw$

$$A = 2 \frac{M \gamma_{mo}}{f_y \cdot d} + dtw$$

$$\text{Let } \frac{d}{tw} = K.$$

$$A = \frac{2 M \gamma_{mo}}{f_y \cdot d} + \frac{d^2}{K}$$

for economical depth $\frac{dA}{dd} = 0$.

$$\frac{dA}{dd} = \frac{2M\gamma_{mo}}{f_y} \left(-\frac{1}{d^2}\right) + \frac{2d}{K} = 0$$

$$\frac{d}{K} = \frac{M\gamma_{mo}}{f_y d^2}$$

$$d^3 = \frac{M\gamma_{mo} K}{f_y}$$

$$d = \left(\frac{M\gamma_{mo} K}{f_y} \right)^{1/3}$$

- In selecting the value of $K = \frac{d}{t_w}$, the following codal provisions are useful.

a) If $\frac{d}{t_w} \leq 67\epsilon$, it may be designed as ordinary beam, where shear buckling is absent, where $\epsilon = \sqrt{\frac{250}{f_y}}$.

b) Minimum web thickness based on serviceability requirement

(i) When transverse stiffeners are not provided:

$$\frac{d}{t_w} \leq 200\epsilon_w, \text{ (web connected to flanges along}$$

both longitudinal edges), $\frac{d}{t_w} \leq 196K_1$.

and $\frac{d}{t_w} \leq 90\epsilon_w$ (web connected to flange along one longitudinal edge only).

(ii) When the transverse stiffeners are provided.

• If $3d \geq C \geq d$: $\frac{d}{t_w} \leq 200\epsilon_w$

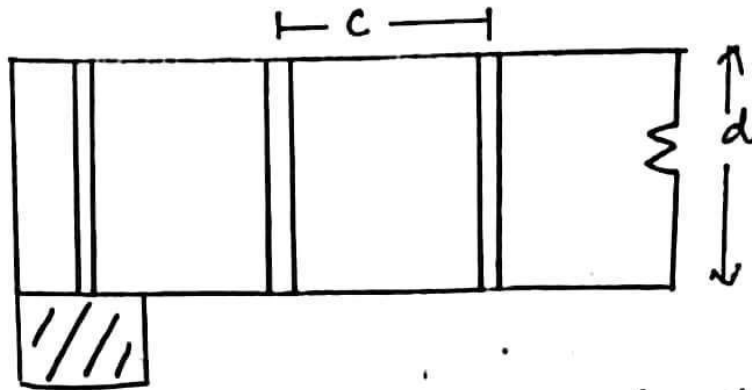
• If $d > C \geq 0.74d$: $\frac{d}{t_w} \leq 200\epsilon_w$

$$c < d \text{ in case}$$

- If $(c < 0.74d)$: $\frac{d}{t_w} \leq 270 \epsilon_w$

- If $c > 3d$: web shall be considered unstiffened

$$\epsilon_w = \sqrt{\frac{250}{f_{yw}}}, \quad f_{yw} \Rightarrow \text{yield stress of web.}$$



$c = c/c$ spacing of intermediate stiffener.

(iii) When transverse stiffeners & longitudinal stiffeners at one level only are provided ($0.2d$ from compression flange).

- If $2.4d \geq c \geq d$: $\frac{d}{t_w} \leq 250 \epsilon_w$

- If $d \geq c \geq 0.74d$: $\frac{c}{t_w} \leq 250 \epsilon_w$

- If $c < 0.74d$: $\frac{d}{t_w} \leq 340 \epsilon_w$

(iv) ~~If~~ When a second longitudinal stiffener is provided (at mid depth).

Here

$$\frac{d}{t_w} \leq 400 \epsilon_w$$

c) Minimum web thickness based on compression flange buckling requirement.

- In order to avoid buckling of the compression flange into the web, thickness shall satisfy the following

(i) when transverse stiffeners are not provided: $\frac{d}{t_w} \leq 345 \epsilon_f^2$

(ii) when transverse stiffeners are provided.

• $C \geq 1.5d$: $\frac{d}{t_w} \leq 345 \epsilon_f^2$

• $C < 1.5d$: $\frac{C}{t_w} \leq 345 \epsilon_f^2$ $\frac{d}{t_w} \leq 345 \epsilon_f$ in code.

$\epsilon_f = \sqrt{\frac{250}{f_{yf}}}$, f_{yf} = yield stress of flange.

d) Guideline for selecting the depth of plate girder (D)

$\frac{D}{L} = \frac{1}{15}$ to $\frac{1}{25}$ for girders in building

$= \frac{1}{2}$ to $\frac{1}{18}$ for highway girders

$= \frac{1}{10}$ to $\frac{1}{15}$ for railway girders

D = overall depth of girder (including flange thickness)

L = Equivalent span of the girder.

Size of flange.

- Assuming moment is resisted by flanges only

$$M = \frac{f_y}{\gamma_{mo}} A_f \cdot d.$$

$$A_f = \frac{M \gamma_{mo}}{f_y \cdot d}$$

$$b_f t_f = \frac{M \gamma_{mo}}{f_y \cdot d} \quad \text{--- (i)}$$

Select $9.46 < \frac{b}{t_f} < 13.6 \epsilon$, so that bending strength can be computed using expression of semi compact section. — (ii).

from (i) & (ii) $b_f \times t_f$ can be computed.

Shear Buckling Resistance of Web.

— For thin webs, it is necessary to check the shear resistance of web for buckling.

$\frac{d}{t_w} > 67 \epsilon$, for a web without stiffener

$\frac{d}{t_w} > 67 \epsilon \sqrt{\frac{K_v}{5.35}}$, for a web with stiffener

here, $K_v = 5.35$, when transverse stiffeners are provided at supports

$$K_v = 4 + \frac{5.35}{\left(\frac{c}{d}\right)^2} \quad \text{for } \frac{c}{d} < 1.0$$

$$= 5.35 + \frac{4}{\left(\frac{c}{d}\right)^2}, \quad \text{for } \frac{c}{d} \geq 1.0$$

c = spacing of transverse stiffeners

d = depth of the web.

Now, the nominal shear strength of the web can be computed by following method. (Simple Post critical Method)

- This method can be used for plate girders with or without transverse stiffener
- According to this method.

$$V_n = V_{cr} = A_v \tau_b$$

τ_b = shear stress corresponding to web buckling which is found as follows.

a) If $\lambda_w \leq 0.8$: $\tau_b = \frac{f_{yw}}{\sqrt{3}}$

f_{yw} = yield stress of web material

b) If $1.2 > \lambda_w > 0.8$: $\tau_b = [1 - 0.8(\lambda_w - 0.8)] \frac{f_{yw}}{\sqrt{3}}$

c) If $\lambda_w \geq 1.2$: $\tau_b = \frac{f_{yw}}{\sqrt{3} \lambda_w^2}$

λ_w = non-dimensional web slenderness ratio for shear buckling.

$$\lambda_w = \left[\frac{f_{yw}}{\sqrt{3} \tau_{cr}} \right]^{1/2}$$

τ_{cr} = elastic critical shear stress of web.

$$= \frac{K_v \pi^2 E}{12(1-\mu^2) \left(\frac{d}{t_w} \right)^2}$$

μ = poisson's ratio of steel = 0.3

Design of connection b/w flange & web plates.

- If "V" is shear force acting on the section, then shear stress at the junction

$$q_w = \frac{VA\bar{y}}{I_z b}$$

$$\text{Hence shear force per unit length} = q_w(b \times 1) = \frac{VA\bar{y}}{I_z}$$

Let weld of throat thickness "t" is provided on both sides then, strength of shop weld per unit length =

$$\text{ultimate} = \frac{2t f_w}{\sqrt{3}} \times \frac{1}{1.25} \quad \text{--- (ii)}$$

from (i) & (ii), throat thickness of weld can be computed. from which size of weld is obtained as

$$S = \frac{t}{0.7}$$

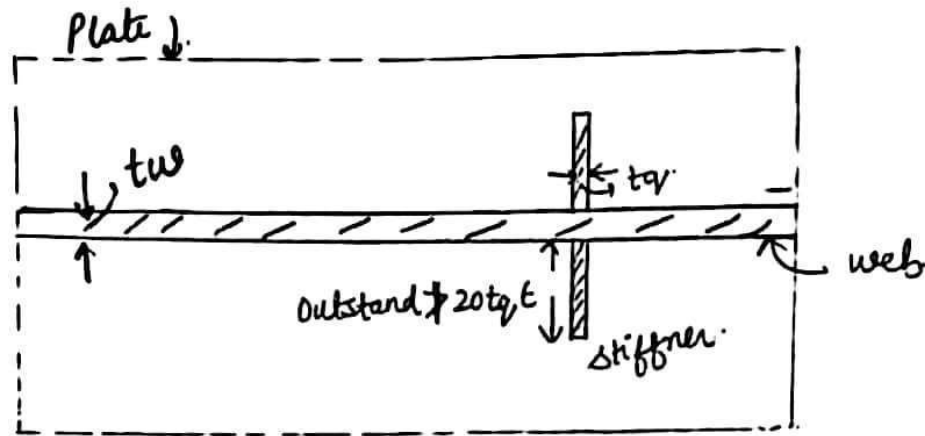
- If weld size comes out to be small; intermittent welds may be provided.

Design of Bearing stiffeners.

- In case of rolled steel sections, the webs are so proportioned that it will safely carry load without buckling or crippling.
- But in plate girder to achieve economy, web are made very thin, requiring stiffeners to transfer the reaction safely.
- Thickness of the stiffeners are kept such that outstand from the face of web is not more than $20t_w$

here t_q is thickness of the stiffener.

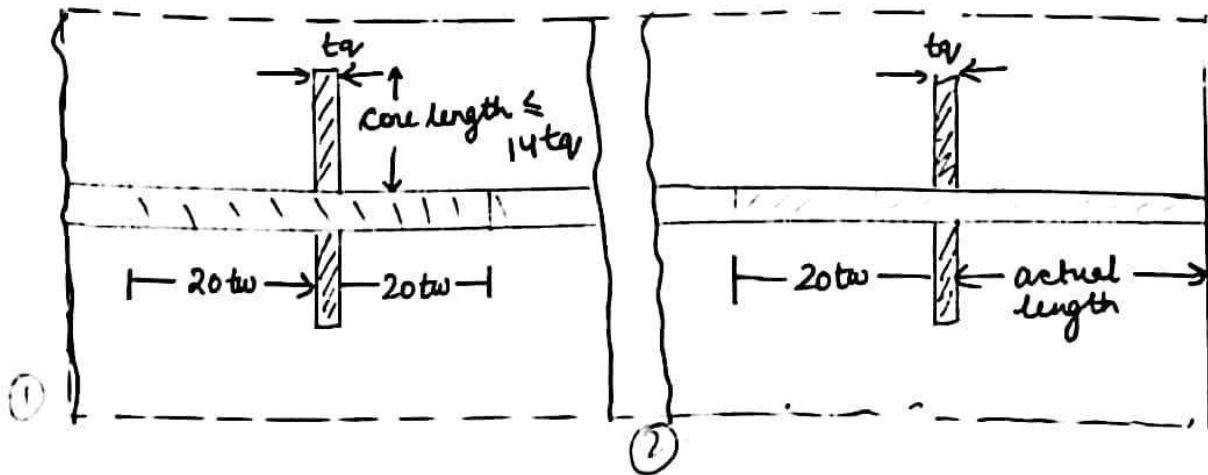
- When the outstand is in b/w $14t_q \leq 20t_q$, the stiffener design is on the basis of core section with an outstand of $14t_q$.



Plan.

- These stiffeners should be checked for
 - a) Buckling resistance
 - b) Bearing capacity
 - c) Torsional resistance.
- a) Buckling resistance of stiffeners.
 - For this purpose, the effective section is full area or core area of the stiffener together with an effective length of web on each side. of $20t_w$.
 - In some cases (in end stiffener) there may not be web on one side of the stiffener or it may be less than $20t_w$, in such cases available web size on that side is to be considered.
- To find Buckling resistance. compute
 - $\kappa = \sqrt{\frac{I_z}{A}}$
 - $\lambda = 0.7 \frac{L}{\kappa}$ $L = \text{length of stiffener}$

- find f_{cd} corresponding to buckling class (E)
- Buckling resistance = $f_{cd} \times A > \text{applied load}$



b) bearing capacity of stiffener.

- The load carrying capacity of stiffener is given by.

$$F_{psd} = \frac{A_g f_{yg}}{0.8 \gamma_{mo}}$$

A_g = area of stiffener in contact with flange.

f_{yg} = yield stress of stiffener.

for safety $F_{psd} > \text{applied load / Reaction}$

c) Torsional Resistance of stiffener.

- After erecting ends of plate girder may have lateral restraint but during transportation & erection, it may not have lateral restraint.

- Hence it must be checked for torsional resistance as

$$I_s \geq 0.34 d_s D^3 T_{cf}$$

I_s = second moment of area of stiffener section about centre line of web.

D = overall depth of beam / girder

T_{cf} = Maximum thickness of compression flange in span under consideration.

$$\alpha_s = 0.006 \quad \text{for } \frac{L_{LT}}{r_y} \leq 0$$

$$= \frac{0.3}{L_{LT} \cdot r_y} \quad \text{for } 50 < \frac{L_{LT}}{r_y} \leq 100$$

$$= \frac{30}{(L_{LT} \cdot r_y)^2} \quad \text{for } \frac{L_{LT}}{r_y} > 100$$

L_{LT} = Effective length for lateral torsional Buckling

r_y = radius of gyration of beam about minor axis

Weld for End stiffeners.

- These welds should be designed to transfer the shear not less than $\frac{t_w^2}{5 b_s}$ (KN/mm)

t_w = web thickness (mm)

b_s = outstand width of the stiffener. (mm)

Design of Intermediate Stiffener

- The design requirements for Intermediate stiffener are as follows:-

- They may be provided on one or both sides of web.
- The spacing of stiffener depend upon the web thickness.
- Outstand of stiffeners should not be more than $20t_w$,
If the outstand is more than $14t_w$ & less than $20t_w$,
the core section width is taken to be $14t_w$.
- Minimum moment of Inertia should satisfy the following.

$$\text{If } \frac{c}{d} \geq \sqrt{2}, \quad I_s \geq 0.75 d t w^2$$

$$\frac{c}{d} < \sqrt{2}, \quad I_s \geq \frac{1.5 d t w^2}{c^2}$$

- Stiffeners not subjected to external load or moments should be checked for a stiffener force.

$$F_q = V - \frac{V_{cr}}{\gamma_{mo}} \leq F_{qd}$$

F_{qd} = design resistance of Intermediate stiffener.

V = factored shear force adjacent to the stiffener.

V_{cr} = shear buckling resistance of web panel.

- Intermediate stiffeners subjected to external load should be designed as Bearing stiffeners; in addition they should satisfy following Interaction equation.

$$\boxed{\frac{F_q - F_x}{F_{qd}} + \frac{F_y}{F_{rd}} + \frac{M_q}{M_{yd}} \leq 1}$$

F_x = external load / reaction at the stiffener

F_{rd} = design resistance of load carrying stiffener.

M_q = moment on the stiffener

M_{yd} = yield moment capacity of the stiffener.

- Intermediate transverse stiffeners not subjected to external load should be connected to the web so as to withstand a shear between each component of stiffener & web not less than $\frac{t w^2}{5 b s}$ (kN/mm).

and if it is subjected to load, then the design is similar to that of bearing stiffeners.

Design Procedure of Plate Girder

- Assume self wt. = $\frac{W}{200}$ to compute factored shear force & Bending moment.
- Decide whether to use or not to use transverse stiffener & assume the value of $K \rightarrow \left(\frac{d}{t_w}\right)$
- Determine economical depth
- Determine the area of flange required to resist moment
- Check the moment capacity of girder.
- Find shear resistance of web using ~~post~~ simple post critical method.
- Design the weld connecting flange plate & web plate.
- Design end bearing stiffeners
- Design the connection of the stiffener
- Design load carrying stiffener if required.
- Design Intermediate stiffener if required.

Q A welded plate girder of span 20m simply supported is subjected to an U.D.L. of 100 kN/m including its self wt. The cross section of girder consist of top & bottom flanges (500mm x 50mm) & web plate of (1000mm x 15mm). The section is safe against flexure & shear. Design the connection b/w flange & web only. $\gamma_{mw} = 1.25$ & Fe 410 grade of steel is used. Use limit state method of design.

Solⁿ Factored ^{max.} shear force $V = \frac{w_u l}{2} = \frac{1.5 \times 100 \times 20}{2} = 1500 \text{ kN}$

Shear force in weld per unit length $q_w = \frac{V A \bar{y}}{I_z}$

$$q_w = \frac{1500 \times 10^3 \times 500 \times 50 (500 + 25)}{\frac{500 \times 100^3}{12} - \frac{(500 - 15) \times 1000^3}{12}} = 1308.86 \text{ N/mm} \quad \text{--- (i)}$$

Let throat thickness be "t".

$$\text{strength of shop weld} = 2t \frac{f_w}{\sqrt{3}} \times \frac{1}{1.25}$$

from (i) & (ii),

$$1308.86 = 2t \times \frac{410}{\sqrt{3}} \times \frac{1}{1.25}$$

$$t = 3.455 \text{ mm.}$$

$$\text{size } s = \frac{t}{0.7} = \frac{3.455}{0.7} = 4.93 \text{ mm.}$$

Min. size of weld required for $t_f = 50 \text{ mm} \Rightarrow s_{\min} = 8 \text{ mm.}$

Lesson 34 Jan 24

Q Design a welded plate girder of span 30m to carry a. super imposed load of 35 kN/m. Avoid use of bracing & intermediate stiffeners. Fe 410 grade of steel.

$$\text{Factored self wt} = \frac{1.5 \times 35 \times 30}{200} = 7.875 \text{ kN.}$$

$$\text{Total factored UDL (w)} = 35 \times 1.5 + \frac{7.875}{30} = 52.7625 \text{ kN/m}$$

$$\text{Max BM, } M = \frac{wL^2}{8} = \frac{52.7625 \times 30^2}{8} = 5935.78 \text{ kN-m.}$$

$$\text{Max SF, } V = \frac{wL}{2} = \frac{52.7625 \times 30}{2} = 791.4325 \text{ kN}$$

- If stiffeners are avoided, $K = \frac{d}{t_w} \leq 67$

$$\text{Economical depth of web } d = \left(\frac{MK \gamma m_0}{f_y} \right)^{1/3}$$

$$= \left(\frac{5935.78 \times 10^6 \times 67 \times 1.1}{250} \right)^{1/3}$$

$$= 1205.04 \text{ mm.}$$

Adopt $d = 1200 \text{ mm}$.

$$t_w \geq \frac{d}{67} = \frac{1200}{67} = 17.91 \text{ mm}$$

adopt $t_w = 18 \text{ mm}$

Thus web plate be of $1200 \text{ mm} \times 18 \text{ mm}$.

- Neglecting moment capacity of web, area of flange A_f is

$$M = \frac{f_y A_f \cdot d}{\gamma_{mo}}$$

$$A_f = \frac{M \gamma_{mo}}{f_y d} = \frac{5935.78 \times 10^6 \times 1.1}{250 \times 1200}$$

$$A_f = b_f t_f = 21764.5 \text{ mm}^2 \quad \text{--- (i)}$$

for semi compact section $\frac{b}{t_f} \leq 13.6$

$$\text{assume } \frac{b}{t_f} = 12 \quad \text{--- (ii)}$$

$$\text{from (i) \& (ii)} \quad 12 t_f \cdot t_f = 21764.5$$

$$t_f = 42.58 \text{ mm}$$

adopt $t_f = 44 \text{ mm}$.

$$\text{Hence, } b_f = \frac{21764.5}{44} = 494.64 \text{ mm}$$

adopt $b_f = 500 \text{ mm}$.

The flange plate be of size $500 \times 44 \text{ mm}$.

- check the moment capacity of girder

$$M_d = \frac{Z_e f_y}{\gamma_{mo}}$$

$$I_{zz} = 2 \left[500 \times \frac{44^3}{12} + 500 \times 44 (600 + 22)^2 \right]$$

$$= 1.704 \times 10^{10} \text{ mm}^4$$

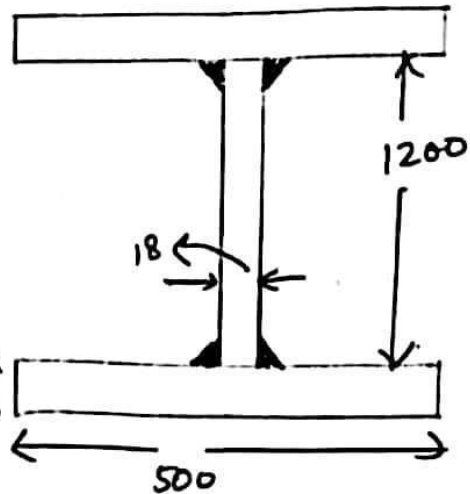
$$y_{max} = 600 + 44 = 644 \text{ mm}$$

$$Z_e = \frac{I_{zz}}{y_{max}} = \frac{1.704 \times 10^6}{644} = 2.64 \times 10^7 \text{ mm}^3$$

$$M_d = 2.64 \times 10^7 \times \frac{250}{1.1} \times 10^{-6}$$

$$M_d = 6010.02 \text{ kN-m} > M = 5935.79 \text{ kNm}$$

OK.



— Shear resistance of web.

$$V_d = \frac{f_y w A_v}{\sqrt{3} \gamma_{mo}} = \frac{250 \times 1200 \times 18}{\sqrt{3} \times 1.1} \times 10^{-3} = 2834.26 \text{ kN} > V = 791.43 \text{ kN} \Rightarrow \underline{\underline{OK.}}$$

Hence stiffeners are required.

— Check for end Bearing

$$F_w = b t_w \frac{f_y w}{\gamma_{mo}}$$

assume $b_1 = 100 \text{ mm}$

$$b = b_1 + 2.5 t_f = 100 + 2.5 \times 44 = 210 \text{ mm}$$

$$F_w = 210 \times 18 \times \frac{250}{1.1} \times 10^{-3} = 859.09 \text{ kN} > V = R = 791.43 \Rightarrow \underline{\underline{OK}}$$

— Hence end stiffener is not required.

→ Design of weld connection b/w flange plate & web plate

$$V = 791.43 \text{ kN}$$

$$\text{Shear force per unit length in junction } q_w = \frac{V A \bar{y}}{I_z}$$

$$q_w = \frac{791.43 \times 10^3 [500 \times 44 \times (600 + 22)]}{1.704 \times 10^{10} + \frac{10 \times 1200^3}{12}} = 551.646 \frac{\text{N}}{\text{mm}} \quad (1)$$

Let "t" be the throat thickness of weld

$$\text{Shear strength of weld} = \frac{2t f_w}{\sqrt{3} \times 1.25} \quad \text{---(ii)}$$

$$\text{from (i) \& (ii)} \quad 551.646 = \frac{2t \times 410}{\sqrt{3} \times 1.25}$$

$$t = 1.45 \text{ mm.}$$

$$\text{Hence } S = \frac{t}{0.7} = \frac{1.45}{0.7} = 2.07 \text{ mm.}$$

Min. thickness of weld required is 8mm

Provide intermittent weld such that

- a) min. weld length = 40mm
- b) Max. unwelded length $12 \times 18 = 216 \text{ mm}$, adopt 210 mm.

Gantry Girder.

- The travelling overhead cranes are commonly used in factories & workshops to lift & move heavy materials & assembled parts from one point to another.
- This crane system consists of
 - a) crane bridge / crane girder
 - b) crab / trolley
 - c) Hoist
 - d) Cabin
- This crane bridge itself is provided with beams to move over the rails which are in turn provided over gantry girder.
- Thus gantry girder is the beam which supports crane girder.
- This gantry girder is supported on the columns with bracket.

The following imposed loads should be considered in the design of gantry girder.

- Vertical loads from the cranes
- Impact load from the crane. (when crane moves so load can be in both dir)
- Longitudinal horizontal force along the crane rail.
- lateral thrust (surge) across crane rail.
(Hoist swings when crab moves so horizontal (lateral) force acts)

Note: 1) The effect of acceleration & braking of the loaded crab & swinging of the suspended load in transverse direction leads to the development of the surge.

2) In calculating the above forces, crane should be positioned such that it gives max. design forces in the girder

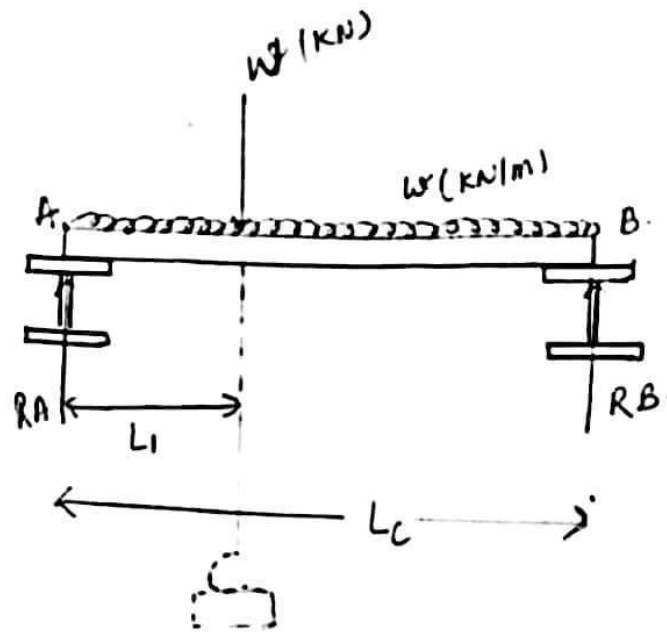
3) The impact load to be considered are as follows:⇒

Type of load	Impact Allowance.
1) Vertical loads.	
a) for electric operational cranes.	25% of max. static wheel load
b) for hand operated cranes	10% of max. static wheel load.
2) Horizontal force Transverse to rails.	
a) for electric operated cranes	10% of wt. of trolley & wt. lifted
b) for hand operated cranes	5% of wt. of trolley & wt. lifted.
3) Horizontal force along the rails.	5% of the static wheel loads

4) Whenever necessary, the design should be checked for earthquake forces & secondary effects like

- Handling
- erection
- Temperature effects
- Fatigue

- Position of crane hook for maximum vertical load on gantry girder.



Lesson 35 Jan 25

- The maximum vertical load on gantry girder is the max. reaction of crane girder
- This would be obtained when the crab is placed as close as possible to the gantry girder (L_1)
- For R_A , $\sum M_B = 0$

$$R_A \cdot L_c = w \cdot L_c \cdot \frac{L_c}{2} + W(L_c - L_1)$$

$$R_A = \frac{1}{L_c} \left[w \frac{L_c^2}{2} + W(L_c - L_1) \right]$$

L_1 = minimum approach of crane hook

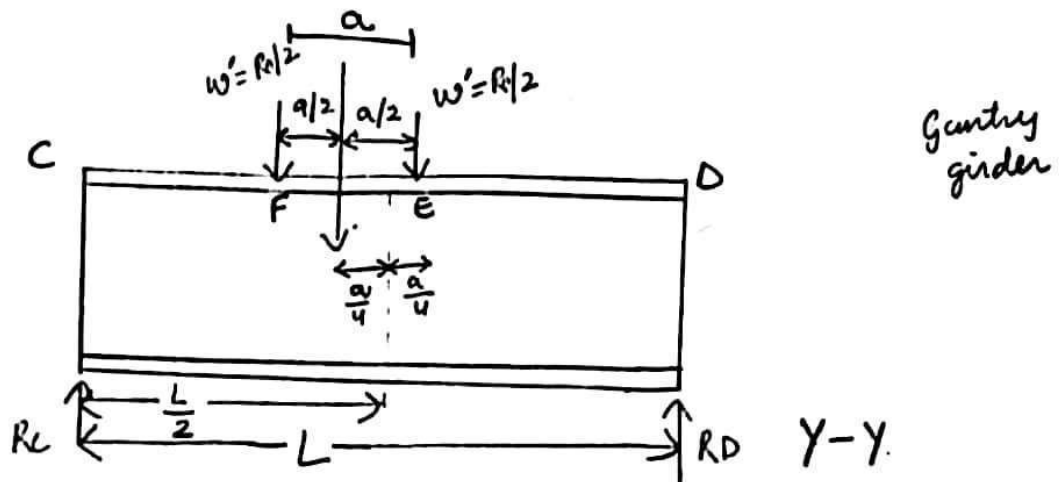
(distance b/w c.g. of gantry girder & trolley)

L_c = span of crane girder

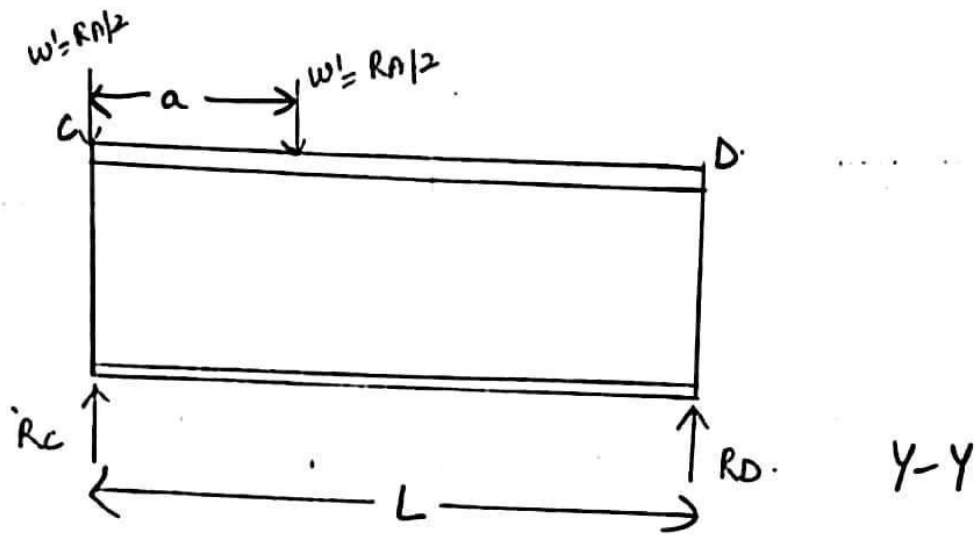
W = weight of crab + weight lifted

w = weight of crane girder per unit length.

Position of crane wheels for Maximum effects on Gantry girder



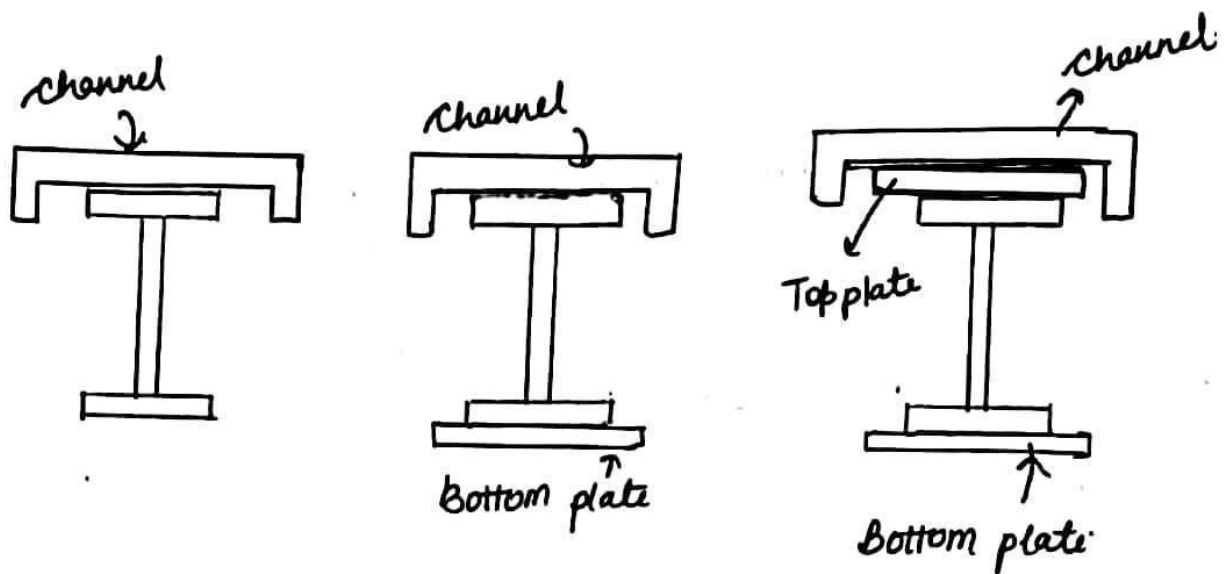
a = distance b/w the crane wheels.



- The R_A calculated in previous section gets equally distributed in wheels (two) of crane girder.
- Hence the moving load on gantry girder is wheel load of $w' = R_A/2$ acting at a distance of " a " (wheel base)
- ⇒ Maximum moment occurs under a load when the line of action of that load & the C.G. of the loads are at equal distance from the centre of span.
- ⇒ Maximum shear force in gantry girder occurs when both wheel loads are on the girder & are positioned as close to supports as possible.

Note: ⇒ If two cranes are operated on a gantry girder, then position of max. effect must be considered accordingly.

- To resist heavy moment, I sections are required
- They can be large rolled section or even built up plate girders.
- Apart from bending moment & shear, these girders are subjected to longitudinal & lateral forces on compression flange.
- Hence, compression flange needs additional strength.
- This is achieved by connecting a channel section on compression flange of the I-section as follows.



- To avoid wear & tear of rails & excessive vibration. Code restricts vertical deflection as follows:-

Type of deflection	Vertical deflection Maximum
A) Vertical deflection.	
(i) Manually operated crane	span/500
(ii) electrically operated crane $\leq 500\text{ kN}$	span/750
(iii) electrically operated crane $> 500\text{ kN}$	span/1000.
B) Lateral Deflection.	
(i) Relative distance b/w rail & crane	10mm

Design procedure of gantry girders.

- In the design of gantry girder it is assumed
 - a) entire section resist vertical loads.
 - b) compression flange with channel resist horizontal forces.
- with suitable positioning of the crane, determine the max. moment & shear force on gantry.
- Add Impact load contribution to it

Note: → Though the position for max. moment due to wheel load is slightly away from the centre of girder, but it is just added to the maximum moment due to UDL on girder to find design moment.

- calculate horizontal bending moment due to surge load.
- calculate shear force due to vertical & horizontal forces.
- select a trial section on the basis of following guidelines.

- a) Economic depth is $\frac{1}{12}^{\text{th}}$ of span.
 - b) Compression flange width may be kept $\frac{1}{30}^{\text{th}}$ of the span
 - c) The moment capacity for vertical load should be 40% more than the moment due to vertical load; so that section can resist combined moment safely.
- Compute I_{zz} , I_{yy} , Z_p , Z_e of the trial section.
 - Check for moment capacity of section selected.
 - Check for buckling resistance.
 - Check for biaxial bending.
 - Check for shear capacity.
 - Check for web buckling & bearing.
 - Check for deflection.
 - Design the welds.

Q Design a simply supported gantry girder to carry one electric overhead crane given.

- span of gantry girder = 6.5m
- span of crane " = 16m.
- crane capacity = 250kN.
- self wt of crane girder = 280kN. excluding trolley.
- self wt. of trolley = 50kN.
- Minimum hook approach = 1m.
- distance b/w wheels = 3.5m.
- self wt. of rails = 0.3kN/m.
- section available are
ISWB:600 & ISMC-300

ISWB:600

$$A = 17038 \text{ mm}^2$$

$$b = 250 \text{ mm}$$

$$t_f = 21.3 \text{ mm}$$

$$t_w = 11.2 \text{ mm}$$

$$I_{zz} = 106198.5 \times 10^4 \text{ mm}^4$$

$$I_{yy} = 4702.5 \times 10^4 \text{ mm}^4$$

ISMC:300.

$$A = 4564 \text{ mm}^2$$

$$b = 90 \text{ mm}$$

$$t_f = 13.6 \text{ mm}$$

$$t_w = 7.6 \text{ mm}$$

$$I_{zz} = 6362.6 \times 10^4 \text{ mm}^4$$

$$I_{yy} = 310.8 \times 10^4 \text{ mm}^4$$

$$C_{yy} = 23.6 \text{ mm}$$

For complete section $I_{zz} = 1207.28 \times 10^6 \text{ mm}^4$

$$Z_e = 333.2 \times 10^4 \text{ mm}^3$$

$$Z_p = 4810.07 \times 10^3 \text{ mm}^3$$

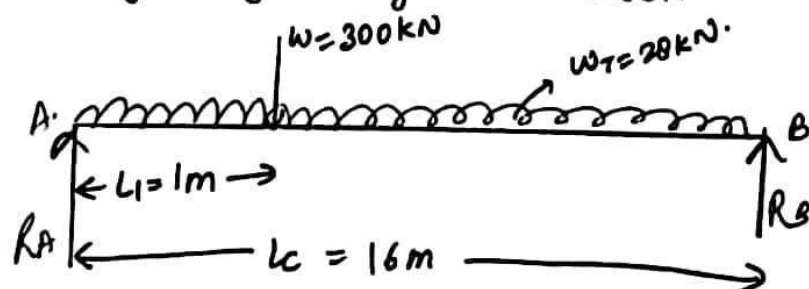
For compression flange. $Z_{ef} = 609.06 \times 10^3 \text{ mm}^3$

$$Z_{pf} = 828.48 \times 10^3 \text{ mm}^3$$

Deflection, web buckling & bracing check is not required.

solⁿ wt. of trolley + lifted load $W = 50 + 250 = 300 \text{ kN}$.

Self wt. of crane girder = 280kN.



For max. reaction on gantry girder, the moving load should be as possible to gantry girder.

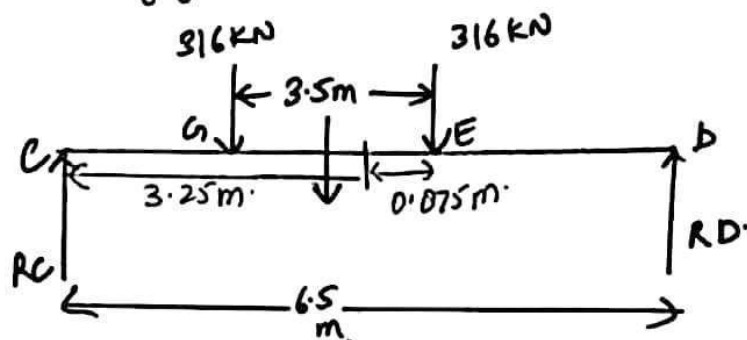
$$R_A = (280 \times 8 + 300 \times 15) \frac{1}{16} = 421.25 \text{ KN.}$$

The load is transferred on gantry girder as two wheel loads

$$W' = \frac{R_A}{2} = \frac{421.25}{2} = 210.625 \text{ KN.}$$

$$\text{Factored wheel load} = 210.625 \times 1.5 = 316 \text{ KN.}$$

- Max moment due to moving load occurs under a wheel when the C.G. of wheel load & the wheel are equidistant from the centre of girder.



$$\sum M_C = 0 \Rightarrow R_D \times 6.5 = 316 [(3.25 + 0.875) + (3.25 + 0.875 - 3.5)]$$

$$R_D = 230.92 \text{ KN.}$$

$$\text{Max } M_E = R_D (6.5 - 3.25 - 0.875) = 548.435 \text{ KN-m} \text{ --- (i)}$$

$$\text{Moment due to impact load} = 0.25 \times 548.435 = 137.1 \text{ KN-m} \text{ --- (ii)}$$

$$\text{Assume self wt. of gantry girder} = \frac{316 \times 2}{6.5 \times 200} = 0.5 \text{ KN/m}$$

$$\text{Dead load due to rail} = 1.5 \times 0.3 = 0.45 \text{ KN/m.}$$

$$\text{Moment due to DL} = (0.5 + 0.45) \times \frac{6.5^2}{8} = 5 \text{ KN-m} \text{ --- (iii)}$$

$$\text{Design moment due to vertical load} = 548.435 + 137.1 + 5$$

$$= 690.5 \text{ KN-m.}$$

- Horizontal force transverse to rail = 10% of wt. of trolley + load.

$$= 0.1 [50 + 250] = 30 \text{ KN}$$

This is distributed over 4 wheels.

$$\text{Horizontal force on each wheel} = \frac{30}{4} = 7.5 \text{ kN}.$$

$$\text{Factored force on each wheel} = 7.5 \times 1.5 = 11.25 \text{ kN}.$$

$$\text{Max. moment due to horizontal force} = \frac{548.435}{316} \times 11.25$$

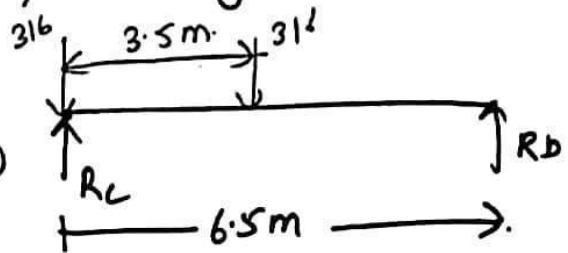
$$M_y = 19.52 \text{ kN-m}.$$

For max. shear force on the girder, the trailing wheel should be just on girder.

$$\sum M_D = 0.$$

$$R_C \times 6.5 = 316 \times 6.5 + 316(6.5 - 3.5)$$

$$R_C = 461.85 \text{ kN} \quad (i)$$



$$\text{Vertical shear due to impact} = 0.25 \times 461.85 = 115.46 \text{ kN} \quad (ii)$$

$$\text{Vertical shear due to self wt} = (0.5 + 0.45) \times \frac{6.5}{2} = 3 \text{ kN} \quad (iii)$$

$$\text{Total vertical shear} = 461.85 + 115.46 + 3 = 580.4 \text{ kN}$$

$$\begin{aligned} \text{By proportioning lateral shear due to surge} &= \frac{461.85}{316} \times 11.25 \\ &= 16.44 \text{ kN} \end{aligned}$$

$$\text{Now, economical depth} = \frac{L}{12} = \frac{6.5 \times 10^3}{12} = 541.67 \text{ mm}.$$

$$\text{width of flange} = \frac{L}{30} = \frac{6.5 \times 10^3}{30} = 216 \text{ mm}.$$

Adopt ISWB 600 with ISMC 300 on compression flange.

$$\text{Now, } \frac{b}{t} \text{ of flange of ISWB 600} = \left(\frac{250 - 11.2}{2} \right) \times \frac{1}{21.3} = 5.4 < 9.4 \epsilon = 9.4$$

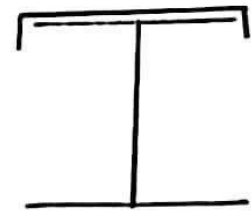
$\Rightarrow$ flange is plastic.

$$\frac{d}{t_w} \text{ of web of ISWB 600} = \frac{600 - 2 \times 21.3}{11.2} \times 49.78 < 84 \epsilon = 84$$

$\Rightarrow$ web is plastic

$$\frac{b}{t} \text{ of flange of channel} = \frac{90-7.6}{13.6} = 6.06 < 9.46 = 9.4.$$

$\Rightarrow$ flange is plastic.



- Hence the entire section is plastic

$$M_d = \frac{f_y Z_p}{\gamma_{mo}} = \frac{250 \times 4810 \times 10^3}{1.1} \times 10^{-6} = 1093.19 \text{ kN-m.}$$

$$\text{also } \frac{1.2 Z_e f_y}{\gamma_{mo}} = \frac{1.2 \times 333.2 \times 10^4}{1.1} \times 250 \times 10^{-6} = 908.72 \text{ kN-m}$$

$$\text{Hence } M_d = 908.72 \text{ kN-m} > M_z = 690.5 \text{ kN-m} \quad \underline{\underline{OK}}$$

$$\text{Now for top flange. } M_{dz} = \frac{f_y Z_p}{1.1} = \frac{250 \times 828.48 \times 10^3 \times 10^{-6}}{1.1}$$

$$M_{dz} = 188.29 \text{ kN-m.}$$

$$\text{also } \frac{1.2 Z_e f_y}{\gamma_{mo}} = \frac{1.2 \times 609.06 \times 10^3 \times 250 \times 10^{-6}}{1.1} = 166.107 \text{ kN-m.}$$

$$M_{dz} = 166.107 \text{ kN-m} > M_z = 19.52 \text{ kN-m.}$$

Check for combined local capacity.

$$\frac{690.5}{908.72} + \frac{19.52}{166.07} = 0.877 < 1 \quad \underline{\underline{OK}}$$

INDUSTRIAL ROOF TRUSS

- Steel buildings are classified as
 - a) Shell structures
 - b) Framed structures
- A framed structure can further be classified as
 - a) Low rise steel structures / single story structure
 - b) High rise steel structure / multi storey structure
- Industrial buildings are low rise steel structures as their height is comparatively low.
- Industrial structures lack interior floor, walls & partitions and their ~~loading~~ roofing system consist of truss with roof coverings.
- The enclosure of an Industrial building may be brick masonry, concrete wall or GI sheet covering which are not load bearing.
- But these enclosure should be strong enough to resist lateral force due to wind or earthquakes.
- Eg. workshop, warehouse, cold storage yards, bangers etc.
- The width of an Industrial building may consist of one or more enclosure termed as AISLE, according to which they can be classified as single, Double, Multi-Aisle Industrial building.
- The length of Industrial building is divided into number of "Bays".
- The length of bay varies in range of 4-8m.
- For developing the Industrial building, the site is selected on the basis of following factors.
 - a) Purpose of Industrial building.
 - b) It must be located on arterial road.
 - c) Local availability of raw material
 - d) Soil condition

- c) Topography & water damage
- b) Facilities like water, electricity, telephone & waste disposal.

- Industrial building consists of following components.

- a) Roof Trusses
- b) Bracings
- c) Gable & columns.
- d) Cladding, sag rods.
- e) Purlins.
- f) Column & their Bases.
- g) Gantry girder.

Roof Trusses.

- These are economical for span more than 6 m.
- The pitch of roof truss (i.e. $\frac{\text{rise}}{\text{span}}$) should be $\frac{1}{4}$ to $\frac{1}{2}$ for

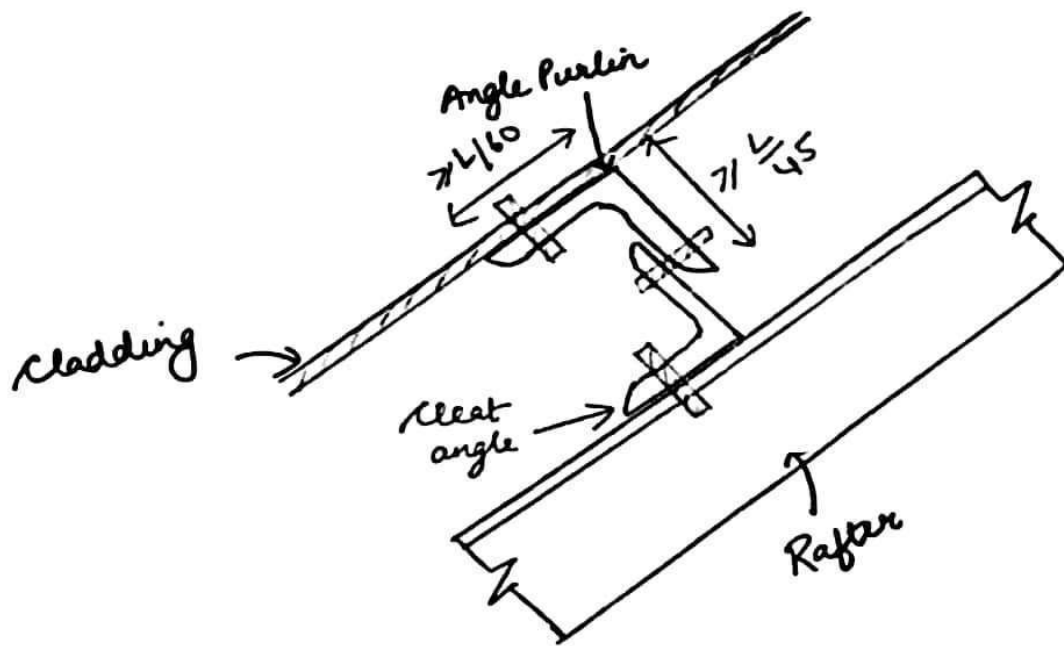
proper drainage

- Spacing of roof trusses is kept $\frac{1}{3}$ - $\frac{1}{6}$ of the span.
- Material used should be light weight.
- Minimum angle section to be provided to avoid damage during transportation & erection is ISA 50x50x8mm

Purlins

- For purlins, angle, channel, I or Z sections can be used.
- Width of angle leg in the plane perpendicular to the roof covering $\geq \frac{L}{45}$
- Width of angle leg in the plane parallel to the roof covering $\geq \frac{L}{60}$

here L = span of purlin.



- Maximum B.M. in purlin is taken to be $M = \frac{Wl^2}{10}$

$W = \text{UDL}$ (load per unit length on the purlin including wind load)

- Required section for purlin can be computed with the help of

$$Z_c = \frac{M}{f_{yc}} \text{ (mm}^3\text{)}$$

Notes - Permissible stress in bending compression is taken to be 165 N/mm^2

- Loads on the trusses are taken as follows.

a) Dead load

If the spacing of truss is 4m & pitch of the truss is $1:4$, then self wt. of truss is $w = \left(\frac{1}{3} + 5\right) \text{ kg/m}^2$ of plan area

$$\text{or } w = 10\left(\frac{1}{3} + 5\right) \text{ N/m}^2 \text{ of plan area.}$$

b) Live load

- If slope of truss is more than 10° , live load is taken as 0.75 kN/m^2
 $LL = 0.75 \text{ kN/m}^2 - 0.02 (\text{kN/m}^2) \text{ (for every degree increase in slope over } 10^\circ)$

out $L \neq 0.4 \text{ kN/m}^2$

c) Snow load

- It is taken as 2.5 kN/m^2 for every 1mm depth of snow
- If slope of truss is more than 50° snow load need not be considered.

d) Wind load

- as discussed in chapter

Q Find the wind pressure for design of a sloping roof of span 10m & pitch $\frac{1}{4}$. The height of the eaves is 5m above the ground. The building is situated in Chennai & its permeability is normal.

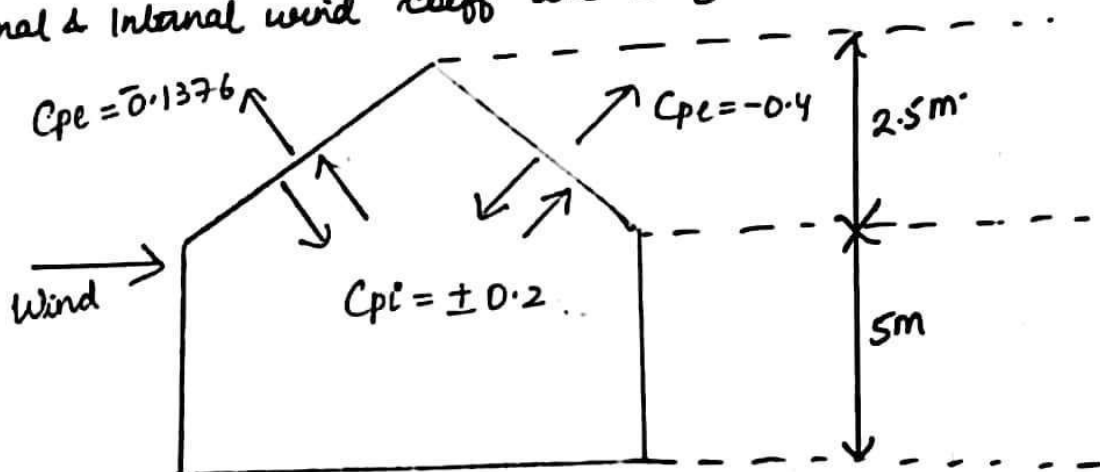
Assume Probability factor $K_1 = 1.0$

Terrain factor $K_2 = 0.8$

Topography factor $K_3 = 1.0$

Basic wind speed at Chennai = 50 m/s .

External & Internal wind coeff are as follows.



Solⁿ

Basic wind speed $V_b = 50 \text{ m/s}$.

Design wind speed $V_z = V_b K_1 K_2 K_3$

$$= 50 \times 1 \times 0.8 \times 1.0$$

$$= 40 \text{ m/s}$$

Design wind pressure $P_z = 0.6 V_z^2$

$$= 0.6 \times 40^2 = 960 \text{ N/m}^2$$

Design wind force per unit area
(pressure) $F_z = C_p z$

For wind ward slope

$$F_z = (-0.1375 + 0.2) 960 = 60 \text{ N/m}^2 \text{ (Downward).}$$

$$\& F_z = (-0.1375 - 0.2) 960 = -324 \text{ N/m}^2 \text{ (upward).}$$

For leeward side

$$F_z = (-0.4 + 0.2) 960 = -192 \text{ N/m}^2 \text{ (upward).}$$

$$\& F_z = (-0.4 - 0.2) 960 = -576 \text{ N/m}^2 \text{ (upward).}$$

Hence design force is 60 N/m^2 (Downward)

576 N/m^2 (upward).

Q Design an I-section purlin for a trussed roof from the data.

Span of roof = 12m

Spacing of truss = 5m.

Spacing of purlin along slope of roof truss = 2m.

Slope of roof truss = 1V: 2H.

Wind load on roof surface normal to roof = 1000 N/m^2

Vertical load from roof sheet = 200 N/m^2

Consider ISLB 125 having wt = 11.9 kg/m as purlin &

$$Z_x = 65 \times 10^3 \text{ mm}^3, Z_y = 11.6 \times 10^3 \text{ mm}^3.$$

Solⁿ wt. of roof sheet = $200 \times 2 = 400 \text{ N/m}$

self wt. of purlin = $11.9 \times 10 = 119 \text{ N/m}$.

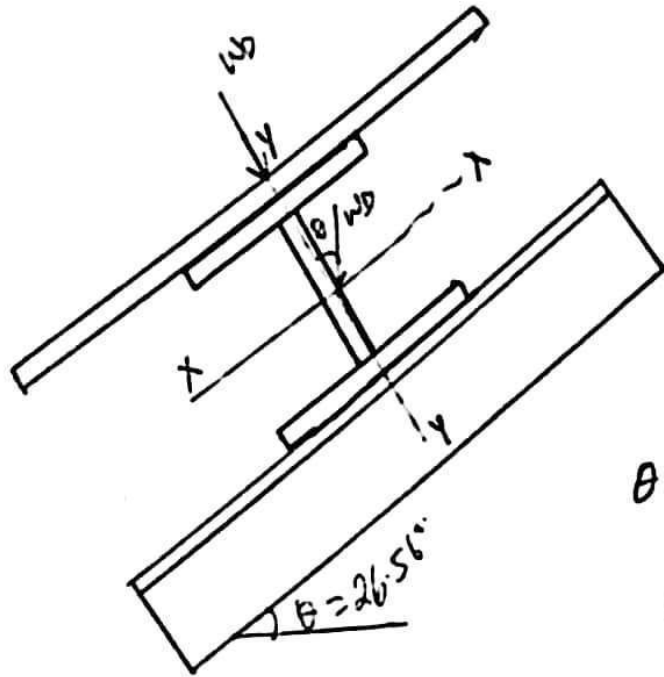
Total vertical load $W_D = 400 + 119 = 519 \text{ N/m}$.

Wind load $W_w = 1000 \times 2 = 2000 \text{ N/m}$.

$$M_x = (W_w + W_D \cos \theta) \frac{l^2}{10}$$

$$= (2000 + 519 \cos 26.56^\circ) \times \frac{5^2}{10} \times 10^3$$

$$M_x = 6160.57 \times 10^3 \text{ Nmm}$$



$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 26.56^\circ$$

$$M_y = (W \sin \theta) \frac{12}{10} = (519 \sin 26.56) \frac{5^2}{10} \times 10^3$$

$$= 580.157 \times 10^6 \text{ N-mm}$$

$$\text{Max. compressive stress} = \frac{M_x}{Z_x} + \frac{M_y}{Z_y}$$

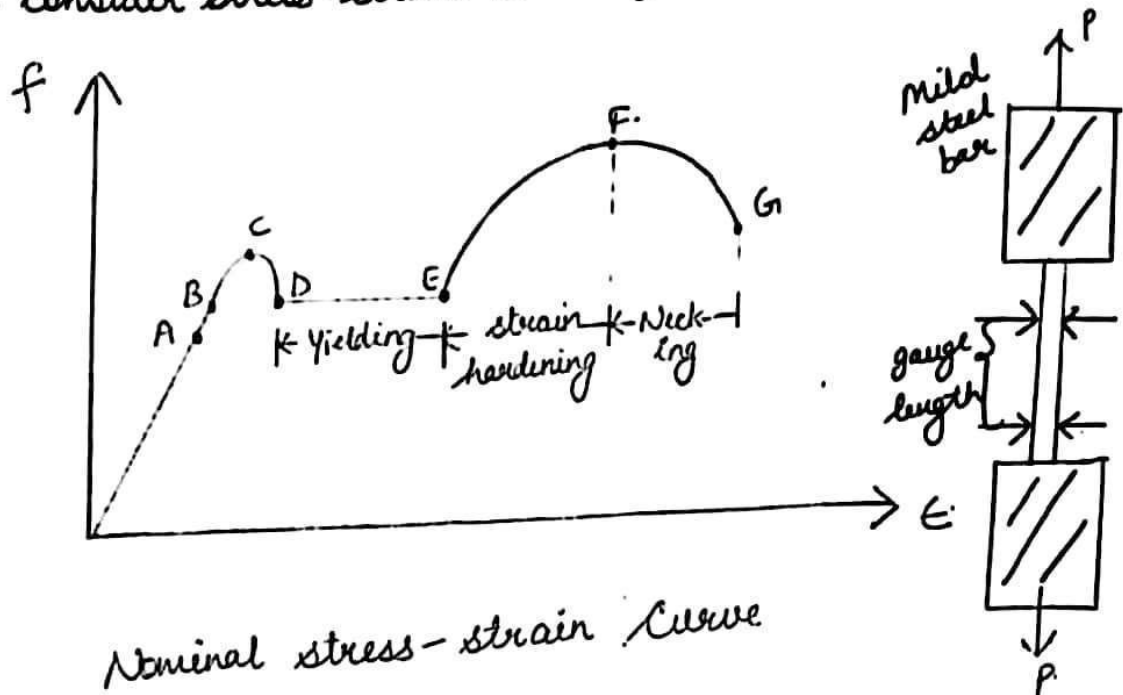
$$= \frac{6160.57 \times 10^3}{65 \times 10^3} + \frac{580.15 \times 10^3}{11.6 \times 10^3}$$

$$= 144.79 \text{ N/mm}^2 < 165 \text{ N/mm}^2$$

OK

Plastic Analysis

- If we consider stress-strain curve for mild steel



Note:- Gauge length is the length over which elongations are measured using extensometer.

- stress strain curve is drawn by conducting strain controlled tension test on Mild Steel bar.

- DE Zone (Yield Plateau): It is the zone in which strain increases without increase in the stress.

- EF Zone (Strain hardening): After this material is yielded, due to change in its structure, it becomes stronger & harder, hence the value of stress increases in strain. This process is termed as strain hardening.

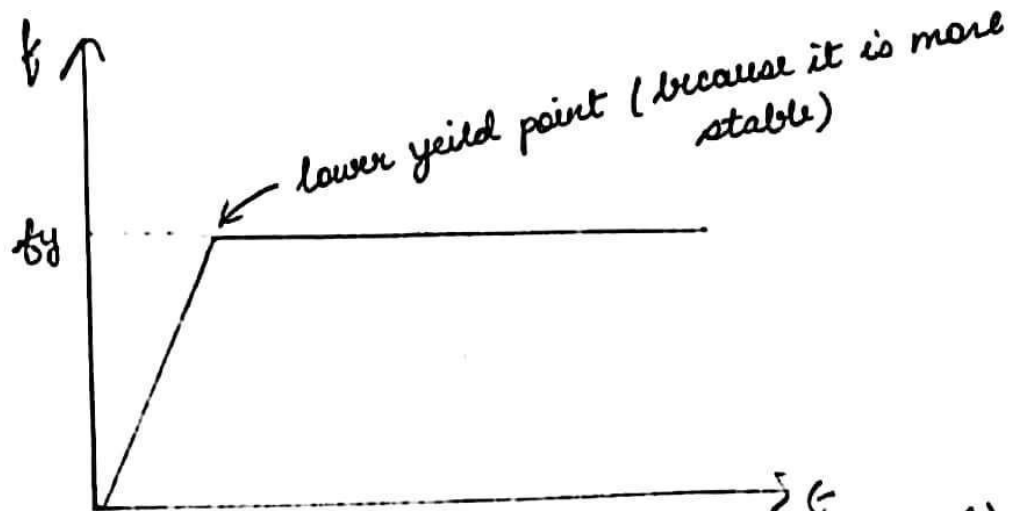
- FG Zone (Necking): In this zone, local reduction in the area of cross-section takes place.

- It is termed as Necking.
- It takes place after ultimate stress point is crossed.

* In plastic analysis, the effect of strain hardening is neglected, as in any structure large deformations

are not permitted.

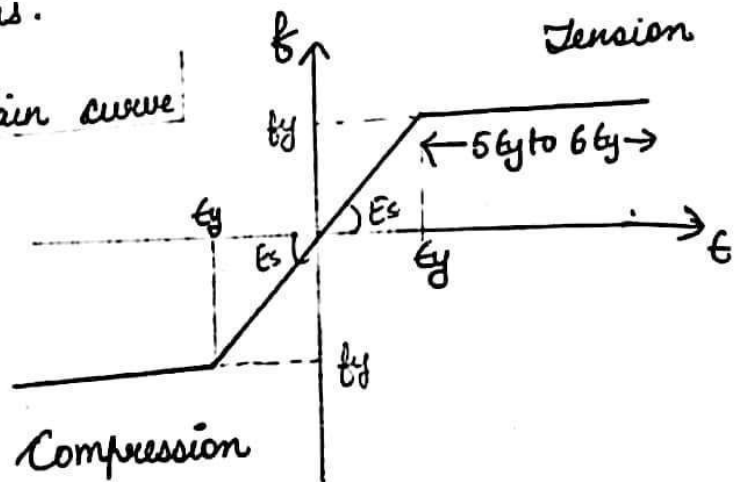
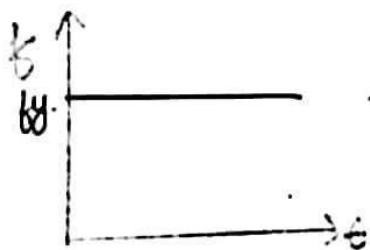
- Hence plastic analysis is based on idealised stress strain curve.



- Characteristics of Ideal Plastic Material (Mild steel).

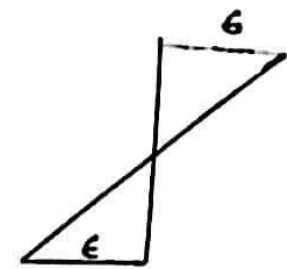
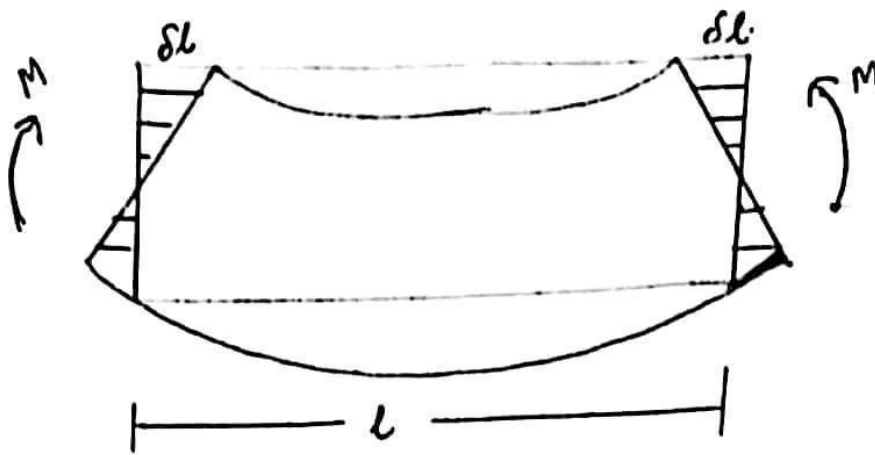
- yield stress (f_y) is same in tension & compression.
- Modulus of Elasticity (E) is same in tension & compression.
- Ideal plastic material obeys Hooke's Law.
- If it is loaded beyond yield stress & unloaded then the residual strain remains.

Note:- Assumed stress strain curve for plastic material



Assumptions in the Plastic Analysis of Beam

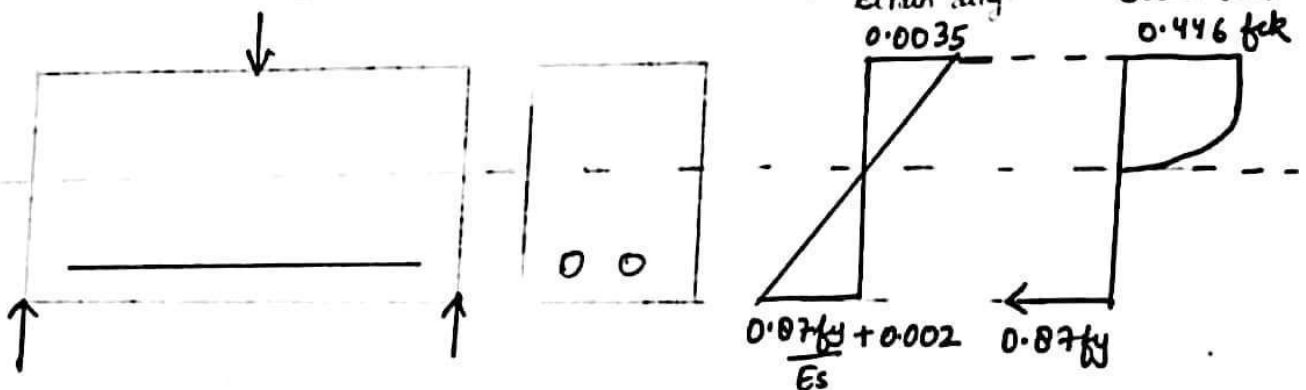
- Plane section before bending remains plane after bending
i.e. strain varies linearly over depth of beam.
(It is termed as Bernoulli's assumption.)



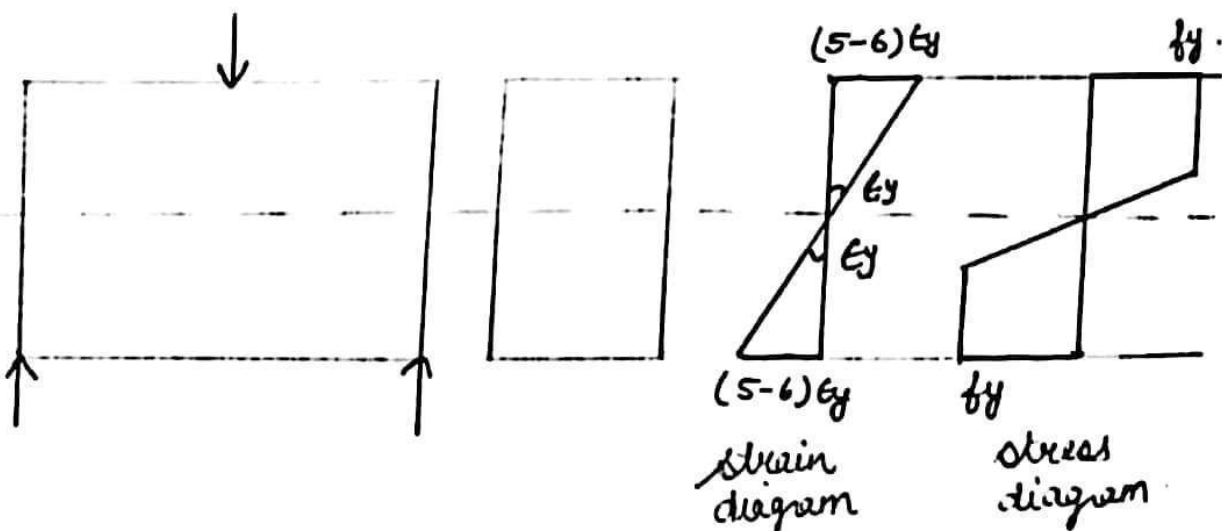
Strain-distribution

- Since δl (deformation) varies linearly over the depth, hence strain $\epsilon = \frac{\delta l}{l}$ also varies linearly.

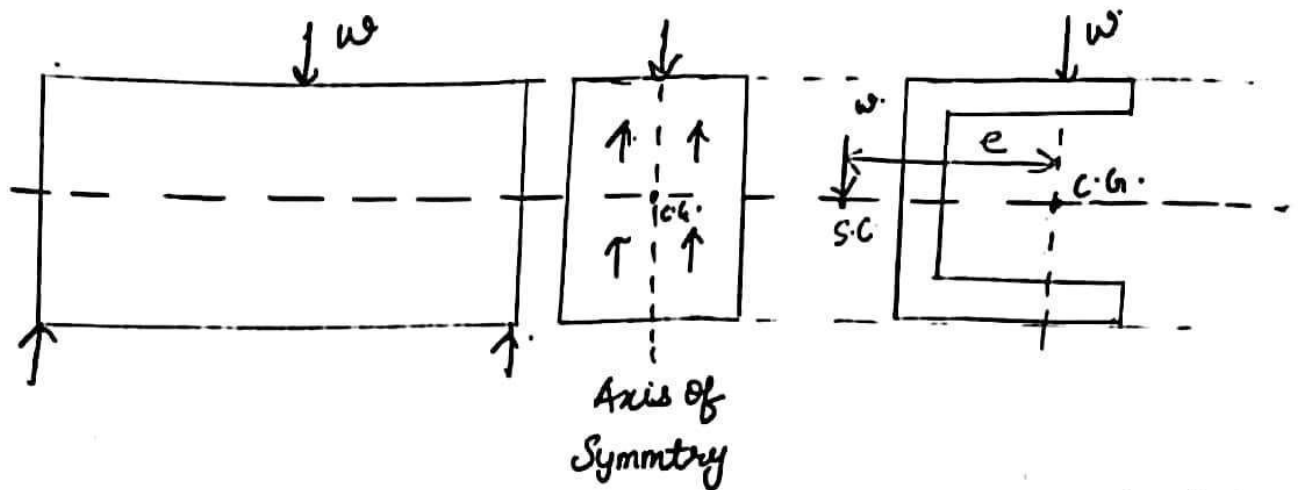
- The above assumption that plane section remains plane before and after bending is valid upto collapse load or Failure load.
- Design of RCC element is strain controlled approach as strain distribution diagram decides load carrying capacity.



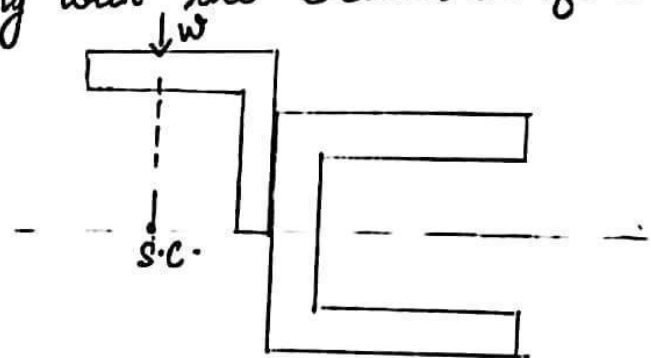
- Design of steel element is stress controlled approach as stress distribution diagram decides load carrying capacity.



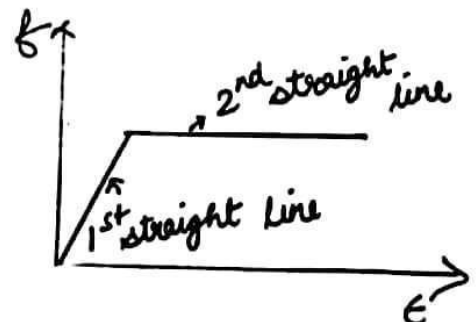
- b) The cross-section must be symmetrical w.r.t plane of loading as if it is not, then twisting moments would be developed in c/s & flexure formula would not be applicable.



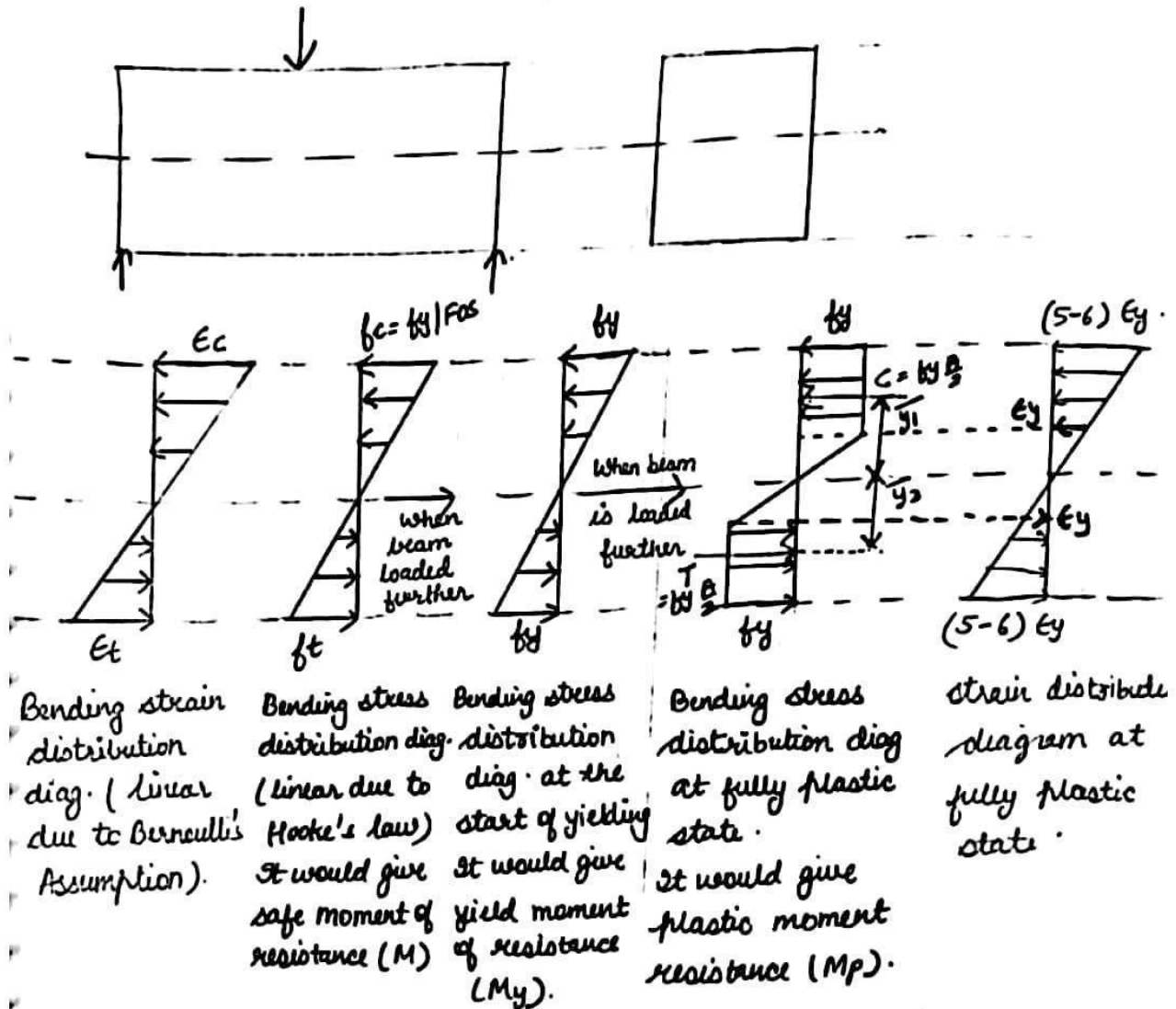
In second case if twisting is required to be eliminated the load must be passed through shear centre (S.C.) by providing an angle along with the channel as follows.



- c) The effect of axial force & shear force i.e. axial & shear deformation are neglected in plastic analysis of beam.
- d) The stress strain relationship is assumed to be bi-linear i.e. it includes two straight lines)



— Plastic moment capacity of section is computed as follows



(i) Safe Moment of Resistance $M = f Z$

f = permissible bending stress which is constant for given material.

$Z = \frac{I}{y}$ = section modulus, which signifies the flexural strength of the beam.

— Safe moment of resistance (M) is independent of length of beam & type of loading.

— M is the property of cross-section & material.

— The expression $M = f Z$ can be applied if the bending stress variation is linear.

(i) Yield Moment of Resistance $M_y = f_y \cdot Z$.

(ii) Plastic Moment of resistance / capacity. $M_p = C \times L \cdot A = T \times L \cdot A$

C = compressive force

T = tensile force on section

$L \cdot A$ = lever arm.

$\Rightarrow$ here M_p cannot be written as $f_y Z$ as variation of stress is not linear at fully plastic stage.

hence.

$$M_p = C \times L \cdot A = f_y \frac{A}{2} (\bar{y}_1 + \bar{y}_2).$$

$$M_p = f_y \left\{ \frac{A}{2} (\bar{y}_1 + \bar{y}_2) \right\}.$$

$$M_p = f_y \cdot Z_p.$$

where $Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$ = Plastic Modulus.

$\bar{y}_1$ & $\bar{y}_2$ are the centroidal distance of compression & tension area of c/s from N.A.

A = total cross-section area

Note: At fully plastic state N.A. cuts the area into two equal parts (area) as follows.

$$\sum F_x = 0 \Rightarrow C = T$$

$$f_y A_c = f_y A_t.$$

$$A_c = A_t \quad \text{--- (i)}.$$

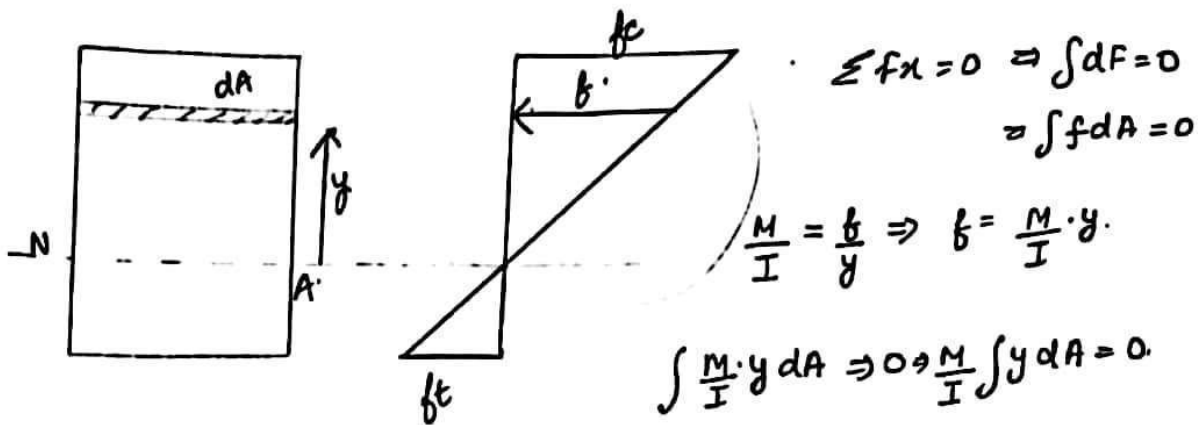
$$A_c + A_t = A \quad \text{--- (ii)}$$

$$\text{from (i) \& (ii)} \quad A_c = A_t = \frac{A}{2}$$

So, plastic area N.A. cuts the area into 2 equal parts.

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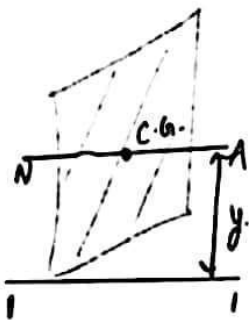
Note: + However elastic NA passes through C.G. of the A/s.



$$\Rightarrow \int y dA = 0$$

$\int y dA$ represents the moment of total area & it is zero only about centroidal axis of the section.

$\Rightarrow$ So elastic N.A. must pass through C.G. of the section.



Moment of area about axis 1-1 = Ay .

Moment of area about N.A. = $A(0) = 0$.

(iv) Shape Factor It is the ratio of plastic moment capacity & yield moment capacity of a cross-section.

$$SF = \frac{M_P}{M_y} = \frac{f_y Z_P}{f_y Z} = \frac{Z_P}{Z}$$

It represents the strength of the beam section beyond yield moment to reach the plastic state.

Hence, higher shape factor implies more reserved strength beyond yield moment to reach plastic state.

- (V) Load Factor

$$LF = \frac{\text{ultimate load}}{\text{working load}} = \frac{\text{Plastic moment capacity of c/s}}{\text{safe moment capacity of c/s}}$$

$\downarrow$ $\downarrow$

RCC steel.

$$LF = FOS \times SF$$

- Load Factor represents the reserved strength of the beam section beyond safe moment to reach fully plastic state.

a) for ductile material,

d

$FOS = \frac{fy}{\bar{\sigma}}$

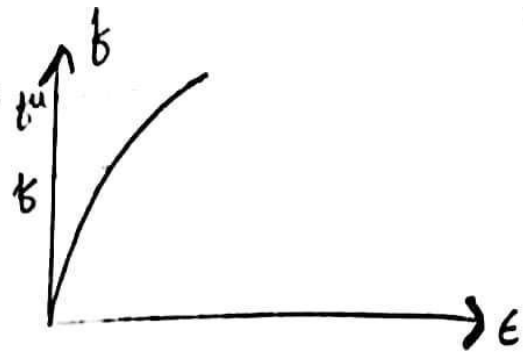
σ

ϵ

b) for brittle materials

It is defined as ratio of ultimate stress to the working stress.

$$FOS = \frac{f_u}{f}$$



c) Margin of safety. (MOS)

- It is defined only for brittle materials

$$MOS = \frac{\text{ultimate stress}}{\text{working stress}} - 1 = FOS - 1 \text{ or } LF - 1$$

vii) Moment - curvature Relationship

- As per flexural formula: $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$

y = distance from NA to extreme fibres.

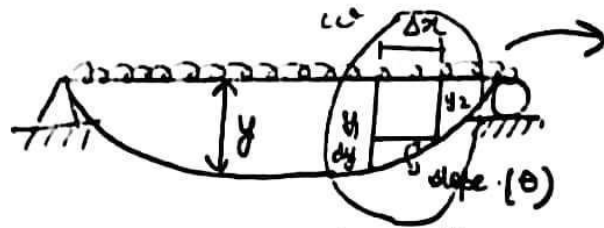
R = Radius of curvature

$\frac{1}{R}$ = curvature = Rate of change of slope.

Now $\frac{1}{R} = \frac{M}{EI} \Rightarrow \text{curvature} \propto \text{moment}$

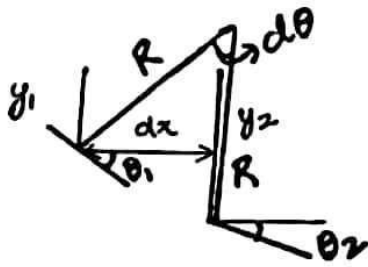
Hence, as moment increases, curvature increases i.e. rate of change of slope increases.

y = deflection = displacement of beam from initial configuration.



$\frac{dy}{dx} = \theta = \text{slope} = \text{Rate of change of deflection along the length of beam.}$

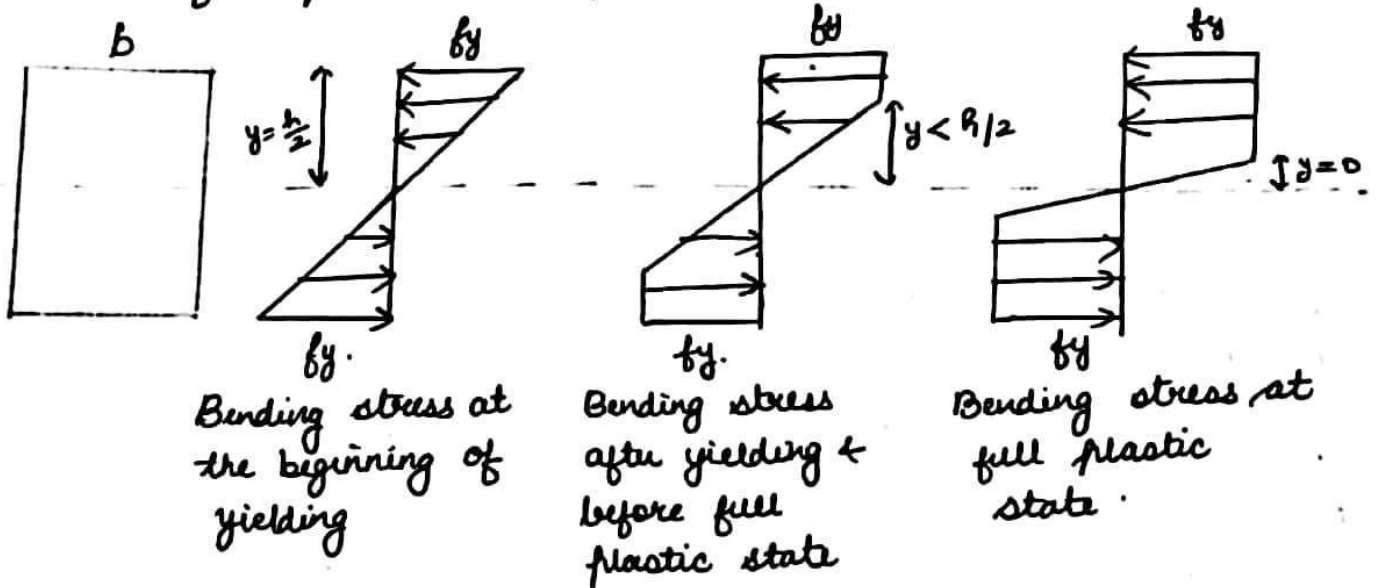
$\frac{d^2y}{dx^2} = \frac{d\theta}{dx} = \frac{1}{R} = \text{curvature} = \text{Rate of change of slope along the length of beam.}$



$$dx = R d\theta \Rightarrow \frac{d\theta}{dx} = \frac{1}{R} = \text{curvature}$$

$$[d\theta = \theta_2 - \theta_1]$$

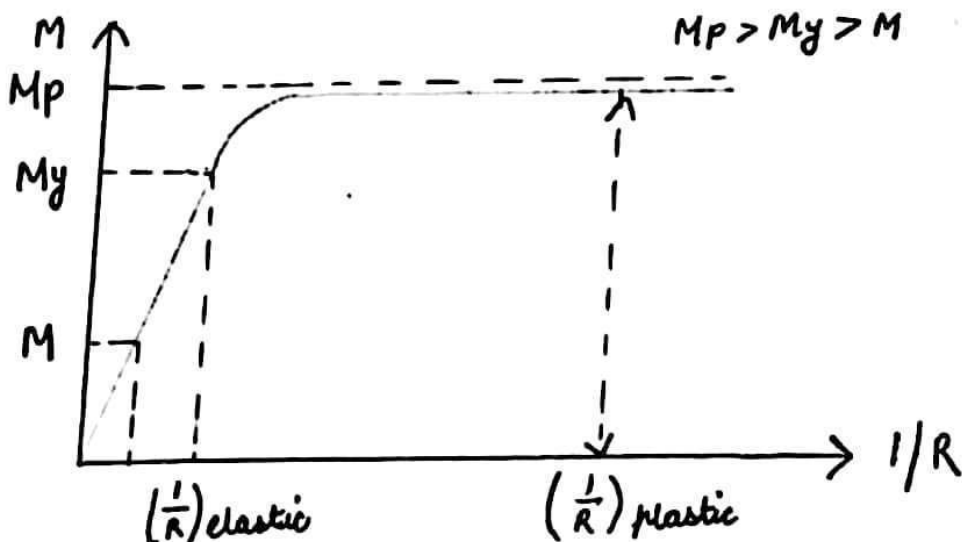
- At the beginning of yielding $\frac{1}{R} = \frac{M_y}{EI}$
- At the full plastic state, curvature is infinity



- full plastic state $\frac{f}{y} = \frac{E}{R} \Rightarrow \frac{1}{R} = \frac{f}{Ey} = \frac{f}{y \times 0} = \infty$

Since, curvature is infinity, it signifies rate of change of slope is very high in fully plastic state when compared with elastic state.

hence moment curvature relationship can be represented as

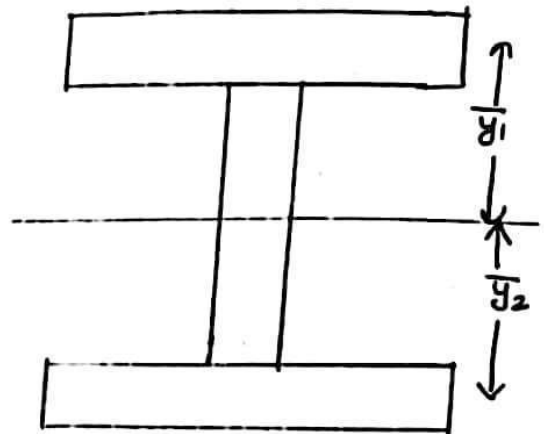


viii) $M_p = f_y \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$ or $f_y Z_p$.

- Plastic moment capacity of cross-section is independent of the length of beam & loading of the beam.

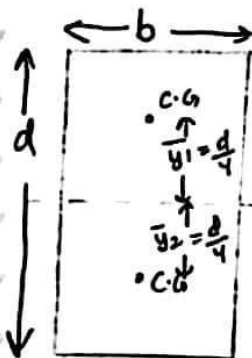
- It depends on type of material (f_y), area of cross-section of that material & distribution of that area ($\bar{y}_1, \bar{y}_2$).
(A)

- Thus, the most efficient cross-section in resisting BM is I-s/c. as for given c/s area $\bar{y}_1$ & $\bar{y}_2$ are maximum for I-s/c.



Q. Compute the S.F. for following s/c.

a) For Rectangular section.



Elastic & Plastic N.A.

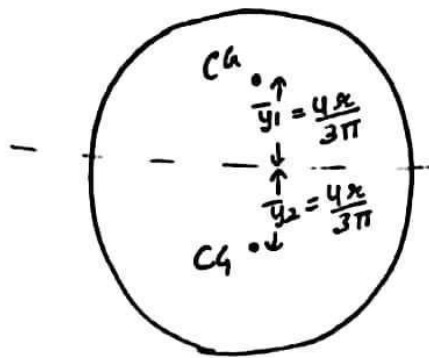
$$Z = \frac{I}{y} = \frac{bd^3}{12 \cdot \frac{d}{2}} = \frac{bd^2}{6}$$

$$\begin{aligned} Z_p &= \frac{A}{2} (\bar{y}_1 + \bar{y}_2) \\ &= \frac{bd}{2} \left(\frac{d}{4} + \frac{d}{4} \right) \\ &= \frac{bd^2}{4} \end{aligned}$$

$$\begin{aligned} S.F. &= \frac{Z_p}{Z} = \frac{\frac{bd^2}{4}}{\frac{bd^2}{6}} \\ &= \frac{6}{4} \end{aligned}$$

$$\boxed{S.F. = 1.5}$$

b) for circular section.



Elastic & Plastic N.A.

$$Z = \frac{I}{y} = \frac{\pi d^4}{64 \cdot \frac{d}{2}} = \frac{\pi d^3}{32}$$

$$Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$$

$$Z_p = \frac{\pi d^2}{4 \times 2} \left(\frac{4r}{3\pi} + \frac{4r}{3\pi} \right)$$

$$Z_p = \frac{\pi d^2}{8} \left(2 \times \frac{4r}{3\pi} \right)$$

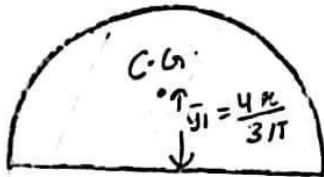
$$= \frac{\pi d^2}{8} \left(2 \times \frac{4 \times d}{2 \times 3\pi} \right)$$

$$Z_p = \frac{d^3}{6}$$

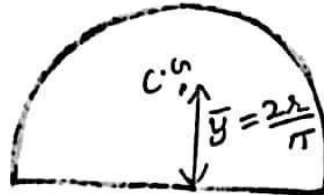
$$S.F. = \frac{Z_p}{Z} = \frac{\pi d^3}{\frac{\pi d^3}{32}} = \frac{d^3}{6 \times \frac{\pi d^3}{32}} = \frac{32}{6\pi}$$

$$S.F. = 1.7$$

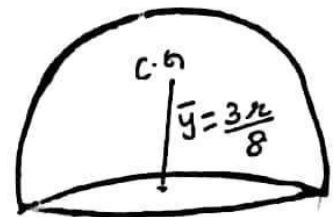
Notes



Semi circular area

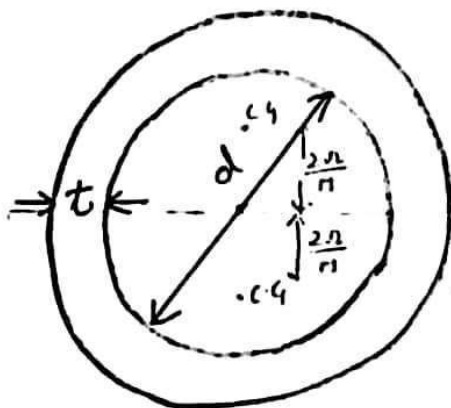


Semi circular line

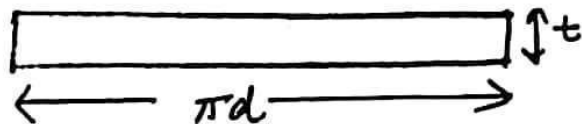


Hemisphere.

c) for thin hollow circular section.



Elastic & Plastic N.A.



$$Z = \frac{I}{y} = \frac{\pi d^3 t}{8 \cdot \frac{d}{2}} = \frac{\pi d^2 t}{4}$$

$$I_{xx} + I_{yy} = I_p \quad (i)$$

$$I_p = A r^2 = \pi d t \cdot \frac{d^2}{4} = \frac{\pi d^3 t}{4} \quad (ii)$$

$$I_{xx} = I_{yy}$$

$$I_{xx} = I_{yy} = \frac{I_p}{2} = \frac{\pi d^3 t}{8}$$

$$Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2) = \frac{\pi d t}{2} \left(\frac{2h}{\pi} + \frac{2x}{\pi} \right)$$

$$= \frac{\pi d t}{2} \cdot \frac{2d}{\pi} = d^2 t$$

$$S.F = \frac{Z_p}{Z} = \frac{d^2 t}{\frac{\pi d^2 t}{4}} = \frac{4}{\pi}$$

$$\boxed{S.F = 1.27}$$

Alternate approach. $Z = \frac{\frac{\pi d^4}{64} - \frac{\pi (d-2t)^4}{64}}{\frac{d}{2}} = \frac{\frac{\pi d^3}{32} - \frac{\pi (d-2t)^3}{32}}{\frac{d}{2}}$

$$Z = \frac{\pi d^3}{32} \left[1 - \frac{(d-2t)^4}{d^4} \right]$$

$$Z = \frac{\pi d^3}{32} \left[1 - \left(1 - \frac{2t}{d} \right)^4 \right]$$

$$= \frac{\pi d^3}{32} \left[1 - \left(1 - 4 \times \frac{2t}{d} + \text{neglected} \right) \right]$$

$$= \frac{\pi d^3}{32} \left[\frac{4 \times 2t}{d} \right]$$

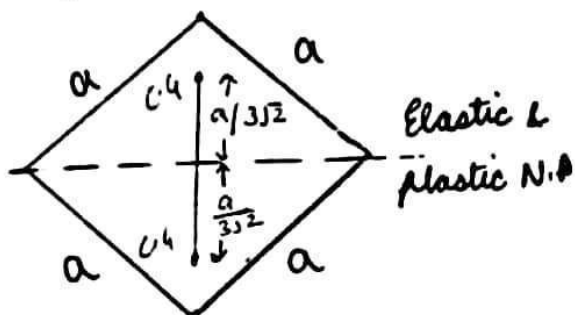
$$= \frac{\pi d^2 t}{4}$$

$$Z_p = \frac{d^3}{6} - \frac{(d-2t)^3}{6} = \frac{d^3}{6} \left[1 - \left(1 - \frac{2t}{d} \right)^3 \right]$$

$$= \frac{d^3}{6} \left[1 - 1 + \frac{6t}{d} \right] = d^2 t$$

$$S.F = \frac{Z_p}{Z} = 1.27$$

d) for diamond section.



$$Z = \frac{I}{y} = \frac{\frac{a^4}{12} \cdot \frac{a}{\sqrt{2}}}{\frac{a}{\sqrt{2}}} = \frac{a^3}{6\sqrt{2}}$$

$$Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$$

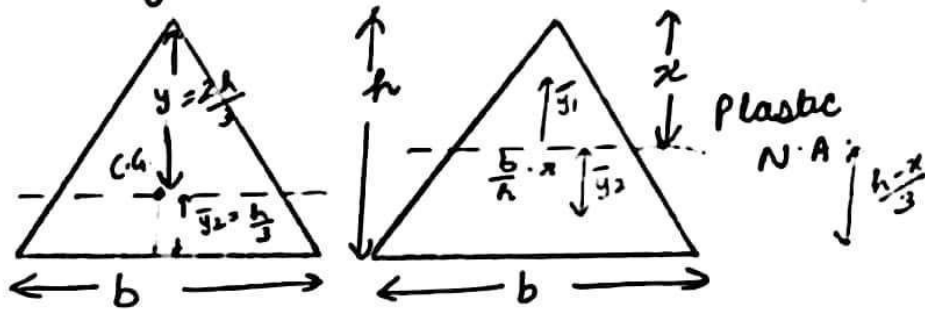
$$= \frac{a^2}{2} \left(\frac{a}{3\sqrt{2}} + \frac{a}{3\sqrt{2}} \right)$$

$$Z_p = \frac{a^3}{3\sqrt{2}}$$

$$S.F. = \frac{Z_p}{Z} = \frac{a^3}{3\sqrt{2} \cdot \frac{a^3}{6\sqrt{2}}} = \frac{6}{3}$$

$$\boxed{SF=2}$$

e) for triangular section.



$$Z = \frac{I}{y_{max}} = \frac{\frac{bh^3}{36} \cdot \frac{2h}{3}}{\frac{2h}{3}} = \frac{3}{36 \times 2} bh^2 = \frac{bh^2}{24}$$

$$\text{for } Z_p: \frac{1}{2} \cdot \frac{b}{h} \cdot x \cdot x = \left(\frac{1}{2} bh \right) \cdot \frac{1}{2}$$

$$x = \frac{h}{\sqrt{2}}$$

$$Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2)$$

$$\left(\frac{1}{2} bh \right) \times \frac{1}{2} \left(\frac{h}{\sqrt{2}} + 0.1548h \right)$$

$$Z_p = 0.09763 bh^2$$

$$S.F. = \frac{Z_p}{Z} = \frac{0.09763 bh^2}{\frac{bh^2}{24}}$$

$$\boxed{SF = 2.34}$$

$$\bar{y}_1 = \frac{x}{3} = \frac{h}{\sqrt{2} \times 3}$$

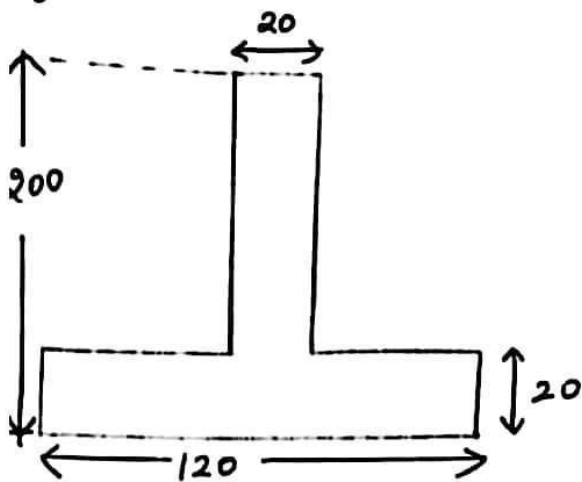
$$\bar{y}_2 = \left(\frac{a+2b}{a+b} \right) \frac{h}{3}$$

$$= \left(\frac{\frac{b}{h} \cdot x + 2b}{\frac{b}{h} \cdot x + b} \right) \left(\frac{h-x}{3} \right)$$

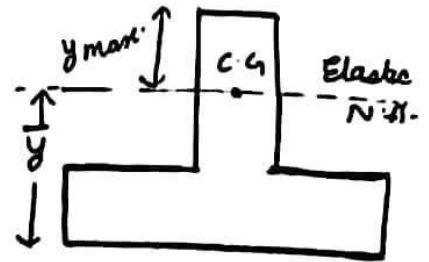
$$= \left(\frac{\frac{b}{\sqrt{2}} + 2b}{\frac{b}{\sqrt{2}} + b} \right) \left(\frac{h - \frac{h}{\sqrt{2}}}{3} \right)$$

$$\bar{y}_2 = 0.1548h$$

f) For T-section.



For Z ,



$$A\bar{y} = A_1\bar{y}_1 + A_2\bar{y}_2$$

$$\bar{y} = \frac{180 \times 20 \times (90 + 20) + 120 \times 20 \times 10}{180 \times 20 + 120 \times 20}$$

$$\bar{y} = 70 \text{ mm}$$

$$y_{max} = 200 - \bar{y} = 130 \text{ mm}$$

$$Z = \frac{I}{y_{max}}$$

$$= \frac{\frac{20 \times 130^3}{3} + \frac{120 \times 70^3}{3} - \frac{120 \times 50^3}{3}}{130}$$

$$= 186.154 \times 10^3 \text{ mm}^3$$

$$\text{For } Z_p, 20 \times x = \frac{180 \times 20 + 120 \times 20}{2}$$

$$x = 150 \text{ mm}$$

$$\bar{y}_1 = \frac{x}{2} = \frac{150}{2} = 75 \text{ mm}$$

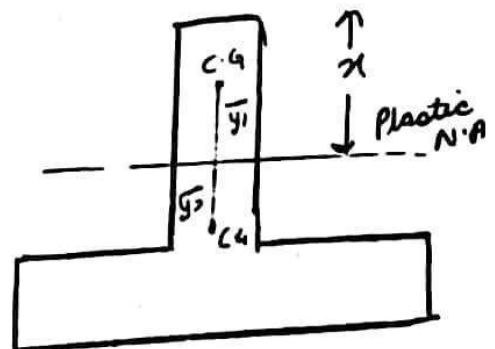
$$\bar{y}_2 = \frac{30 \times 20 \times 15 + 120 \times 20 \times (30 + 10)}{30 \times 20 + 120 \times 20}$$

$$\bar{y}_2 = 35$$

$$Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2) = 20 \times 150 (75 + 35) = 330 \times 10^3 \text{ mm}^3$$

$$SF = \frac{Z_p}{Z} = \frac{330 \times 10^3}{186.154 \times 10^3} = 1.77$$

$$\boxed{SF = 1.77}$$

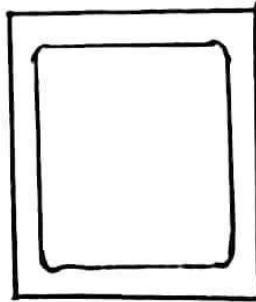
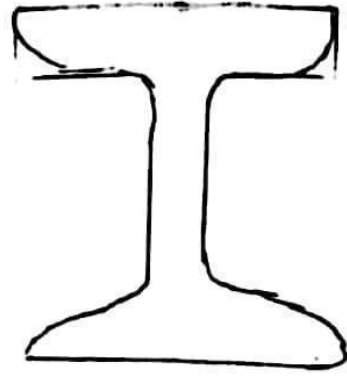


g) for Rolled I-section.

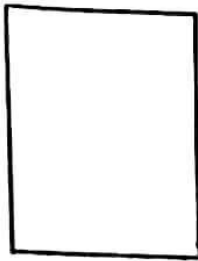
$$SF = 1.12 - 1.14$$

h) for Rolled Rectangular hollow section.

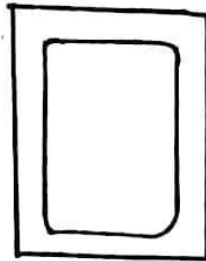
$$SF = 1.12$$



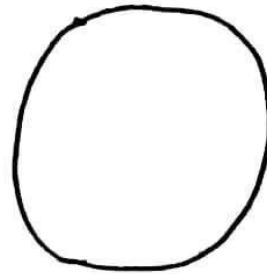
Note: $\Rightarrow$



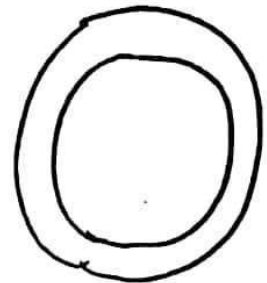
$$SF = 1.5$$



$$SF = 1.12$$



$$SF = 1.7$$

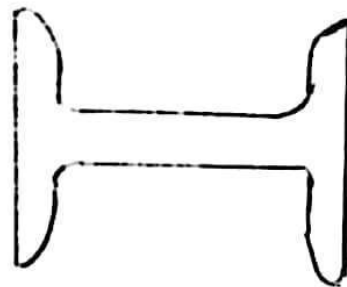


$$SF = 1.27$$

- In comparison to thin hollow ~~box~~ sections, reserved strength beyond yield moment is more for solid section.
- In comparison to thin hollow box section, reserved strength of thin hollow circular section is more beyond yield moment.

i) for horizontal I-section

$$SF = 1.5$$



Q Shape factor for rectangular section is 1.5. If permissible bending stress is $0.67 f_y$ then compute load factor.

$$LF = FOS \times SF = \frac{f_y}{f} \times SF = \frac{f_y}{0.67 f_y} \times SF = \frac{1.5}{0.67} = 2.23$$

Q. If yield moment capacity of section in above case is 12 kN-m compute plastic moment capacity.

$$SF = \frac{M_p}{M_y} \Rightarrow M_p = SF \times M_y$$

$$= 1.5 \times 12 = 18 \text{ kN-m}$$

Q. If same moment capacity is 12 kN-m, then what is plastic capacity

$$M_p = LF \times M_y$$

$$= 2.23 \times 12 = 26.76 \text{ kN-m}$$

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Q. shape factor for I-s/c is 1.12. FOS in bending is 1.5. If the permissible stress is increased by 20%. Then what would be its load factor.

$$\text{Load factor} = \frac{M_p}{M} = \frac{\frac{b_y}{b} \frac{Z_p}{Z}}{1.2} = \left(\frac{b_y}{b} \right) \left(\frac{Z_p}{Z} \right)$$

If permissible stress is increased by 20%.

$$LF = \frac{\frac{b_y}{b} \left(\frac{Z_p}{Z} \right)}{1.2} = \frac{1}{1.2} \left(\frac{b_y}{b} \right) \left(\frac{Z_p}{Z} \right) = \frac{FOS \times SF}{1.2} = \frac{1.5 \times 1.12}{1.2}$$

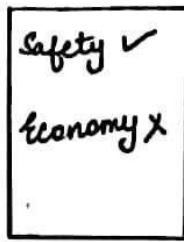
$$LF = 1.4$$

Note: 1) With increase in permissible stress, load factor decreases.

2) If wind & earthquake loads are considered, permissible stresses in structural steel are increased by 33% and in bolts & welds, permissible stresses are increased by 25%.
As with increase in permissible stress, LF decreases - that economises the designing.



When WL & EL are not considered

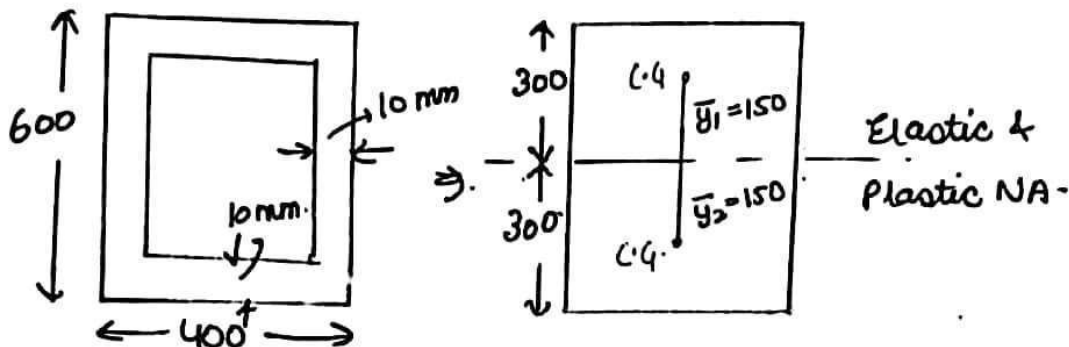


When WL & EL are considered by but permissible stresses are not increased.



When WL & EL are considered & permissible stresses are also increased.

Q A thin hollow box section of outer dimensions 400×600 mm and thickness of 10 mm is used as beam section, find its Z_p .

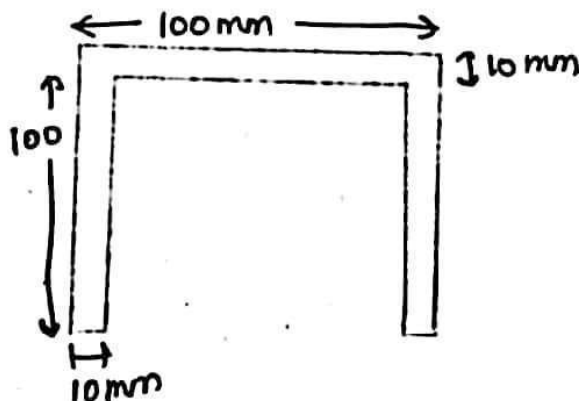


$$Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2) = \left(\frac{600 \times 10}{2} \right) \times 2 (150 + 150) +$$

$$\left(\frac{400 \times 10}{2} \right) \times 2 \times (300 + 300)$$

$$Z_p = 4.2 \times 10^6 \text{ mm}^3$$

Q Compute M_p for the given section.



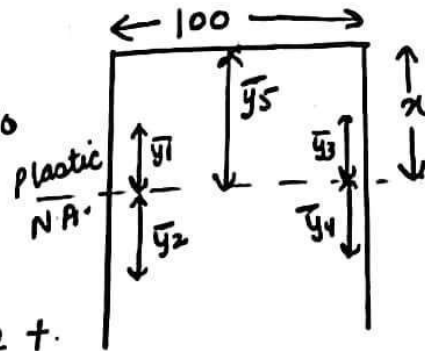
$$M_p = f_y Z_p$$

$$Z_p = A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3 + A_4 \bar{y}_4$$

for plastic NA, $A_L = A_T$

$$100 \times 10 + 2 \times x \times 10 = 2 \times (100 - x) \times 10$$

$$x = 25 \text{ mm}$$



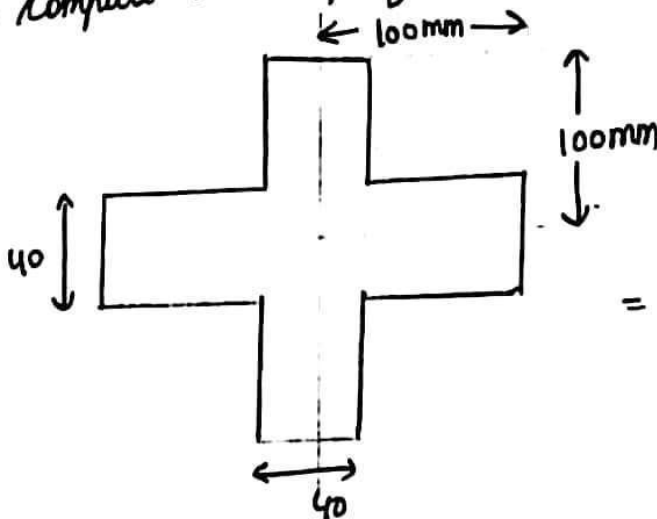
$$Z_p = \left(25 \times 10 \times \frac{25}{2} \right) \times 2 + \left(75 \times 10 \times \frac{75}{2} \right) \times 2 +$$

$$100 \times 10 \times 25$$

$$Z_p = 87500 \text{ mm}^3$$

$$M_p = 87500 \text{ ft}$$

B Compute the shape factor for the given section



$$SF = \frac{Z_p}{Z}$$

$$Z = \frac{I}{y}$$

$$= \frac{40 \times 200^3}{12} + \frac{160 \times 40^3}{12}$$

$$100$$

$$= 275200 \text{ mm}^3$$

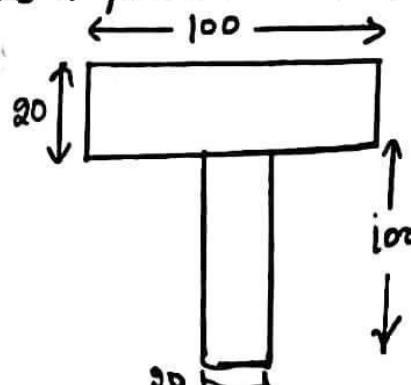
$$Z_p = \frac{A}{2} (\bar{y}_1 + \bar{y}_2) = \frac{200 \times 40}{2} (50 + 50) + \frac{160 \times 40}{2} (10 + 10)$$

$$= 464000$$

$$SF = \frac{464000}{275200} = 1.686$$

B Compute the distance b/w elastic & plastic NA for given s/c

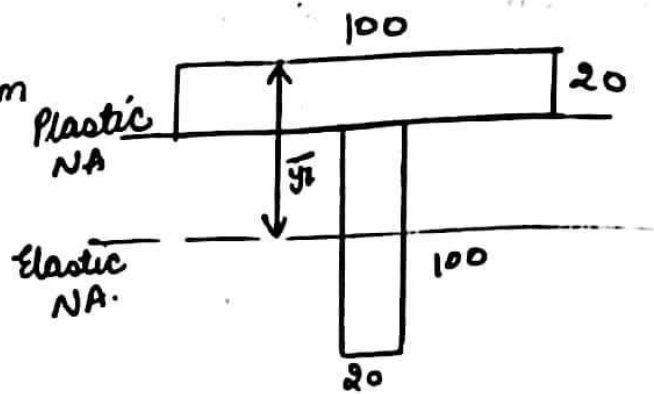
Plastic NA cuts the entire area into two equal parts, hence it passes through bottom of flange.



Location of plastic NA
from top of flange = 20 mm
for elastic NA.

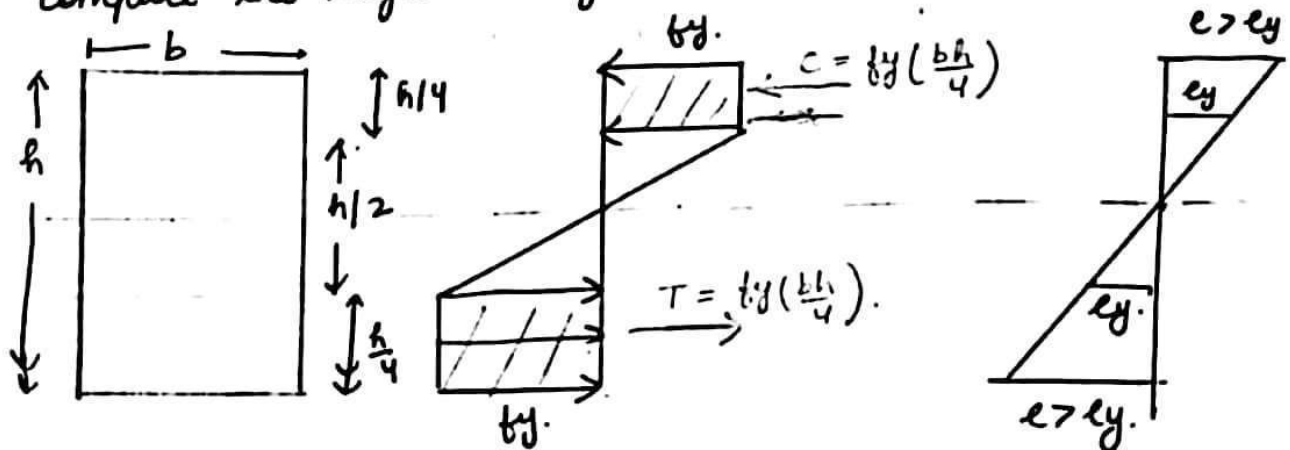
$$\bar{y} = \frac{100 \times 20 \times 10 + 100 \times 20 \times 70}{2 \times 100 \times 20}$$

$$= 40 \text{ mm.}$$



Hence distance b/w plastic & elastic N.A. = 20.

Q A rectangular section of width "b" & depth "h" is stressed to f_y upto a depth of $\frac{h}{4}$ from both ends (Top & bottom) under the action of moment "M". Compute the magnitude of M.



$$M = M_{\Delta} + M_{\square}$$

$$M_{\Delta} = f_y Z = f_y \left[\frac{b \left(\frac{h}{2} \right)^2}{6} \right] = f_y \frac{bh^2}{24}.$$

$$M_{\square} = C \times LA = f_y \left(\frac{bh}{4} \right) \left(\frac{3h}{4} \right) = f_y bh^2 \times \frac{3}{16}.$$

$$M = f_y bh^2 \left[\frac{1}{24} + \frac{3}{16} \right] \Rightarrow M = 0.229 f_y bh^2.$$

Note: Also compute the moment required to produce a strain of $2\epsilon_y$.

solⁿ same $M = 0.229 f_y bh^2$

~~Ans~~ : ~~same as before~~

Q In the same case if yield moment is M_y , find the moment required to produce strain of $2\epsilon_y$.

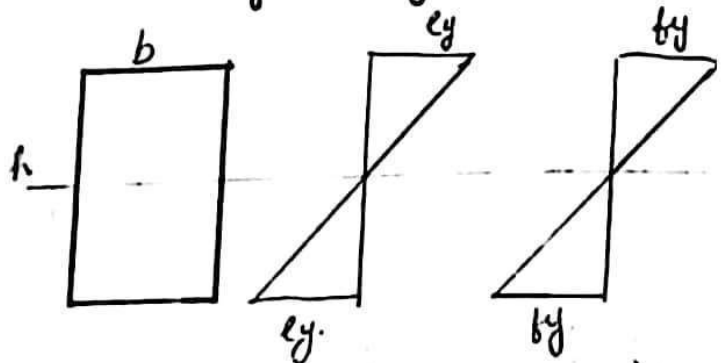
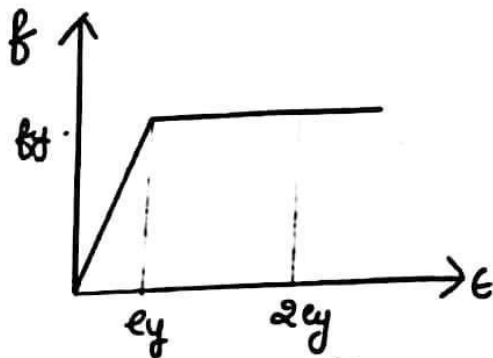
Moment required to produce the strain of $2\epsilon_y$.

$$M = 0.229 f_y b h^2.$$

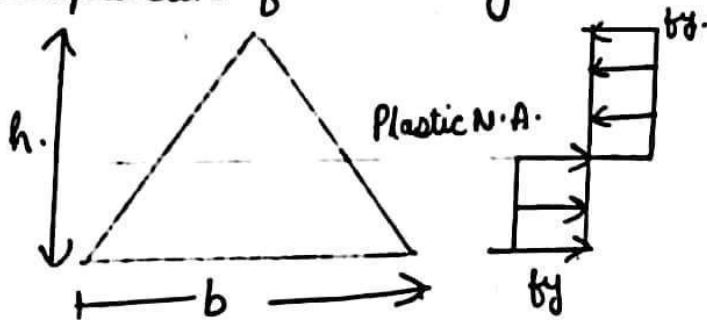
$$M_y = f_y Z = f_y \frac{b h^2}{6}.$$

$$6 M_y = f_y b h^2.$$

$$M = 0.229 \times 6 M_y = 1.375 M_y.$$



Q When a Δ section becomes a plastic hinge, the compressive force acting on the section.



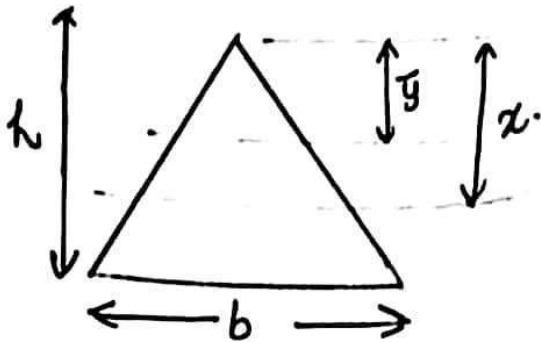
$$C = f_y \cdot \frac{A}{2}.$$

$$= f_y \frac{1}{2} \left(\frac{1}{2} b h \right).$$

$$C = f_y \frac{b h}{4}.$$

If section is converted to plastic hinge, bending stress distribution across the section becomes rectangular at full plastic state.

Q Also compute the difference b/w elastic & plastic N.A.



- Elastic NA is at distance of $\bar{y} = \frac{2}{3}h$ ($0.66h$) from vertex of Δ .

- Let plastic NA is at distance " x " from vertex of Δ .

$$A_c = A_t = A/2.$$

$$\frac{1}{2} \left(\frac{b}{h} \cdot x \right) x = \left(\frac{1}{2} bh \right) \frac{1}{2}.$$

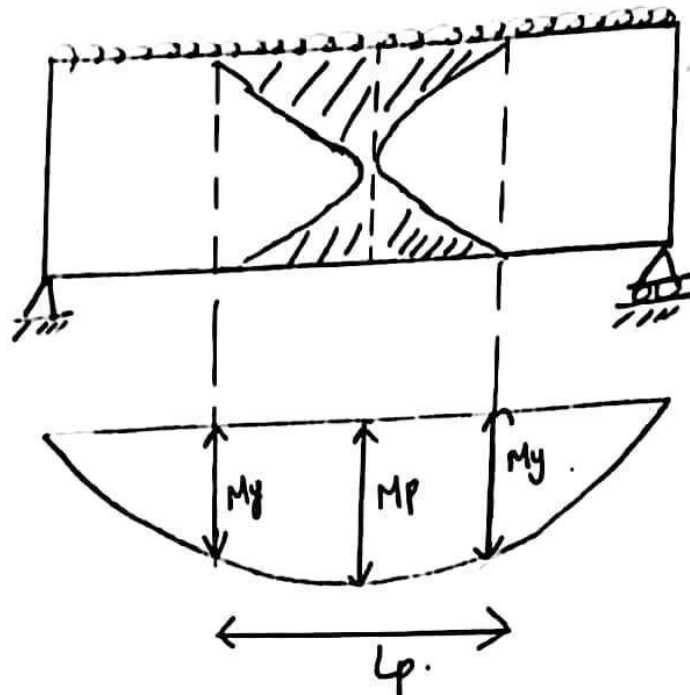
$$x = \frac{h}{\sqrt{2}} = 0.707h.$$

$$\begin{aligned} \text{Difference b/w plastic \& elastic N.A} &= 0.707h - 0.66h \\ &= 0.04h. \end{aligned}$$

PLASTIC HINGE

- If the cross-section of the beam is subjected to uniform/constant bending stress " f_y " throughout its depth, then it cannot take any further loading.
- If additional load is applied then the beam would rotate at that section.
- Or the beam rotates after applied BM reaches M_p value.
- Hence, plastic hinge means stress distribution becoming rectangular i.e. hinge behaviour is observed at full plastic state.
- Moment capacity of mechanical hinge is zero, whereas moment capacity of plastic hinge is equal to M_p .
- Hence, plastic hinge can be thought of as a rusted hinge in which upto M_p , there is resistance against rotation, but the instant applied BM, becomes equal to M_p , its moment resisting capacity becomes equal to zero.

- Plastic hinge can be defined as yielded zone due to flexure in a structure in which infinite rotation can take place at constant resisting moment of M_p .



L_p : length of plastic

- However for calculation purpose plastic hinge is assumed to be at 1 section where moment applied is equal to M_p .

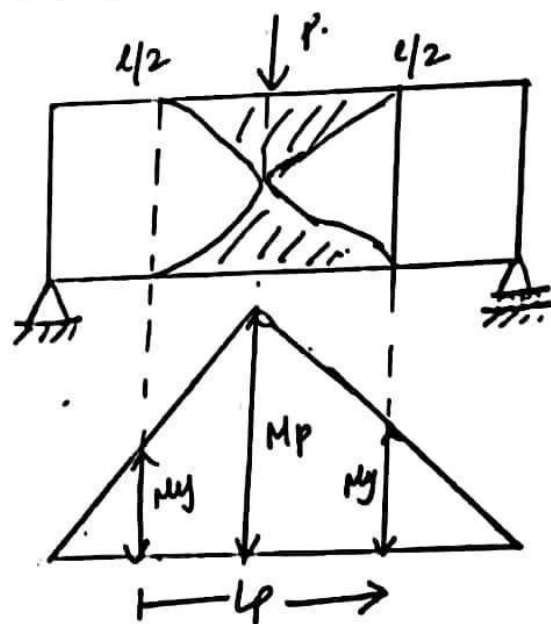
(Case 1) Simply supported beam with conc. load.

$$\frac{M_y}{\frac{l - L_p}{2}} = \frac{M_p}{l/2}$$

$$l - L_p = l \frac{M_y}{M_p}$$

$$L_p = l - \frac{l}{SF}$$

$$L_p = l \left(1 - \frac{1}{SF} \right)$$



Case 2) simply supported beam with UDL.

if $x = \frac{l}{2}$, $y = M_p$.

$$M_p = a \left(\frac{l}{2} \right)^2 - (i).$$

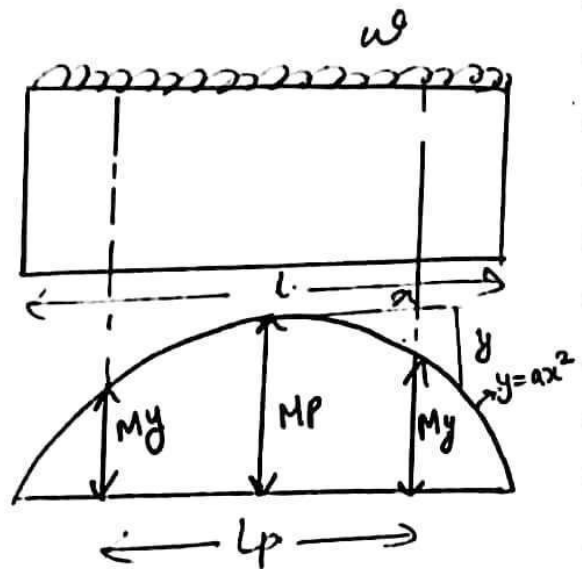
if $x = \frac{l_p}{2}$, $y = M_p - M_y$.

$$M_p - M_y = a \left(\frac{l_p}{2} \right)^2 - (ii)$$

$$\frac{M_p - M_y}{M_p} = \frac{l_p^2}{l^2}$$

$$1 - \frac{M_y}{M_p} = \left(\frac{l_p}{l} \right)^2$$

$$l_p = l \sqrt{1 - \frac{1}{SF}}$$



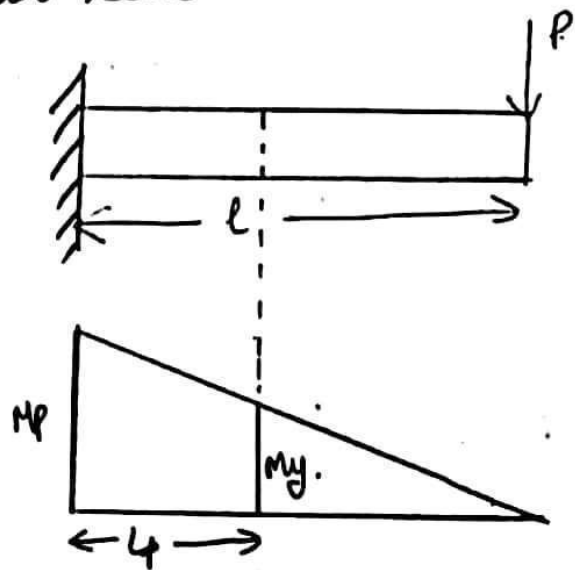
Case 3) cantilever beam with conc. load at free end.

$$\frac{M_p}{l} = \frac{M_y}{l - l_p}$$

$$\frac{l - l_p}{l} = \frac{M_y}{M_p}$$

$$l_p = l - \frac{l}{SF}$$

$$l_p = l \left(1 - \frac{1}{SF} \right)$$



Note: Length of plastic hinge depends on type of loading & type of cross-section.

- On the span of beam, plastic hinge forms first at the section subjected to max. curvature (curvature $\propto$ moment)

$$\text{curvature} \propto \frac{1}{R} = \frac{M}{EI}$$

- Due to formation of plastic hinge one after the other redistribution of moment takes place & because of this load carrying capacity of structure becomes greater than the load at which first hinge is formed.

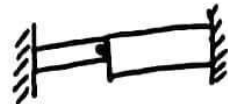
- Plastic hinge is expected to form at

a) point of location of concentrated load



In actual, no hinge is formed, its only meant that at this point along the whole depth stresses = f_y .

b) at section of sudden change in geometry



c) at fixed ends.

d) at point of zero shear in span subjected to distributed loading (for const. c/s).

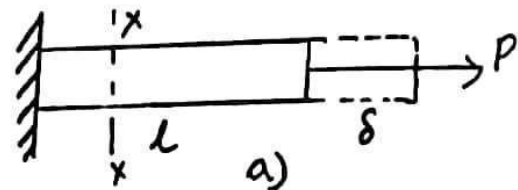
¶ Calculation of Collapse Load.

i) Mechanism (In elastic analysis).

a) stress at x-x, $f = \frac{P}{A}$

- here stress

" $f = \frac{P}{A}$ " develops after elongation of " δ ".



b) Here stress at x-x, $f = 0$

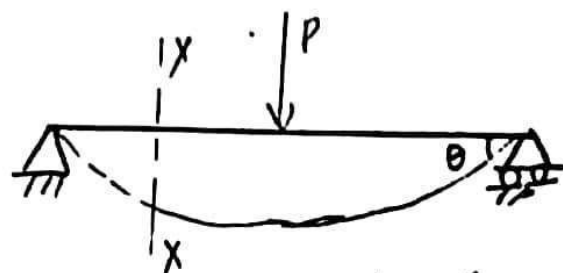
- here, without developing any stresses bar translates hence it is termed as "Rigid Body Translation".



b) Unstable / Mechanism.

c) stress at x-x, $f = \frac{M}{Z}$

- here beam rotates by " θ " after developing bending stress " f ".

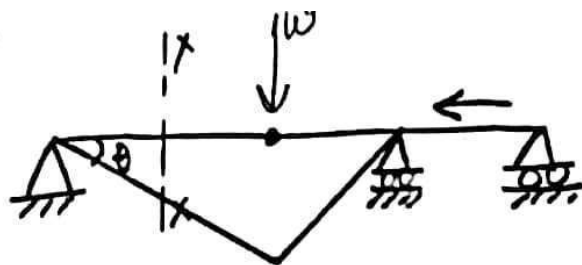


c) stable structure

d)

stress at $x-x$, $f = 0$

- without developing bending stress, beam rotates hence it is termed as Rigid Body Rotation.



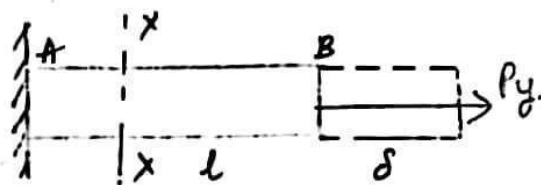
Unstable / Mechanism
Eg → selfie stick.

- Without developing any stress, if a body translates or rotates due to action of load, then it is termed as Rigid Body Translation & Rigid Body rotation respectively.
- Mechanism implies rigid body translation or rigid body rotation due to loads or the structure is unstable.

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(ii) Mechanism (In plastic Analysis)

a)



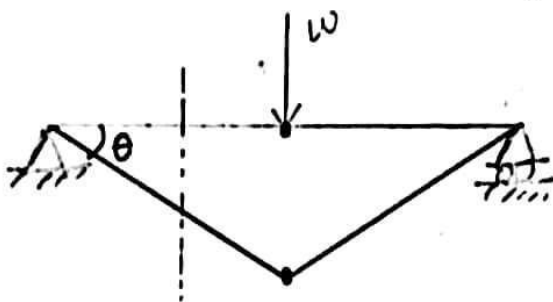
stress at $x-x$, $f_y = \frac{P_y}{A}$

- Here bar elongates without increase in stress of " f_y " after

reaching stress of " f_y "

- This can be idealised as Rigid Body Translation.
 $\delta \text{ inc} \rightarrow \text{no change in } f_y$

b)



- here after applied BM reaches M_p value, it rotates without increase in load or BM

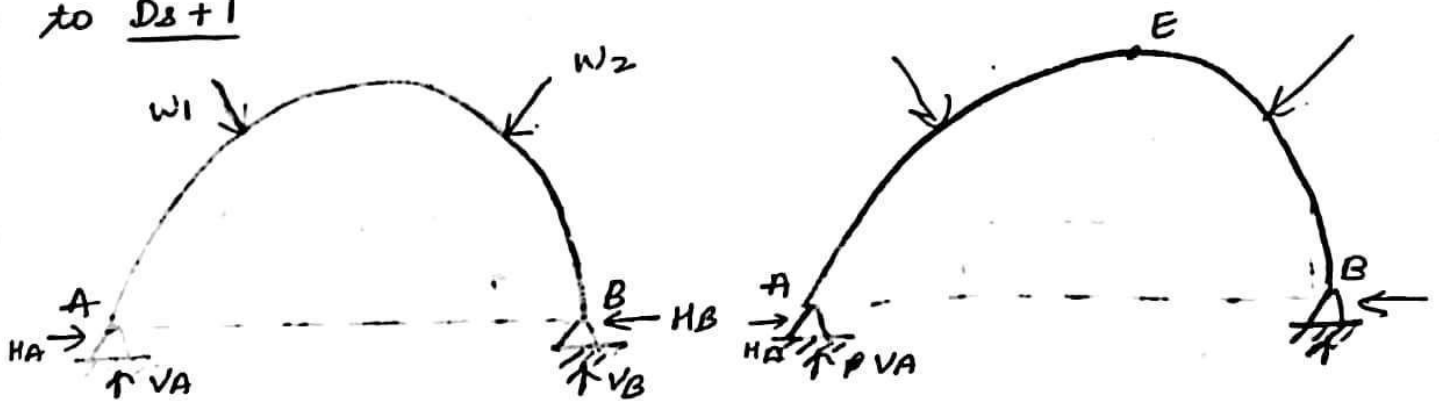
- This can be idealised as Rigid Body Rotation.
 $\theta \text{ inc} \rightarrow \text{no change in } M_p$. (because no change in load)

- Hence, if the segments of the beam b/w the plastic hinge are able to move ^{or rotate} without increase in load/BM, then it is termed as MECHANISM.

- Load required to form Mechanism is termed as COLLAPSE LOAD.

(iii) In plastic analysis to make a statically Indeterminate structure as a determinate structure, no. of plastic hinge required are equal to D_s (degree of static Indeterminacy)

- In plastic analysis, to form a mechanism (to make the structure unstable) in a statically Indeterminate structure, no. of plastic hinge required are equal to $D_s + 1$

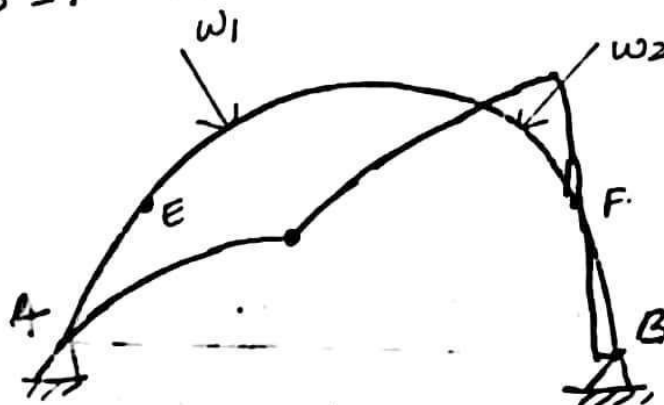


$$D_s = r - s$$

$$= 4 - 3 = 1$$

$$D_s = 4 - (3 + 1)$$

$$= 0.$$



Mechanism/ Unstable

$$D_s = r - s$$

$$= 4 - (3 + 2)$$

$$= -1$$

2 Plastic hinges are reqd to make struct unstable.

i.e. $D_s + 1$

(iv) In plastic analysis following conditions must be satisfied.

a) Equilibrium condition

When structure is about to collapse, it is still in equilibrium, hence

$$\sum F_x = 0, \sum F_y = 0, \sum M = 0$$

(This condition is also satisfied in elastic analysis of structure)

b) Mechanism Condition (Unstable Condition)

- When the structure is about to collapse, sufficient no. of plastic hinges must developed so that a part or entire structure must transform into a mechanism leading to collapse.

c) Yield Condition

- When the structure is about to collapse, BM at any section should not exceed plastic moment capacity of the cross-section i.e. applied $BM \leq M_p$.

Note:- Apart from above 3 conditions, Compatibility condⁿ must also be satisfied in plastic analysis.

- Out of all the above equations equilibrium & Compatibility conditions are valid for both Elastic & Plastic analysis of structure.
- If the above 3 conditions are satisfied, then we get the unique value of load called COLLAPSE LOAD.
- Depending on 1st & 2nd conditions are satisfied or 1st & 3rd conditions are satisfied, following method of analysis are available.

- A) Kinetic Method / Kinematic Theorem / Upper Bound Theorem (UBT)
 B) Static Method / Equilibrium Method / Lower Bound Theorem (LBT)

- Characteristics of UBT

- It satisfies equilibrium & mechanism conditions.
- It states, collapse load found by assuming a mechanism will always be greater than or equal to true collapse load.

i.e. $P \geq P_u$

P = calculated collapse load

P_u = true collapse load.

- Characteristics of LBT

- It satisfies equilibrium & yield conditions.
- It states the collapse load found on the basis of any collapse BM (in which this $BM \leq M_p$) will always be less than or equal to true collapse load.

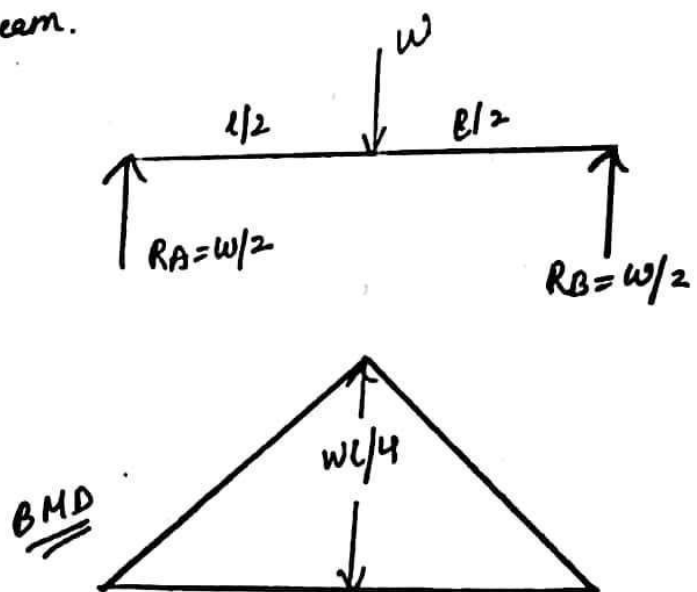
i.e. $P \leq P_u$

Calculation of the collapse load by Kinematic Method

A) Simply supported Beam.

- $D_s = 2 - 2 = 0$
 (determinate str)

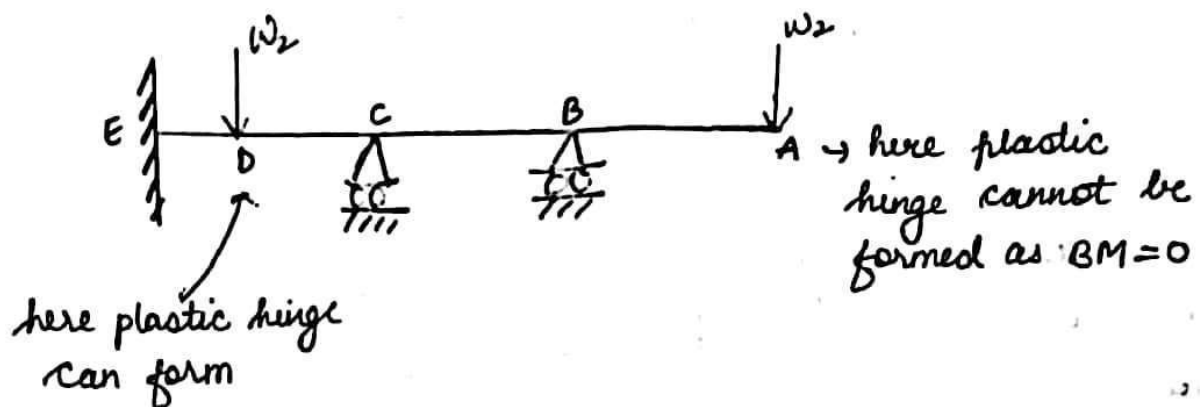
- No. of plastic hinges req.
 at collapse. $n = D_s + 1$
 $= 0 + 1$
 $= 1$



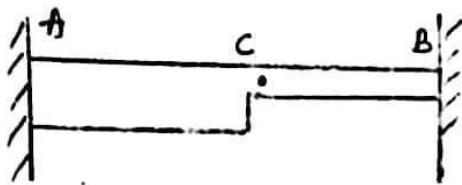
Note:- No. of plastic hinge sufficient for statically determinate structure to form mechanism is only "one".

— Locations where plastic hinge can form

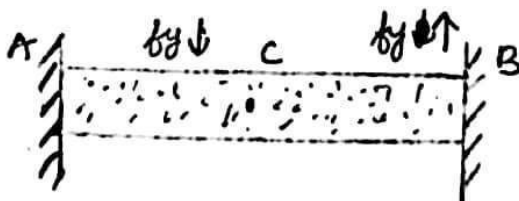
- at location of max. BM.
- at location of zero SF under distributed load.
- at fixed supports & rigid joint
- under point load in supported span, but not at free end.



- Whenever c/s changes, plastic hinge is developed in weak member at the interface.



- Whenever material changes plastic hinge is developed in weaker section.



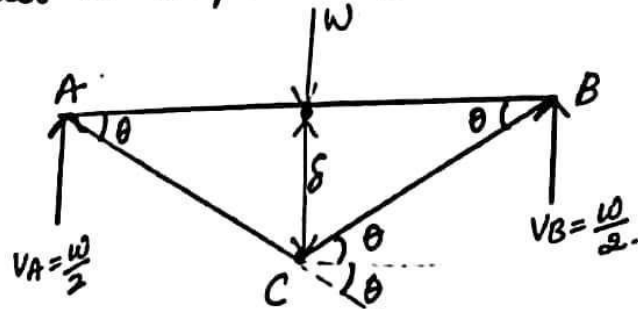
- At collapse condition b/w plastic hinges, elastic deflection and corresponding elastic strain energy are neglected i.e. segments of beam b/w plastic hinges are assumed to be rigid & straight.

- Principle of VIRTUAL WORK is applied now to compute collapse load.

• According to this principle, total virtual work done by all the forces to keep the beam in equilibrium is "0".

Note: Here virtual parameter is displacement.

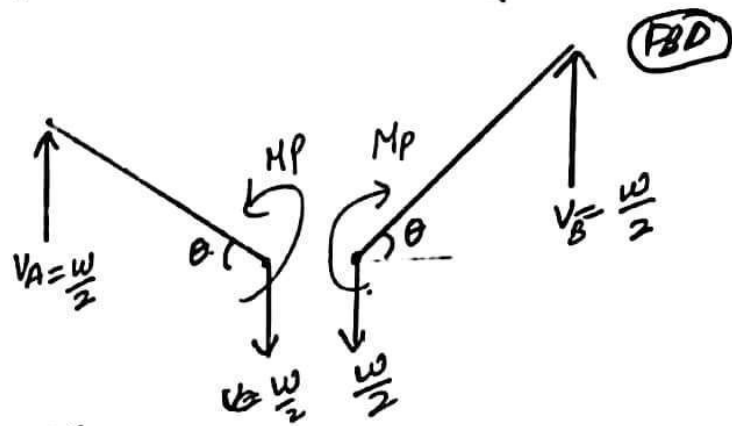
(i) $\delta = \frac{L}{2} \theta$
(virtual displacement)



- As per principle of virtual work done,
Total work done = $U = 0$

External work done (U_e) + Internal work done (U_i) = 0

It is always -ve
as M_p & θ are in opp dirⁿ



⇒ hence, external work done = Internal work done

$$\underbrace{\frac{W}{2} \left(\frac{L}{2} \theta \right)}_{\text{at C in CA}} + \underbrace{\frac{W}{2} \left(\frac{L}{2} \theta \right)}_{\text{at C in CB}} + \underbrace{W \times \frac{WL^3}{48EI}}_{\text{Neglected as it is elastic def.}} = \underbrace{\text{strain energy}}_{\text{neglected}} + \underbrace{M_p \theta}_{\text{at C in CA}} + \underbrace{M_p \theta}_{\text{at C in CB}}$$

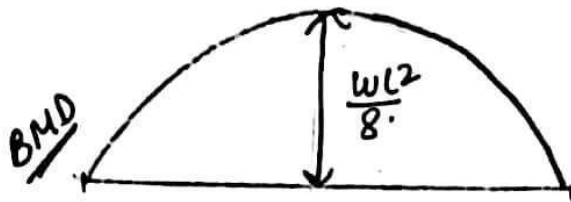
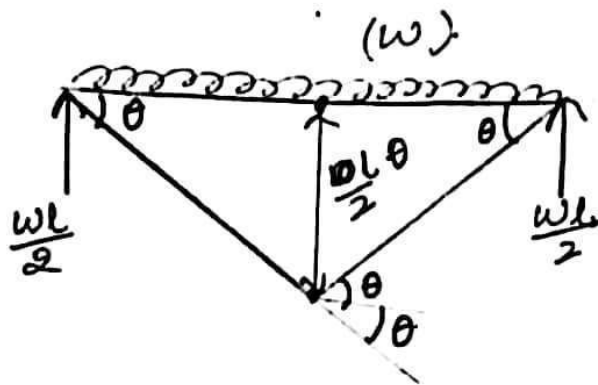
$$W \frac{L}{2} \theta = 2 M_p \theta$$

$$\boxed{W_c = \frac{4 M_p}{L}}$$

or $W \times \frac{L}{2} \theta = M_p (\theta + \theta)$

$$W = \frac{4 M_p}{L}$$

(ii)



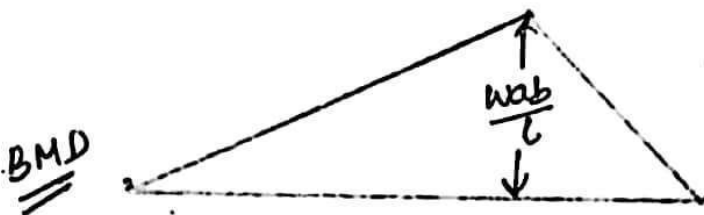
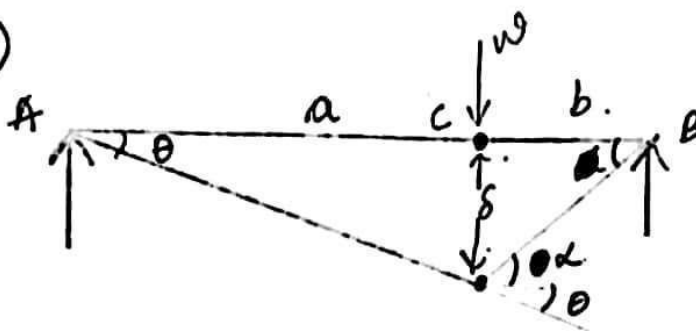
$$u_e = u_i$$

Intensity of
 $udl \times$
 area of
 displacement
 diag. = $M_p(\theta + \theta)$

$$w \times \frac{1}{2} l \times \frac{l}{2} \theta = M_p(2\theta)$$

$$w_u = \frac{8 M_p}{l^2}$$

(iii)



$$\delta = a\theta = b\alpha$$

$$u_e = u_i$$

$$w a \theta = M_p(\theta + \alpha)$$

$$w a \theta = M_p\left(\theta + \frac{a\theta}{b}\right)$$

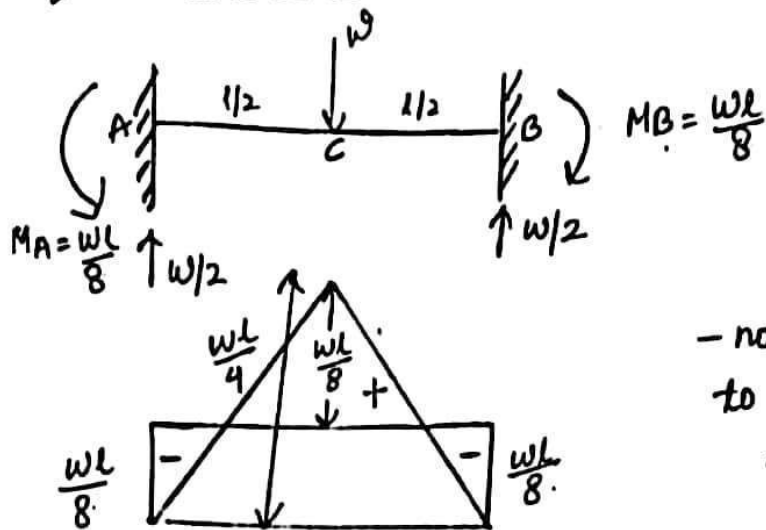
$$w a = M_p + \frac{a}{b} M_p$$

$$w = \frac{M_p}{a} + \frac{M_p}{b}$$

$$w = \frac{M_p(a+b)}{ab}$$

$$w_u = \frac{M_p l}{ab}$$

B) Fixed Beams



$$D_o = 4 - 2 = 2$$

- no. of plastic hinge req. to form mechanism

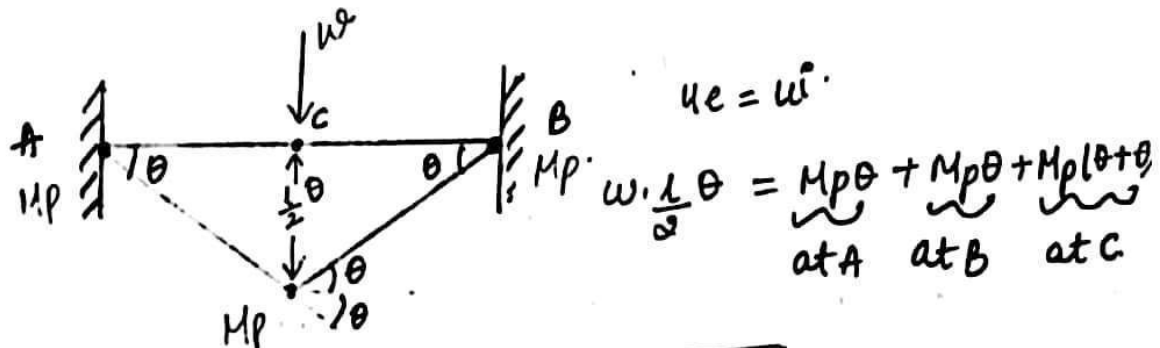
$$n = 2 + 1 = 3$$

- from elastic analysis,

$$\text{max +ve BM} = \text{max -ve BM} = \frac{wL}{8}$$

- from plastic analysis,

since max +ve & BM = max -ve BM, all 3 hinges will be developed simultaneously. at A, B & C



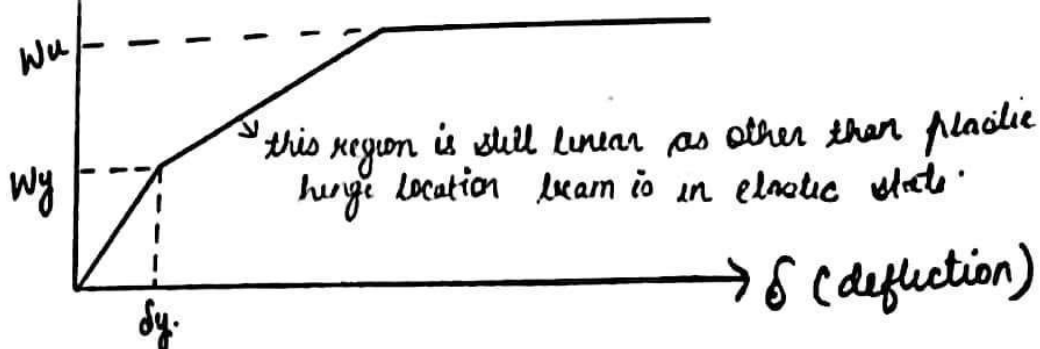
$$u_c = \dot{w}$$

$$w \cdot \frac{l}{2} \theta = \underbrace{M_p \theta}_{\text{at A}} + \underbrace{M_p \theta}_{\text{at B}} + \underbrace{M_p \theta}_{\text{at C}}$$

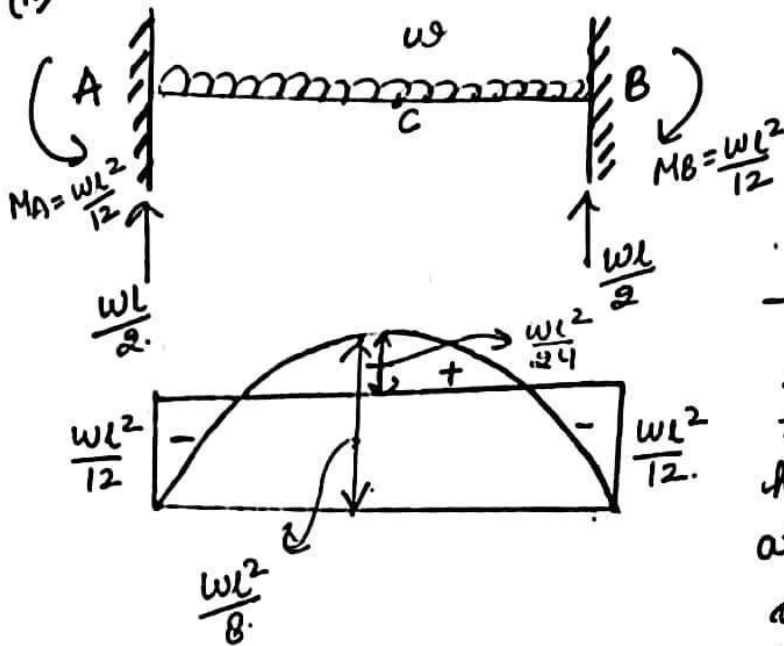
$$w_u = \frac{8M_p}{l}$$

w (applied load)

Load-deflection Curve



(ii)



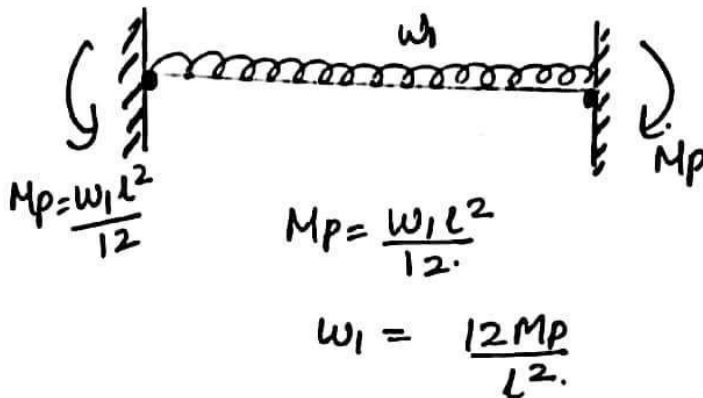
$$- n=3$$

- from elastic analysis

$$\max -ve BM = 2 \max +ve BM$$

- from plastic analysis,

since $\max -ve BM$ is more than $\max +ve BM$, 2 plastic hinges would develop first at A & B simultaneously. and when beam is loaded further, third hinge would develop at centre & mechanism would form.



$$M_p = \frac{w_1 L^2}{12}$$

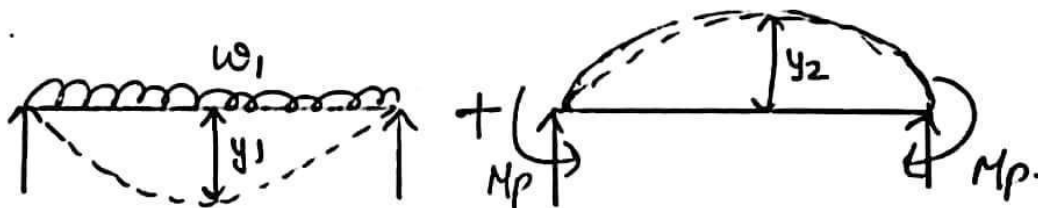
$$w_1 = \frac{12 M_p}{L^2}$$

→ At UDL of $w_1 = \frac{12 M_p}{L^2}$, two plastic hinges would form at

A & B and beam behaves like a simply supported beam with constant moment M_p at support.

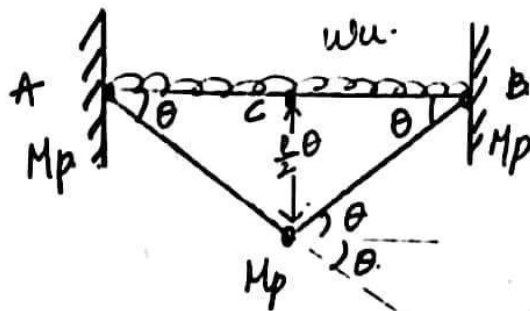
→ at this stage, deflection at centre is given by.

$$y_c = \frac{5}{384} \frac{w_1 L^4}{EI} - \frac{M_p L^2}{8EI} \quad \left\{ \text{using principle of superposition} \right\}$$



- Here principle of superposition is applied so other than plastic hinge location at A & B, beam is in elastic state only.

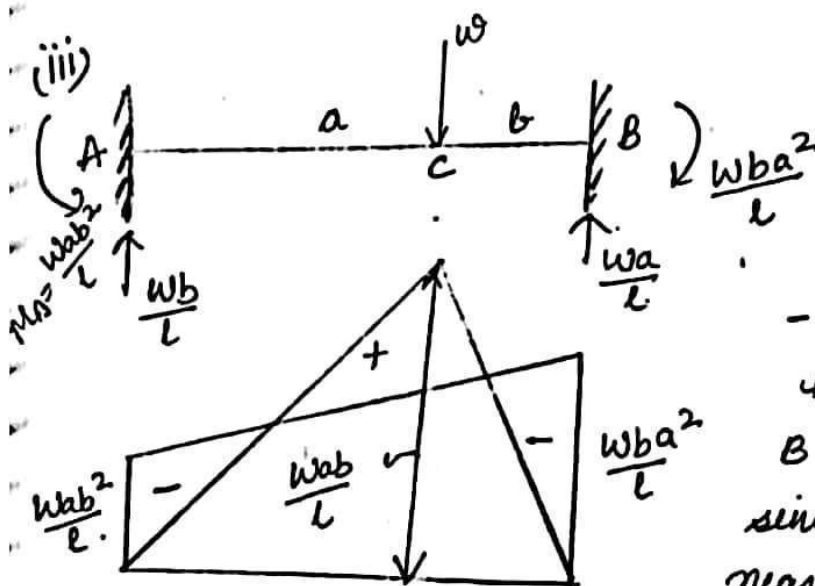
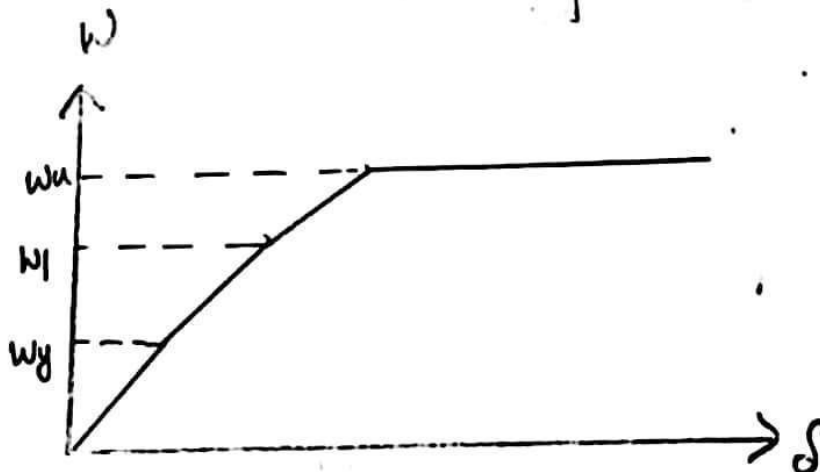
→ Now the load at which (W_u) collapse mechanism develop.



$$U_e = U_i$$

$$W_u \times \frac{1}{2} \times \frac{L}{2} \times \frac{L}{2} \theta = M_p \theta + M_p \theta + M_p (\theta + \theta)$$

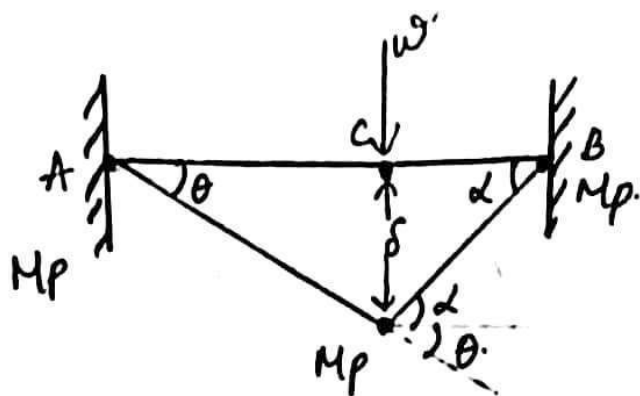
$$W_u = \frac{16 M_p}{L^2}$$



$$D_s = 2$$

$$n = 2 + 1 = 3$$

- sequence of plastic hinge development is $B \rightarrow C \rightarrow A$
 since the load is nearer to support B, max -ve BM at B is max, so first hinge at B



- When the beam is loaded further, second hinge is formed at C & then with further load increment, it is formed at A for collapse.

For collapse load, $U_e = U_i$

$$W a \theta = M_p \theta + M_p \alpha + M_p (\theta + \alpha)$$

$$W a \theta = M_p \theta + M_p \frac{a \theta}{b} + M_p \left(\theta + \frac{a \theta}{b} \right)$$

$$W a \theta = M_p \left[1 + \frac{a}{b} + 1 + \frac{a}{b} \right]$$

$$M_p \left[\frac{2(a+b)}{b} \right]$$

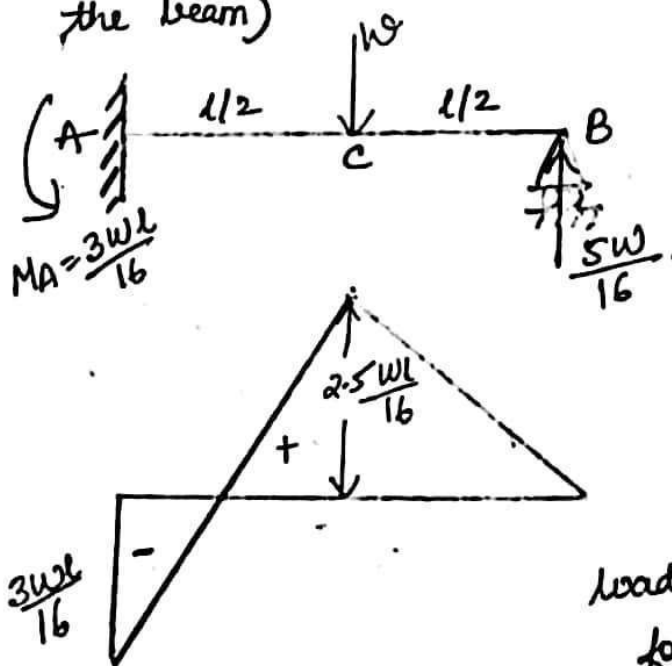
$$W_u = \frac{2 M_p l}{a b}$$

$$\delta = a \theta = b \alpha$$

$$\alpha = \frac{a \theta}{b}$$

C) Propped Cantilever.

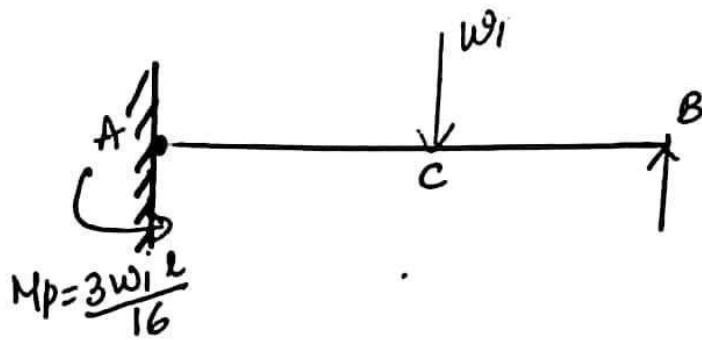
(The purpose of prop is to reduce BM & deflection in the beam)



$$- D \delta = 3 - 2 = 1$$

$$- n = 1 + 1 = 2$$

- Since max -ve BM is more than max +ve BM, first hinge would be developed at A (fixed end) & when beam is further loaded, next hinge is formed at C for collapse.



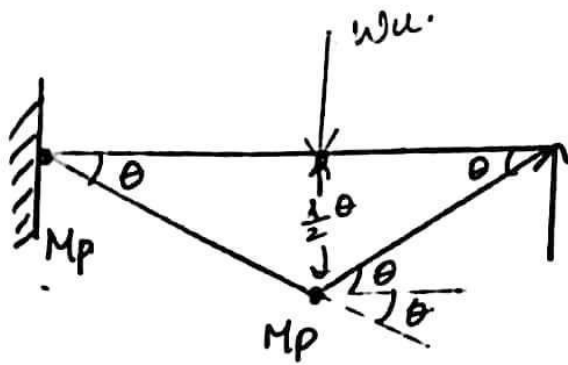
load at which first plastic hinge is formed.
 $W_1 = \frac{16 M_p}{3l} = 5.33 \frac{M_p}{l}$

$$R_B = \frac{5W_1}{16}$$

for collapse load.

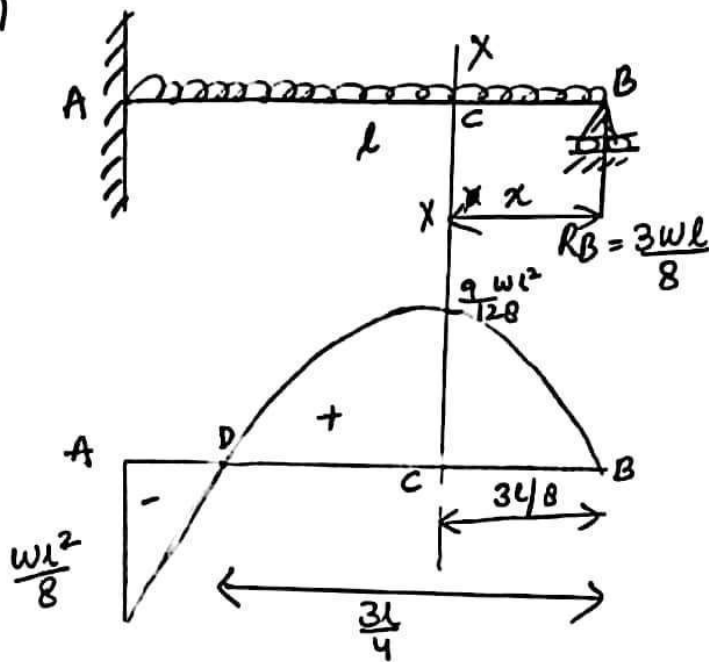
$$u_e = u_c'$$

$$W_u \frac{l}{2} \theta = M_p \theta + M_p (\theta + \theta)$$



$$W_u = \frac{6 M_p}{l}$$

(ii)



$$-D\delta = 1$$

$$-n = D\delta + 1 = 2$$

- from elastic analysis

• Max -ve BM, i.e. BMA.

$$BMA = RB \times l - w \times l \times \frac{l}{2}$$

$$\Rightarrow \frac{3wl^2}{8} - \frac{wl^2}{2}$$

$$BMA = -\frac{wl^2}{8}$$

Max +ve BM, i.e. BMC.

$$\Rightarrow BMC \Rightarrow RB \times \frac{3l}{8} - w \times \frac{3l}{8} \times \frac{3l}{8} \times \frac{1}{2} \quad \text{for location of C, } SFC = 0$$

$$= \frac{3wl}{8} \times \frac{3l}{8} - \frac{w \times 9l^2}{8 \times 8 \times 2}$$

$$\Rightarrow \frac{9wl^2}{8 \times 8} - \frac{9wl^2}{8 \times 8 \times 2}$$

$$\frac{9wl^2}{8 \times 8} \left(1 - \frac{1}{2}\right)$$

$$\Rightarrow \frac{9wl^2}{8 \times 8 \times 2} = \frac{9wl^2}{128}$$

$$-RB + wx = 0$$

$$x = \frac{RB}{w} = \frac{3wl}{8 \times w} = \frac{3l}{8}$$

$$x = \frac{3l}{8}$$

For location of point of contraflexure $BM = 0$ at D

$$RB \times x - w \times x \times \frac{x}{2} = 0$$

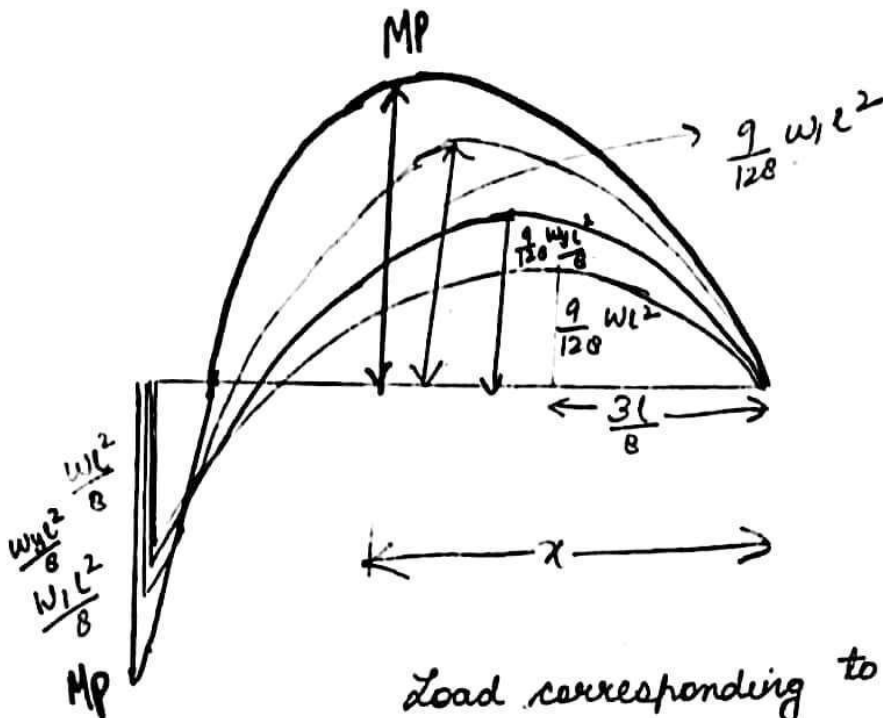
$$\frac{3wl}{8} x = \frac{wx^2}{2} \quad \Rightarrow x = \frac{3l}{4}$$

- from plastic Analysis.

- since max -ve BM is more than +ve BM, first plastic hinge is developed at fixed support (A) & when the beam is further loaded a second plastic hinge is NOT developed at point C i.e. at d/s of $\frac{3l}{8}$ from

propped end as it shifts towards the left with increase in loading from.

$$w \rightarrow w_1 \rightarrow$$

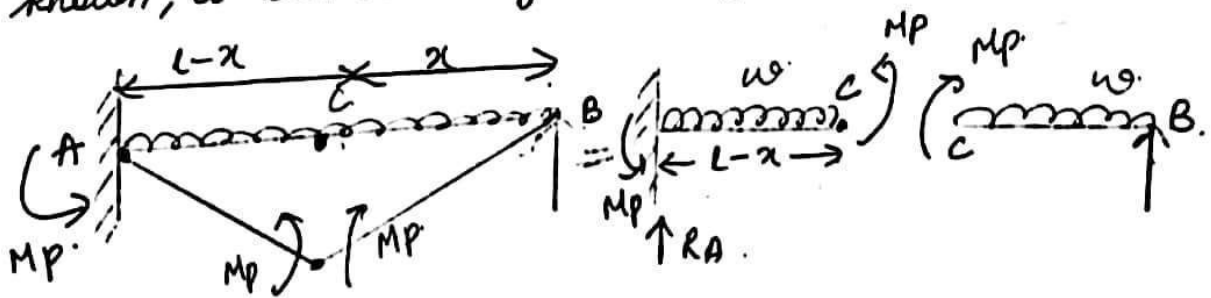


Load corresponding to first hinge.

$$\frac{w_1 L^2}{8} = M_p.$$

$$w_1 = \frac{8 M_p}{L^2}$$

- Since the location of second plastic hinge is not known, it can be analysed using FBD concept.



$$\sum M_A = 0$$

$$\Rightarrow +M_p - w(l-x)\frac{(l-x)}{2} + M_p = 0$$

$$M_p = \frac{w(l-x)^2}{4} \quad \text{--- (i)}$$

$$\sum M_B = 0 \Rightarrow M_P - w x \cdot \frac{x}{2} = 0$$

$$M_P = \frac{w x^2}{2} \quad - (ii)$$

from (i) & (ii)

$$\frac{w(l-x)^2}{2} = \frac{w x^2}{2}$$

$$\frac{(l-x)^2}{2} = x^2$$

$$l-x = \sqrt{2} x$$

$$l = \sqrt{2} x + x$$

$$l = x(1+\sqrt{2})$$

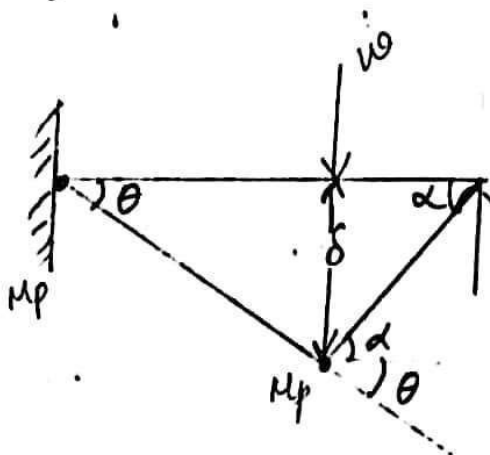
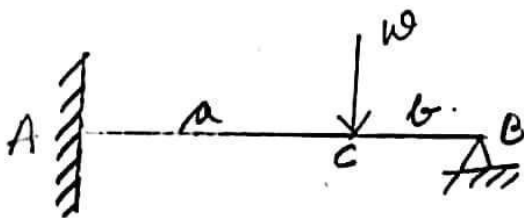
$$x \Rightarrow 0.414 l$$

from eq. & ii) $M_P = \frac{w_u (0.414 l)^2}{2}$

$$w_u = \frac{2 M_P}{0.171 l^2} \Rightarrow \frac{11.66 M_P}{l^2}$$

$$w_u = 11.66 \frac{M_P}{l^2}$$

(iii)



$$- D\delta = 1$$

$$- n = 1+1 = 2$$

$$- a\theta = b\alpha$$

$$- u_e = u_i$$

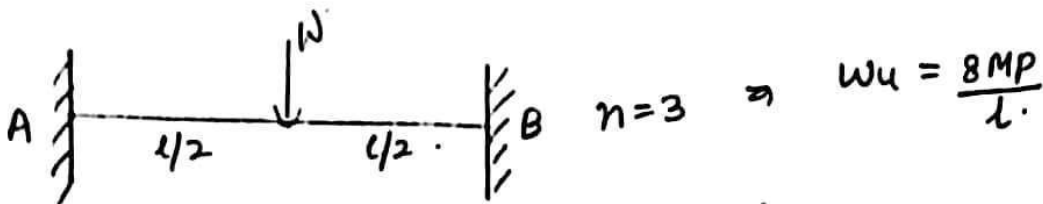
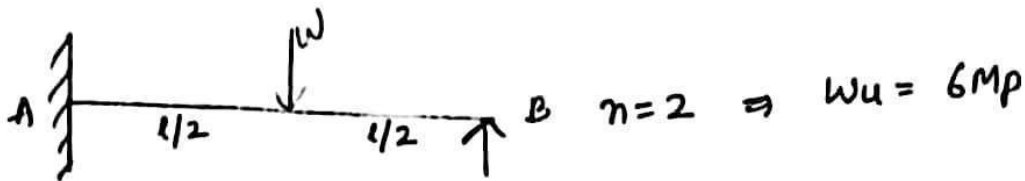
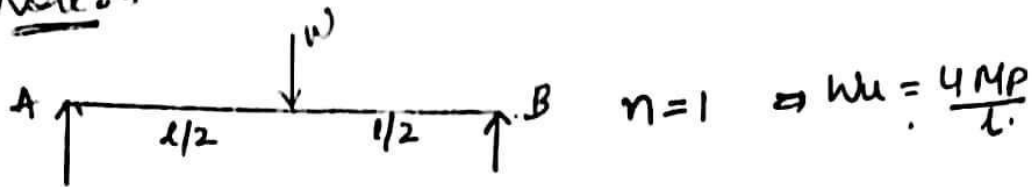
$$w a \theta = M_P \theta + M_P (\theta + \alpha)$$

$$w a \theta = M_P \theta + M_P (\theta + \frac{a}{b} \theta)$$

$$w a = M_P + M_P + M_P \frac{a}{b}$$

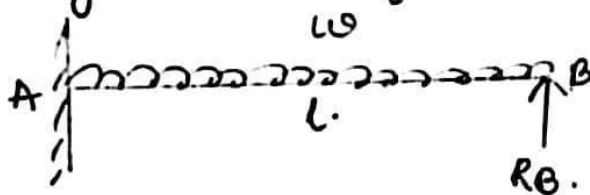
$$w_u = \frac{M_P (l+b)}{ab}$$

Note:-



Hence, as the no. of plastic hinges increases, collapse load also increases.

Q A propped cantilever is subjected to UDL of $w \text{ KN/m}$ throughout its length, compute its prop reaction



from elastic analysis.

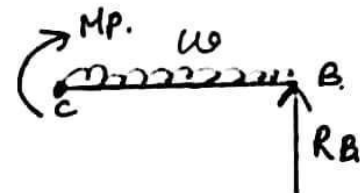
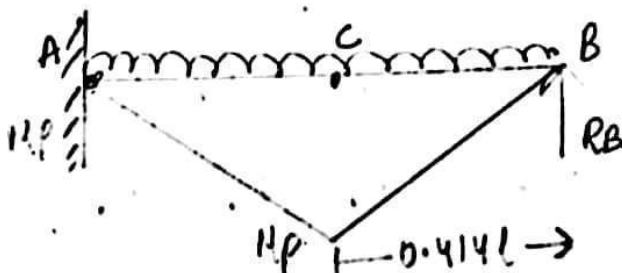
$$\delta_B = 0$$

$$\delta_1 - \delta_2 = 0$$

$$\frac{wL^4}{8EI} - \frac{R_B L^3}{3EI} = 0$$

$$\boxed{R_B = \frac{3wL}{8}}$$

Q A propped cantilever is subjected to collapse UDL of $w \text{ KN/m}$ throughout its length. Compute its prop reaction



a) $\sum F_y = 0$

$$R_B = w \times 0.414l$$

$$= 0.414wL$$

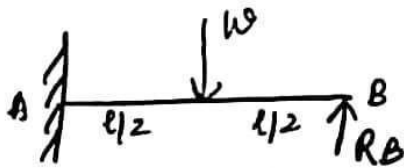
or b) $\sum M_C = -M_P$

$$R_B \times 0.414l - w \times (0.414l) \frac{(0.414l)}{2} = +M_P$$

$$R_B (0.414l) - \frac{w(0.414l)^2}{2} = - \cancel{10.1212} \frac{w(0.414l)^2}{2}$$

$$R_B = 0.414wl$$

Q. A propped cantilever is subjected to a central point load w . Compute the prop reaction.



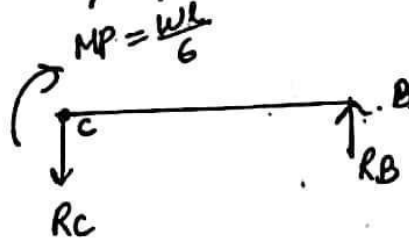
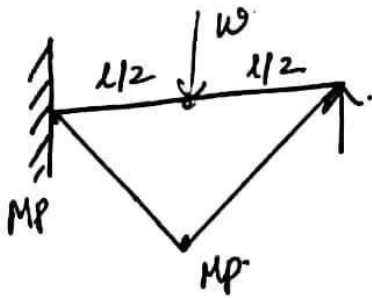
$$\delta_B = 0$$

$$\delta_1 - \delta_2 = 0$$

$$\frac{w(l/2)^3}{3EI} + \frac{w(l/2)^2}{2EI} \times \frac{l}{2} - \frac{R_B l^3}{3EI} = 0$$

$$R_B = \frac{5}{16} w$$

Q. A propped cantilever is subjected to a central collapse load w , find the prop reaction?

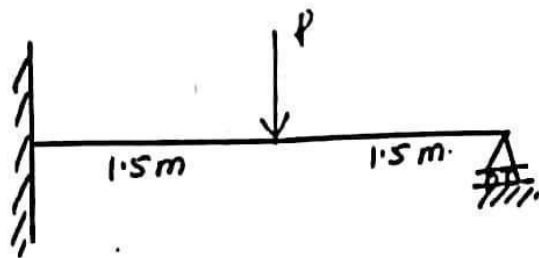


$$\sum M_C = 0$$

$$R_B \times \frac{l}{2} - \frac{wl}{6} = 0$$

$$\boxed{R_B = \frac{w}{3}}$$

Q. Magnitude of P is increased till collapse & M_P of the steel beam section is 90 kN-m . What is value of R prop reaction if $P = 80 \text{ kN}$. Using elastic analysis



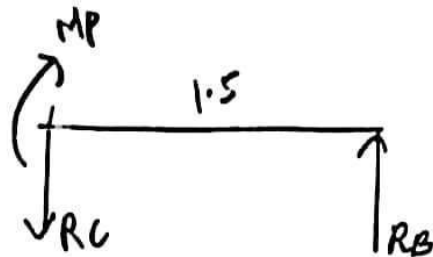
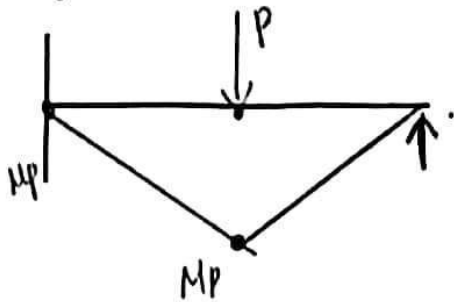
$$R_B = \frac{5}{16} w_0 \cdot$$

$$= \frac{5}{16} \times 80$$

$$= 25 \text{ kN}$$

Note:

In same case what is the prop reaction using plastic analysis.



$$\sum M_C = 0$$

$$R_B \times 1.5 = M_p$$

$$R_B = \frac{90}{1.5} = 60 \text{ kN}$$

Here collapse load is not $P = 80 \text{ kN}$ & is given by.

$$\sum F_y = 0 \Rightarrow R_C = R_B = 60 \text{ kN}$$

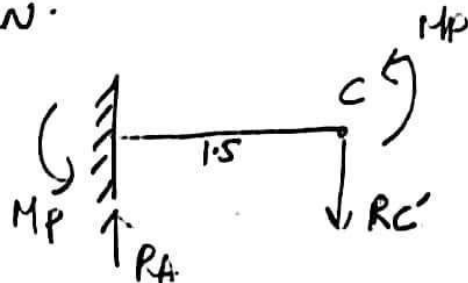
$$\text{also } \sum M_A = 0$$

$$R_C' \times 1.5 - M_p - M_p = 0$$

$$R_C' = \frac{2M_p}{1.5}$$

$$R_C' = 2 \times \frac{90}{1.5} = 120 \text{ kN}$$

$$W_u P = 60 + 120 = 180 \text{ kN}$$

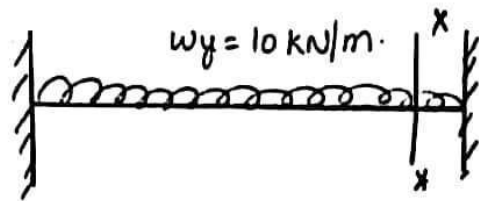


Note: Also, the load at which, plastic hinge would be formed at fixed end is given by.

$$M_p = \frac{3 P L}{16}, \quad P = \frac{16 M_p}{3 L} = \frac{16 \times 90}{3 \times 3} = 160 \text{ kN}$$

Q A steel beam of rectangular section is clamped at both ends. Plastic deformation is just observed when UDL on the beam is 10 kN/m , at the instant of collapse what is the load on the beam.

Soln



For rectangular s/c

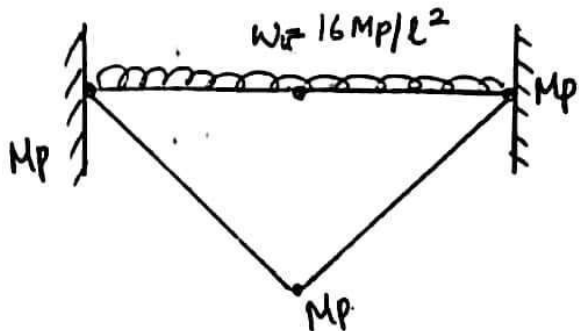
$$SF = 1.5$$

$$SF = \frac{M_p}{m_y}$$

$$M_p = SF m_y$$

$$= 1.5 \times 10$$

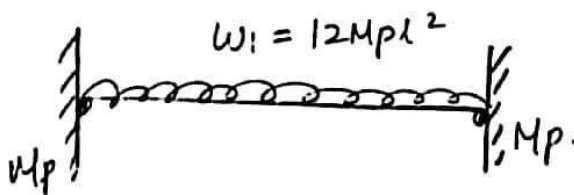
$$M_p = 15 \text{ kN-m}$$



$$m_y = \frac{w_y l^2}{12}, M_p = \frac{w_u l^2}{16}$$

$$SF = \frac{w_u l^2}{16 \cdot w_y l^2} \times 12$$

$$SF \Rightarrow 1.5 = \frac{3}{4} \frac{w_u}{10}$$



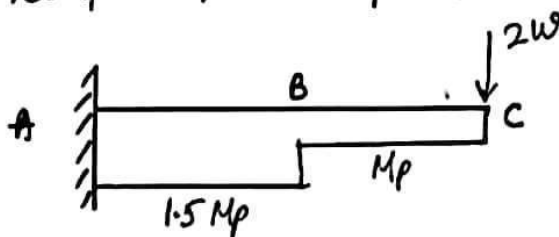
Note: Load at which first two hinges are formed $w_1 = \frac{12 M_p}{l^2}$

$$w_u = 20 \text{ kN/m}$$

$$w_1 = \frac{12}{l^2} \cdot \frac{w_u l^2}{16} = \frac{12 \times 20}{16} = 15 \text{ kN/m}$$

Q Compute the collapse load for the given beam.

Q)



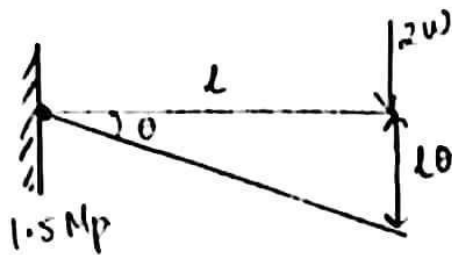
$$D_B = 2 - 2 = 0$$

$$n = D_B + 1 = 1$$

- Plastic hinge are formed at max. BM location, at fixed ends or where c/s changes, hence in this case

Plastic hinge can develop at either A or B.

Case 1) Plastic hinge at A

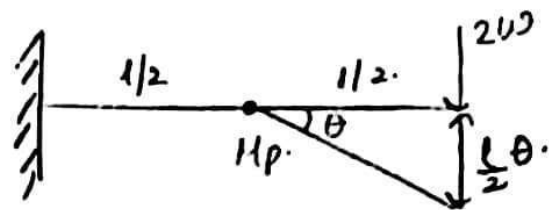


$$u_e = u_i$$

$$2w \times l\theta = 1.5M_p\theta$$

$$W_u = \frac{0.75M_p}{l}$$

Case 2) Plastic hinge at B



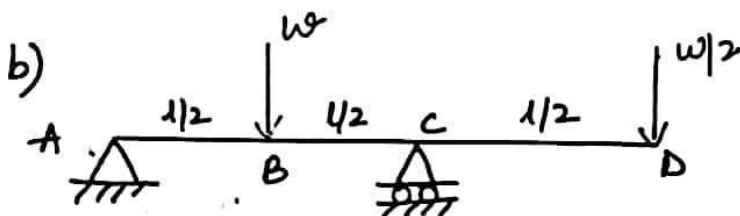
$$u_e = u_i$$

$$2w \frac{l}{2} \theta = M_p \theta$$

$$W_u = \frac{M_p}{l}$$

Hence $W_u = 0.75 \frac{M_p}{l}$

Correct failure mechanism is first mechanism & true collapse load is $0.75 \frac{M_p}{l}$.



$$-Ds = 2 - 2 = 0 \Rightarrow n = 0 + 1 = 1$$

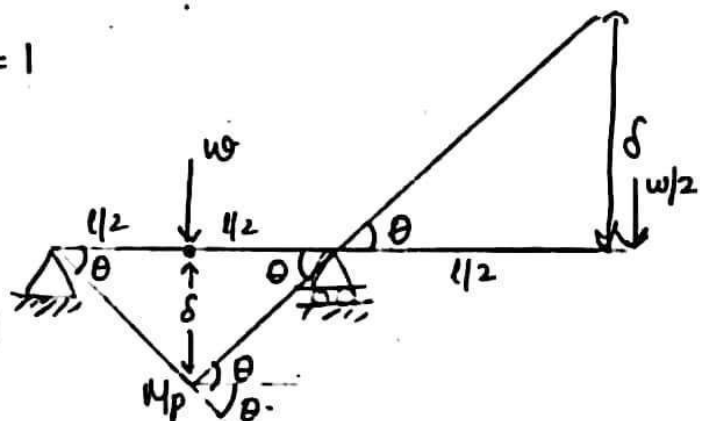
Case 1) Plastic hinge at "B".

$$u_e = u_i$$

$$w \cdot \frac{l}{2} \theta - \frac{w}{2} \cdot \frac{l}{2} \theta = M_p(\theta + \theta)$$

$$\frac{wl\theta}{4} = M_p(2\theta)$$

$$W_u = \frac{8M_p}{l}$$

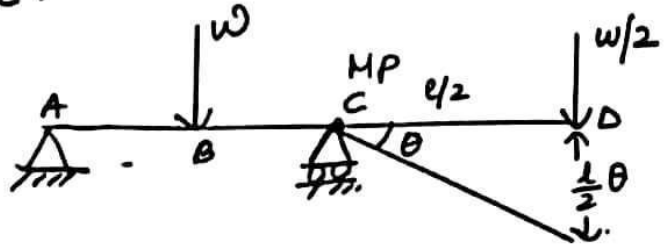


Case 2) plastic hinge at C.

$$u_c = u_c^0$$

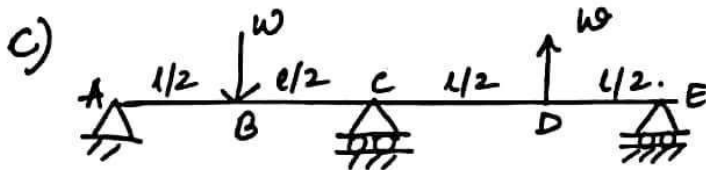
$$\frac{w}{2} \cdot \frac{l}{2} \theta = M_p \theta$$

$$w u = \frac{4 M_p}{l}$$



Hence, correct failure mechanism is case (2) & ~~same~~
true collapse load = $\boxed{\frac{4 M_p}{l}}$

Note:- In plastic analysis, all the loads must be applied simultaneously & increased gradually till collapse.
(as principle of superposition cannot be applied when material is loaded beyond elastic range).



$$D \delta = 1 - 5$$

$$= 3 - 2 = 1.$$

$$\Rightarrow n = 1 + 1 = 2.$$

Case 1) plastic hinges at B & D.

$$w \cdot \frac{l}{2} \theta + w \times \frac{l}{2} \theta = M_p (\theta + \theta)$$

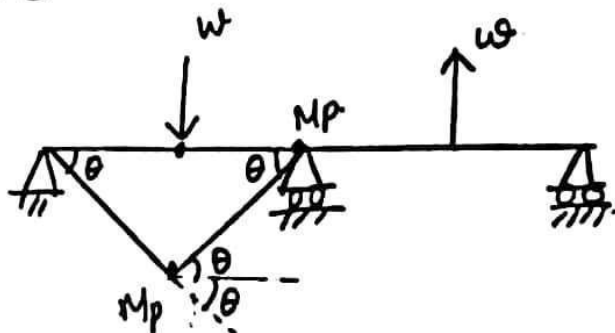
$$+ M_p (\theta + \theta)$$

$$w u = \frac{4 M_p}{l}$$

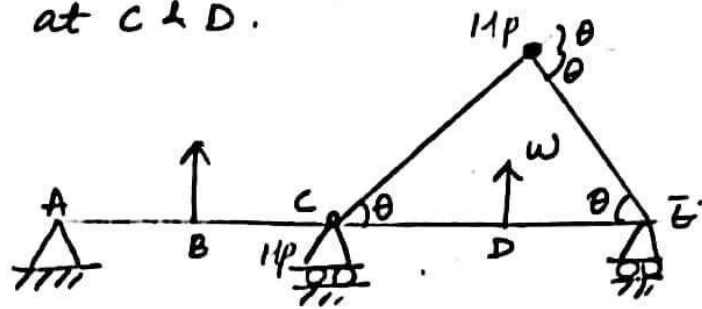
Case 2) plastic hinge at B & C

$$w u = \frac{6 M_p}{l}$$

This case is similar to prop cantilever.



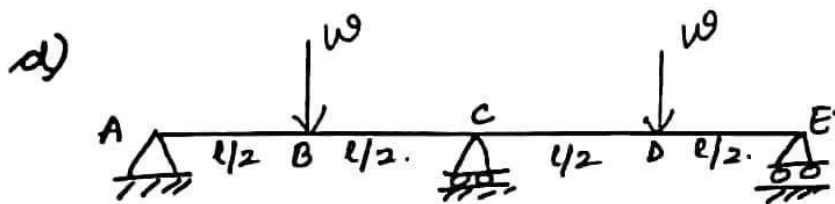
Case 3) plastic hinges at C & D.



$$W_u = \frac{6Mp}{l}$$

— Hence, correct failure mechanism is first mechanism, with true collapse load as $W_u = \frac{4Mp}{l}$.

Note:- If external work done is more, then collapse load required will be less

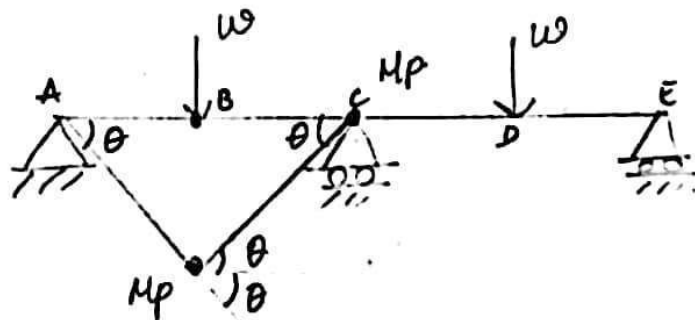


$$D_s = 3 - 2 = 1$$

$$n = 1 + 1 = 2.$$

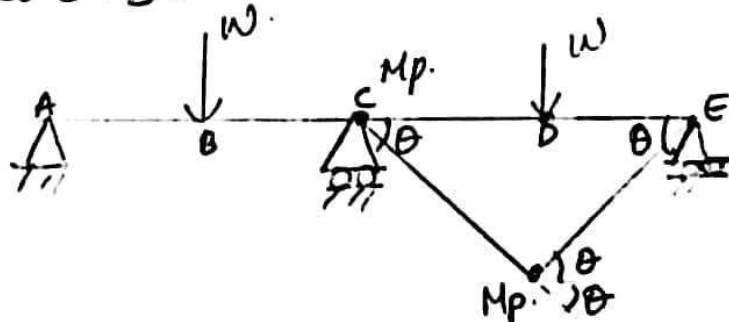
Case 1) plastic hinge at B & C.

$$W_u = \frac{6Mp}{l}$$



Case 2) plastic hinge at C & D.

$$W_u = \frac{6Mp}{l}$$



Case 3) plastic hinges at B + D.

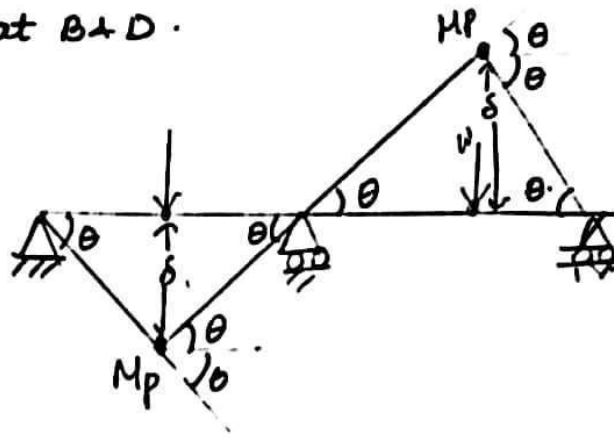
$$U_e = U_i$$

$$w \frac{l}{2} \theta + w \frac{l}{2} \theta = M_p(\theta + \theta)$$

$$+ M_p(\theta + \theta)$$

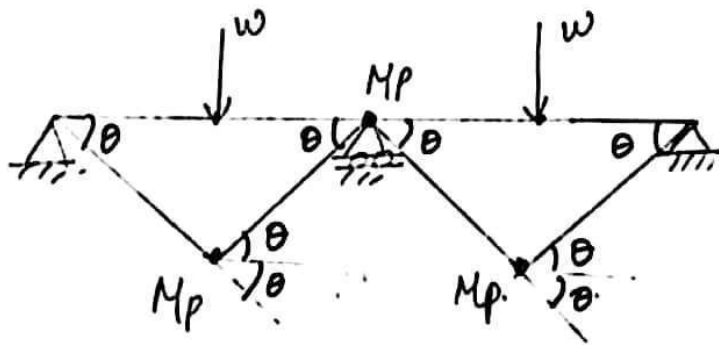
$$M_p \cdot \theta = 0$$

$$M_p \neq 0 \Rightarrow \theta = 0$$



Hence, true collapse load is $\frac{6M_p}{l}$.

- If external work done is zero, then θ is zero. $\Rightarrow$ This mechanism is not possible.
- If any continuous beam is subjected to downward loading only then failure mechanism of individual span will decide true load (collapse) & not $(D_s + 1)$ plastic hinges.
- If two mechanisms have same collapse load, then two mechanisms happens simultaneously.



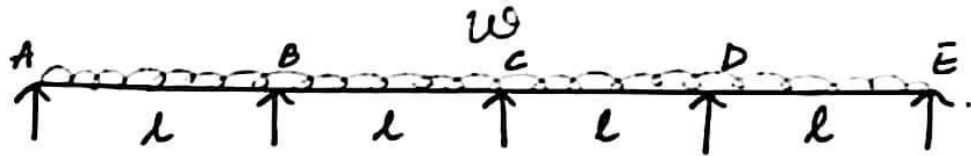
Here, over complete mechanism, 3 plastic hinges will be formed i.e. $\geq D_s + 1$

Note - If mechanism forms at $(D_s + 1)$ plastic hinges $\Rightarrow$ complete collapse

if mechanisms forms at $< (D_s + 1)$ plastic hinges $\Rightarrow$ partial collapse

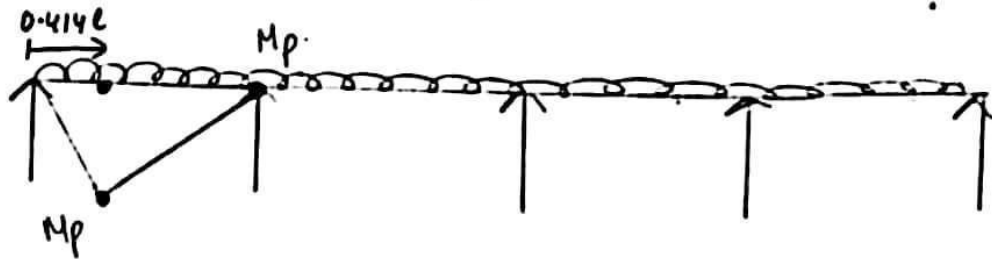
if mechanisms forms at $> (D_s + 1)$ plastic hinges $\Rightarrow$ over complete collapse

e)



Solⁿ since it is continuous beam, subject to a downward loading, only failure mechanism of individual span will decide the true collapse load & not $(Ds+1)$ plastic hinges

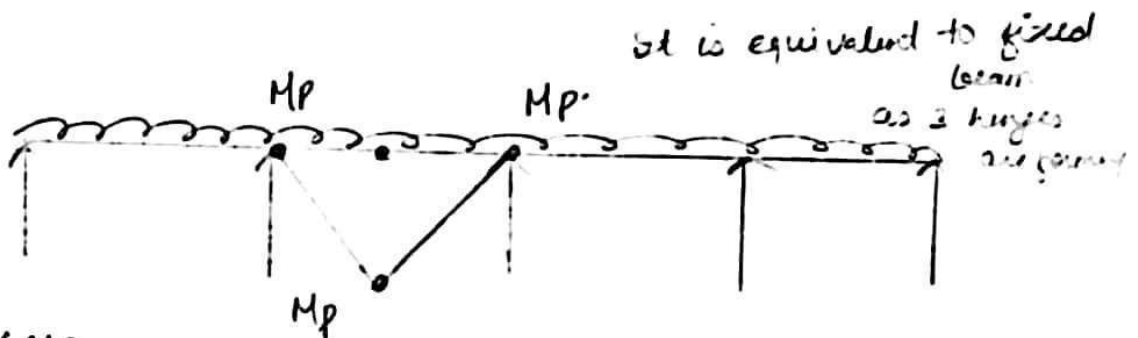
Case 1) Mechanism in AB & DE span



$$W_u = \frac{11.66 M_p}{l^2}$$

It is equivalent to fixed & prop cantilever as 2 hinges

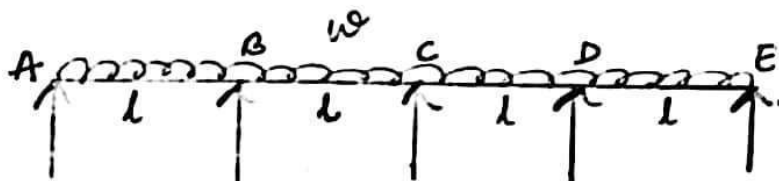
Case 2) Mechanism in BC & CD



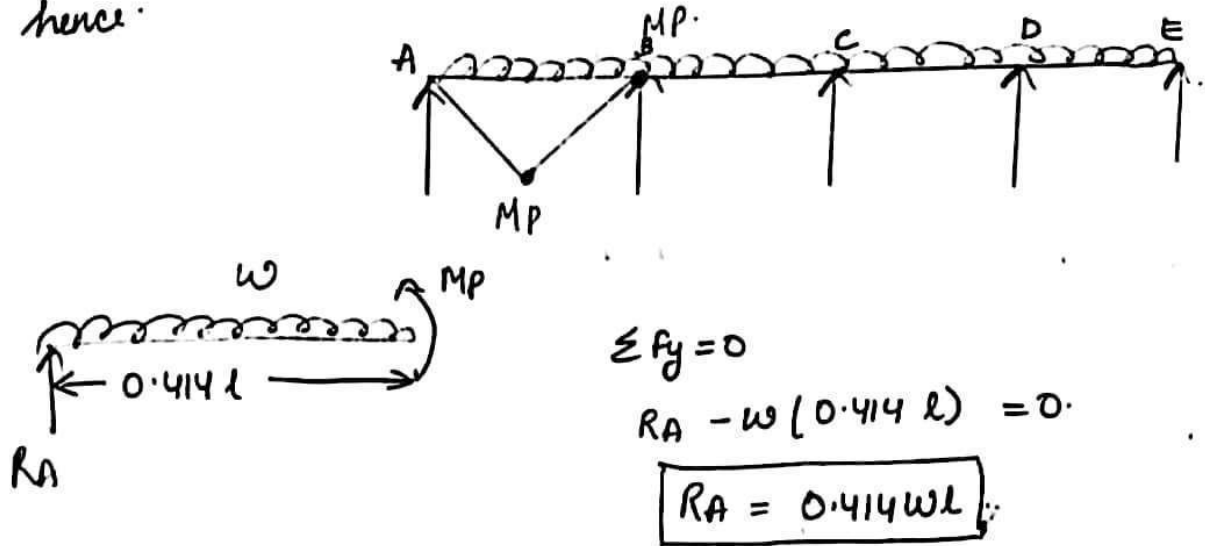
$$W_u = \frac{16 M_p}{l^2}$$

so, true collapse load is $\frac{11.66 M_p}{l^2}$

Q A continuous beam is subjected to a collapse UDL as given. Compute the reaction at support A.

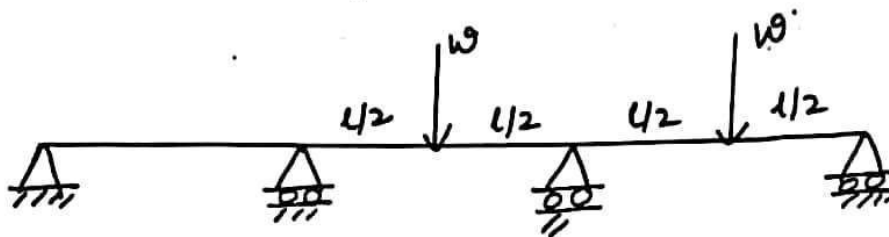


Solⁿ In this case, end span governs the collapse mechanism, hence.



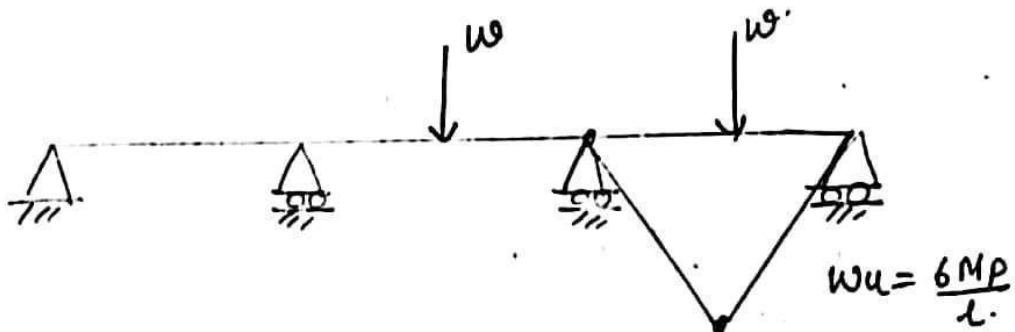
Q Compute the Collapse load.

a)

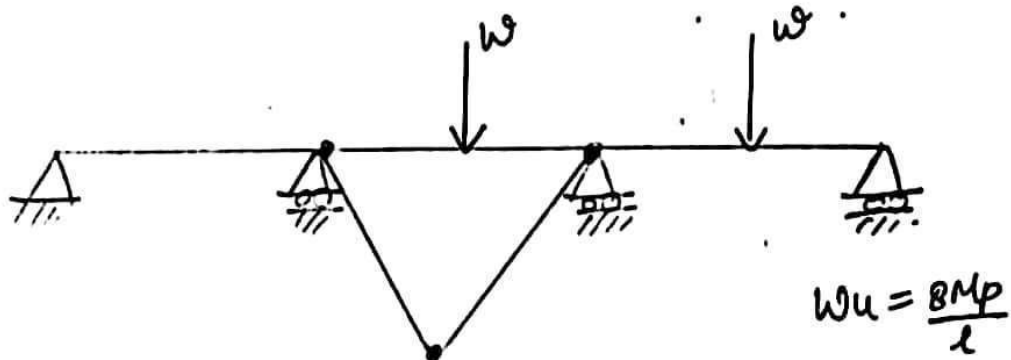


Solⁿ

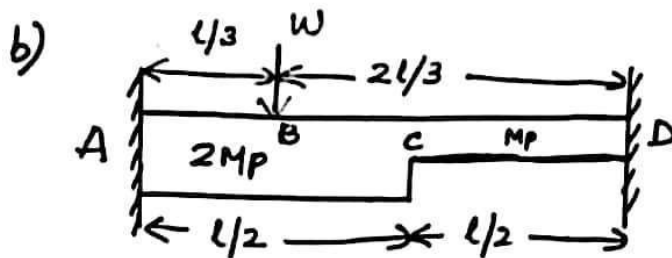
Case 1)



Case 2)



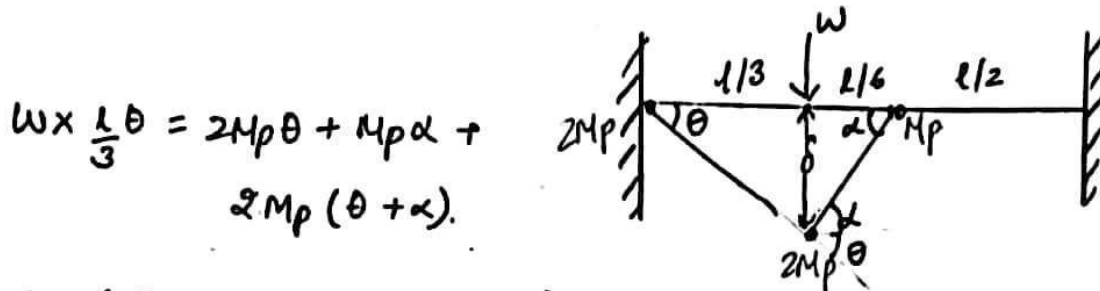
So, true collapse load = $\frac{6MP}{l}$



$$D.S = 4 - 2 = 2.$$

$$n = 2 + 1 = 3$$

Case (1) Plastic hinges at A, B, C.



$$w \times \frac{l}{3} \theta = 2M_p \theta + M_p \alpha + 2M_p (\theta + \alpha).$$

$$w \times \frac{l}{3} \theta = 2M_p \theta + M_p (2\theta) + 2M_p (\theta + 2\theta)$$

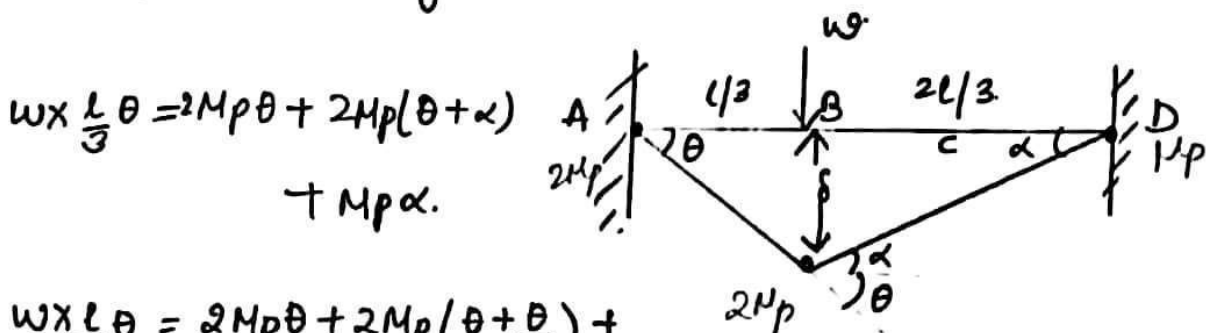
$$w \times \frac{l}{3} \theta = 10M_p \theta$$

$$w_u = \frac{30M_p}{l}$$

$$\frac{l}{3} \theta = \frac{l}{6} \alpha.$$

$$\theta = \frac{\alpha}{2} \rightarrow \alpha = 2\theta$$

Case (2) Plastic hinges at A, B, D.



$$w \times \frac{l}{3} \theta = 2M_p \theta + 2M_p (\theta + \alpha) + M_p \alpha.$$

$$w \times \frac{l}{3} \theta = 2M_p \theta + 2M_p (\theta + \frac{\theta}{2}) + M_p (\frac{\theta}{2})$$

$$w_u = \frac{16.5M_p}{l}.$$

$$\frac{l}{3} \theta = \frac{2l}{3} \alpha.$$

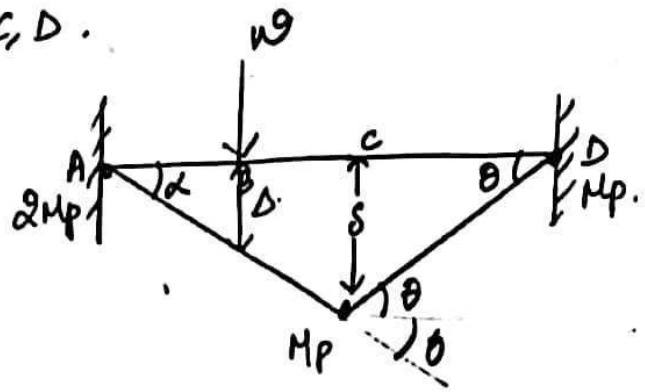
$$\theta = 2\alpha.$$

$$\alpha = \frac{\theta}{2}.$$

Case 3) Plastic hinges at A, C, D.

$$W \times \frac{1}{3} \theta = 2M_p \theta + M_p \theta + M_p (\theta + \theta)$$

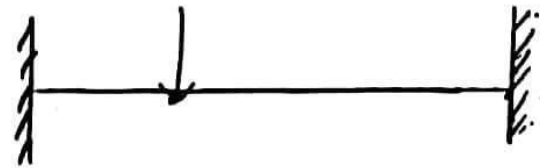
$$W_u = \frac{15M_p}{l}$$



Case 4). Plastic hinges at B, C, D.

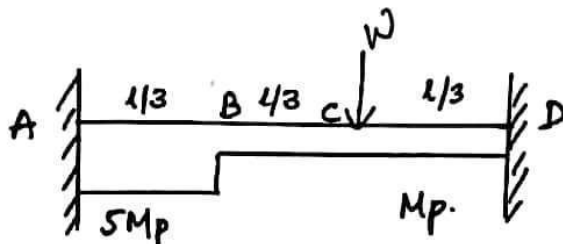
Not possible.

at B, rotation is not possible.



Hence, true collapse load. $W_u = \frac{15M_p}{l}$.

c)



Case 1) Plastic hinge at A, B, D.

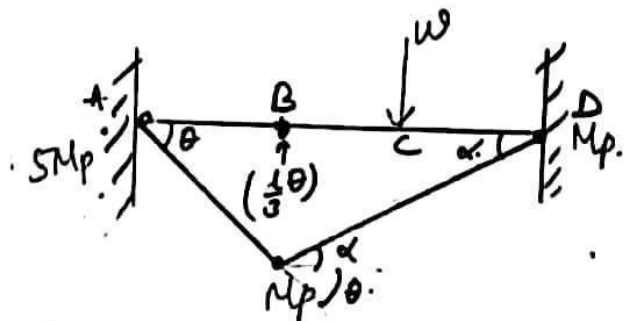
$$U_e = U_i$$

$$\frac{1}{3} \theta = \frac{2l}{3} \alpha$$

$$W \left(\frac{l}{3} \right) \left(\frac{1}{3} \right) \theta = 5M_p \theta + M_p \left(\theta + \frac{\theta}{2} \right) + M_p \frac{\theta}{2}$$

$$W_u = 10.6 \frac{M_p}{l}$$

$$W_u = 4.2 \frac{M_p}{l}$$

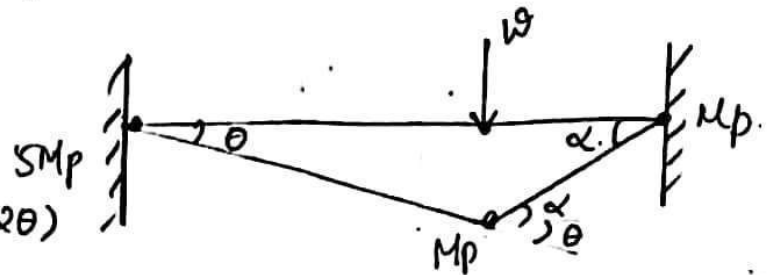


Case 2) Plastic hinge at A, C, D.

$$u_e = u_i$$

$$W \times \frac{2l}{3} \theta = 5M_p \theta + M_p(\theta + 2\theta) + M_p(2\theta).$$

$$W_u = \frac{15M_p}{l}.$$



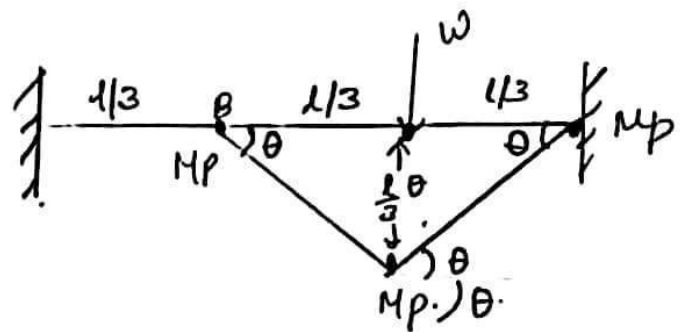
$$\frac{2l}{3} \theta = \frac{l}{3} \alpha.$$

$$2\theta = \alpha.$$

Case 3) Plastic hinge at B, C, D.

$$W \times \frac{l}{3} \theta = M_p \theta + M_p(\theta + \theta) + M_p \theta$$

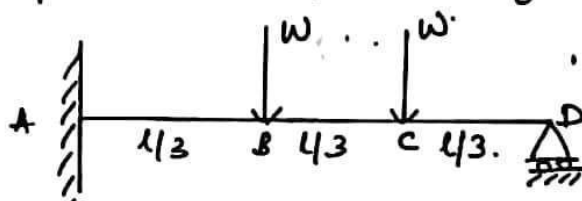
$$W_u = \frac{12M_p}{l}.$$



→ True collapse load = $\frac{12M_p}{l}$.

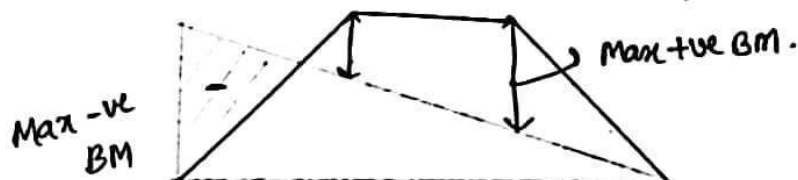
Lesson 43 Feb 3.

Q Compute the collapse load for the following



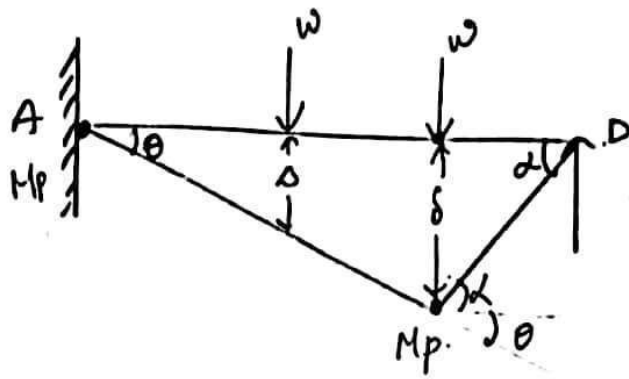
$$Ds = (2+1)\theta - 2 = 1$$

$$n = 1+1 = 2.$$



Qualitative BMD.

⇒ Hence, plastic hinge would be formed at pt "A" & "C", i.e. the point of max -ve BM & max +ve BM.



$$u_e = u_i$$

$$w\delta + w\delta = M_p\theta + M_p(\theta + \alpha)$$

$$\frac{2l}{3}\theta = \frac{1}{3}\alpha$$

$$\alpha = 2\theta$$

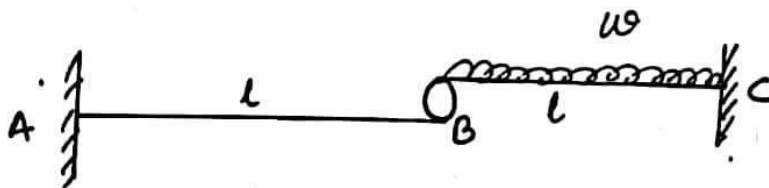
$$\frac{\delta}{\frac{2l}{3}} = \frac{\Delta}{\frac{l}{3}}$$

$$\Delta = \frac{\delta}{2}$$

$$w \times \frac{2l}{3} \times 2\theta + w \times \frac{l}{3} \times 2\theta = M_p\theta + M_p(\theta + 2\theta)$$

$$W_u = \frac{4M_p}{l}$$

b)

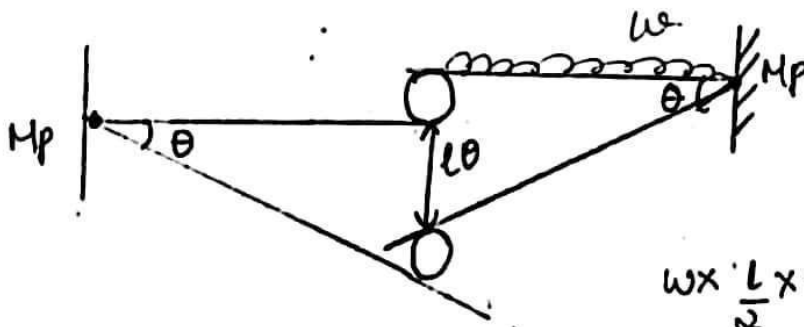


$$D.S = r - s$$

$$= 3 - 2 = 1$$

$$n = 1 + 1 = 2$$

Case 1) Mechanism in AB & BC.



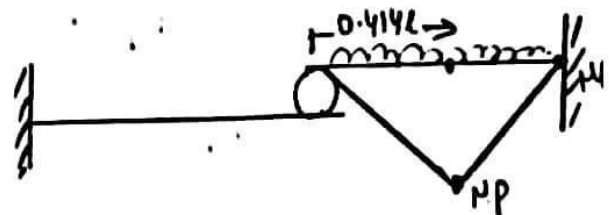
$$u_e = u_i$$

$$w \times \frac{l}{2} \times l\theta = M_p\theta + M_p\theta$$

$$W_u = \frac{4M_p}{l^2}$$

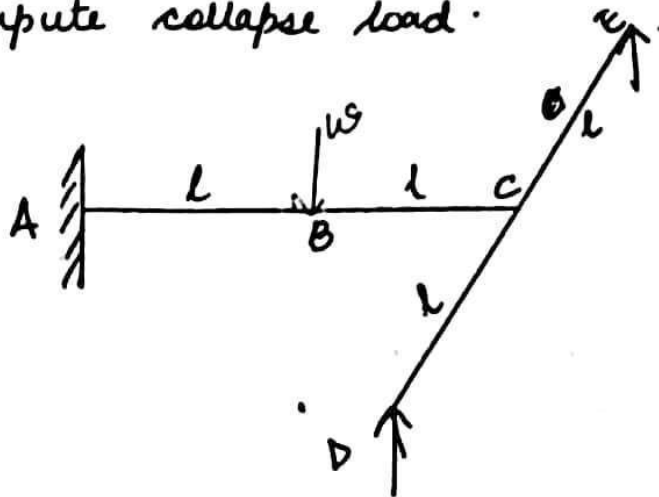
Case 2) Mechanism in BC.

$$W_u = 11.656 \frac{M_p}{l^2}$$



- Hence true collapse load $w_u = \frac{4M_p}{l^2}$

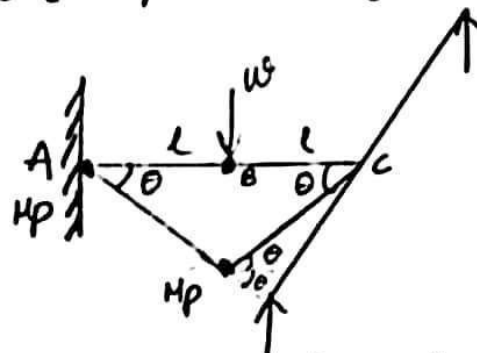
c) If the load w is supported by a cantilever resting on a simply supported beam as given. If M_p of simply supported beam is $\frac{3}{4} M_p$ of cantilever beam, Compute collapse load.



Case 1) Mechanism in AC alone (i.e. plastic hinge at A + B)

$$w \times l\theta = M_p\theta + M_p(\theta + \theta)$$

$$w_u = \frac{3M_p}{l}$$



Case 2) Mechanism in AC + DE (i.e. plastic hinge at A + C)

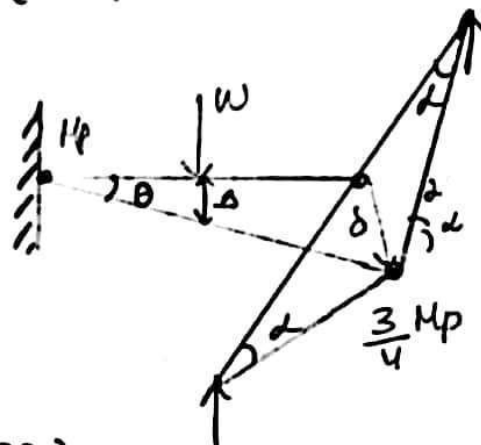
$$w \times \Delta = M_p\theta + \frac{3}{4} M_p(\alpha + \alpha)$$

$$\delta = 2l\theta = l\alpha$$

$$\alpha = 2\theta$$

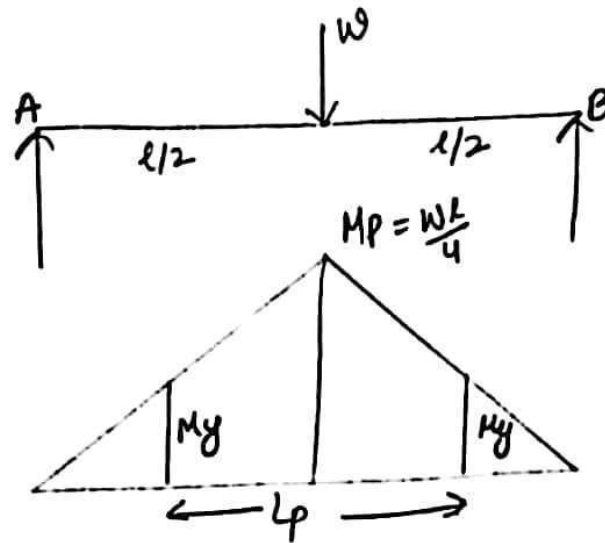
$$w \times l\theta = M_p\theta + \frac{3}{4} M_p(2\theta + 2\theta)$$

$$w_u = \frac{4M_p}{l}$$



Hence, true collapse load = $\frac{3M_p}{l}$

Q A simply supported beam of rectangular c/s is subjected to a central point load w . Compute the plastic length



$$\frac{M_P}{\frac{l}{2}} = \frac{M_y}{\frac{l - L_p}{2}}$$

$$\frac{l - L_p}{l} = \frac{M_y}{M_P}$$

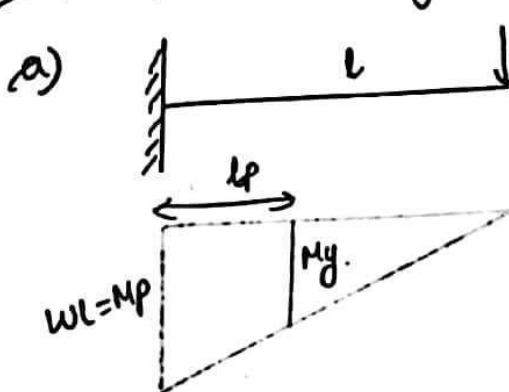
For rect. c/s. $SF = 1.5$

$$L_p = l \left(1 - \frac{1}{SF} \right)$$

$$= l \left(1 - \frac{1}{1.5} \right)$$

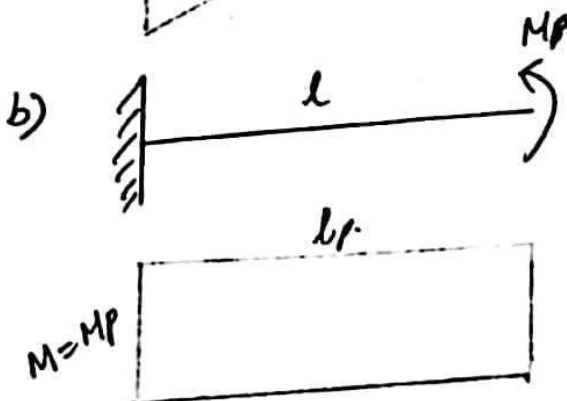
$$L_p = 0.33 L$$

Q Compute the length of plastic hinge for given cases.



$$\frac{M_P}{l} = \frac{M_y}{l - L_p} \Rightarrow l - L_p = \frac{M_y \cdot l}{M_P}$$

$$L_p = l \left(1 - \frac{1}{SF} \right) = l \left(1 - \frac{1}{1.5} \right) = \frac{l}{3}$$

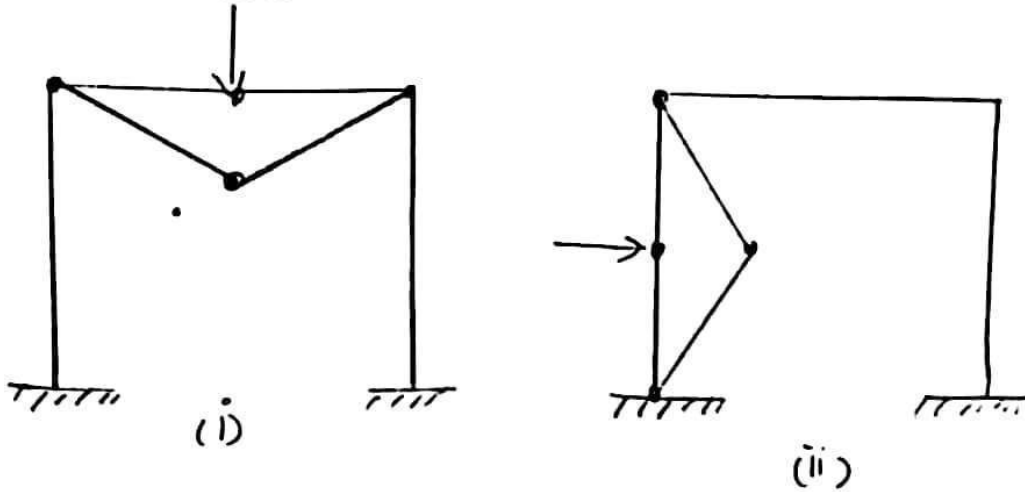


$$L_p = l \Rightarrow \text{since } M = M_P$$

Plastic Analysis of Frames.

- In plastic analysis of frames, the mechanism developed may be any of the following.

A) Beam Mechanism.



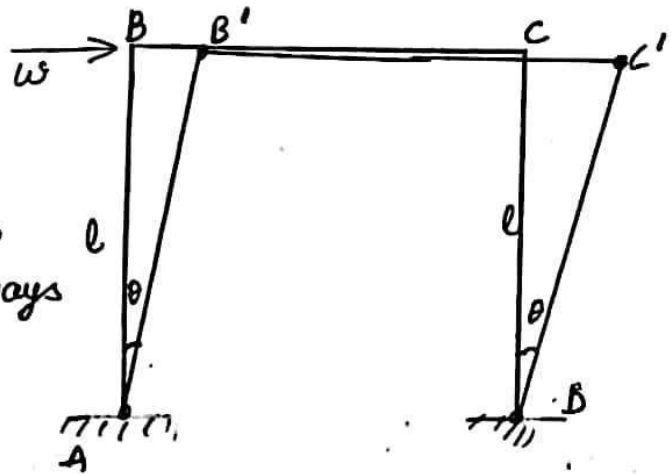
- In portal frame, if mechanism forms in only in one member (i.e. either in a column or in a beam), then it is called beam mechanism.

B) Sway Mechanism / Bay Mechanism

$$\text{here } BB' = CC' = l\theta$$

(as axial deformation is neglected)

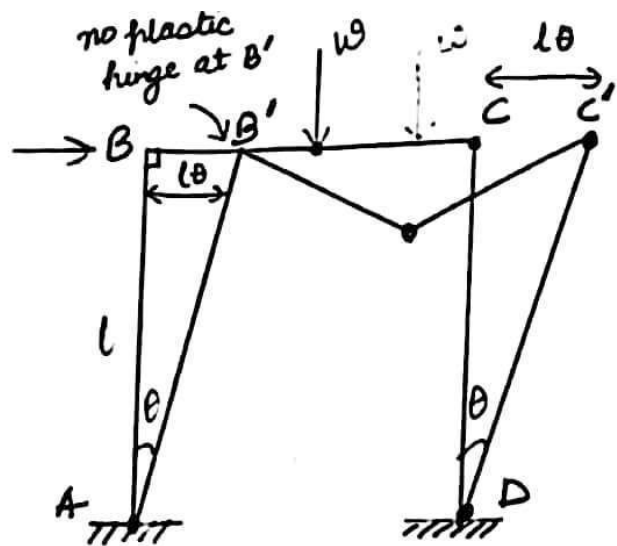
- In portal frame, if due to horizontal load, frame sways to either left or right, then it is called Sway Mechanism.



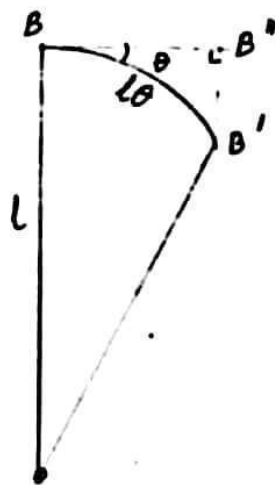
Note $\Rightarrow$ The above two mechanisms are also called Independent or Primary or elementary mechanism.

C) Combined Mechanism

If beam & sway mechanisms are combined, then it is termed as combined mechanism.



Note:-



$$BB' = l\theta$$

$$BB'' = BB' \cos \theta$$

$$BB'' = l\theta \cos \theta$$

$$BB'' \approx l\theta$$

$$B'B'' = BB' \sin \theta$$

$$B'B'' = l\theta \cdot \theta = l\theta^2 \approx 0$$

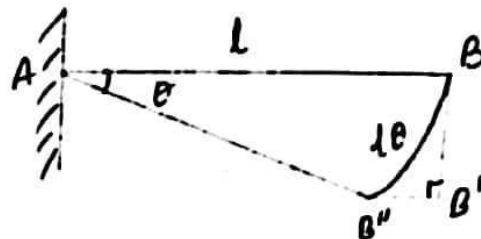
If θ is small.

$$\Rightarrow \cos \theta \approx 1, \sin \theta \approx \theta, \theta^n \approx 0 \quad (n > 1)$$

As θ is very small, downward displacement of joint "B" is neglected i.e. $B'B'' \approx 0$.

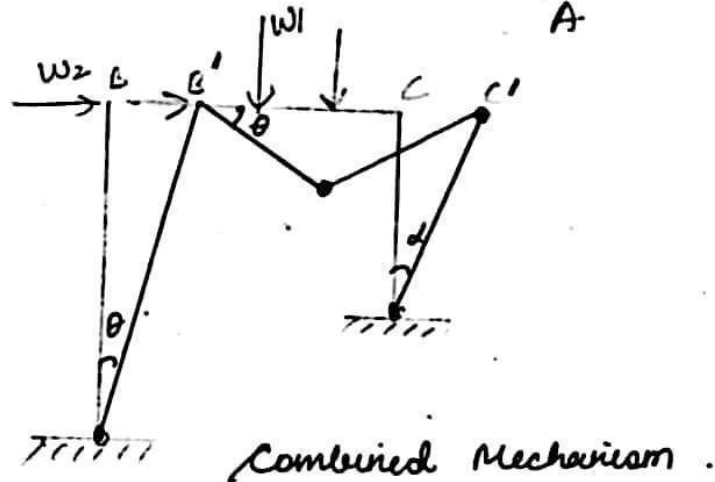
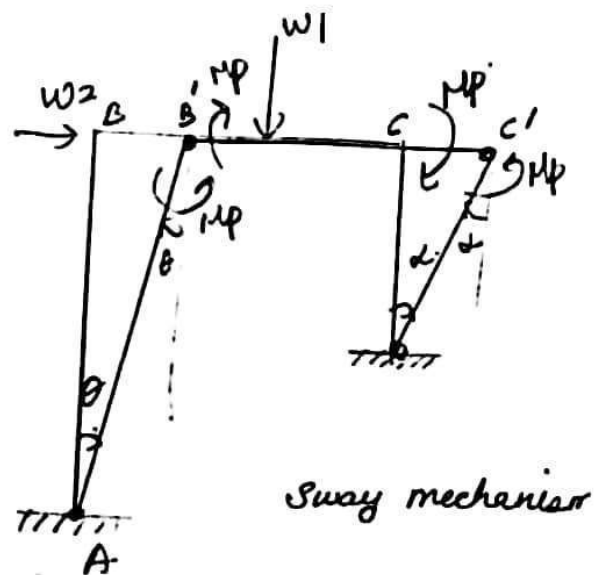
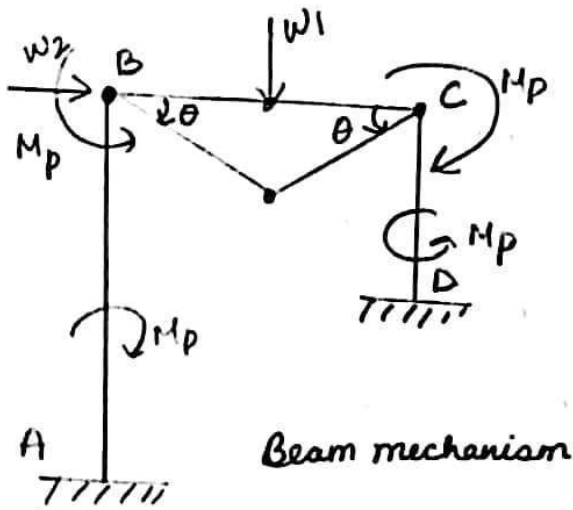
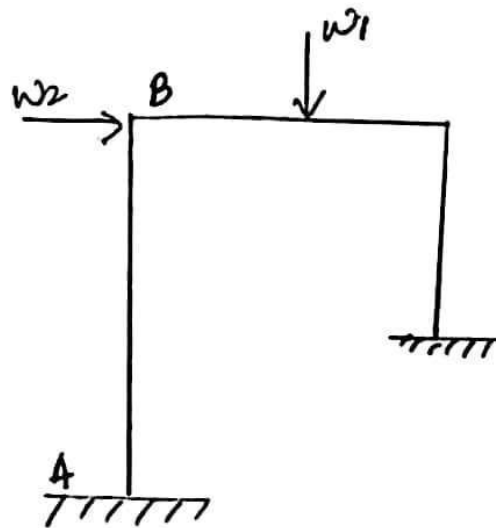
→ Hence, it is assumed that virtual displacements are always perpendicular to the initial configuration of the member.

For eg. →



$$B'B'' = l\theta \cdot \theta = l\theta^2 \approx 0$$

Note 2).



- In beam mechanism, at joint B in the beam, M_p is in anticlockwise direction & in sway mechanism at joint B, in the beam, M_p is in clockwise direction, hence net moment at B in the beam in combined mechanism is very less, so plastic hinge is not developed at B in the combined mechanism.

- In beam mechanism, at joint C in the beam, M_p is in clockwise direction & in sway mechanism, at joint C in the beam, M_p is also in clockwise direction, so net moment at C in the combined mechanism is very high.

⇒ If portal frame sways to the right in the combined mechanism, then plastic hinge is not developed at left corner & if the frame sways to the left in the combined mechanism, the plastic hinge is not developed at right corner.

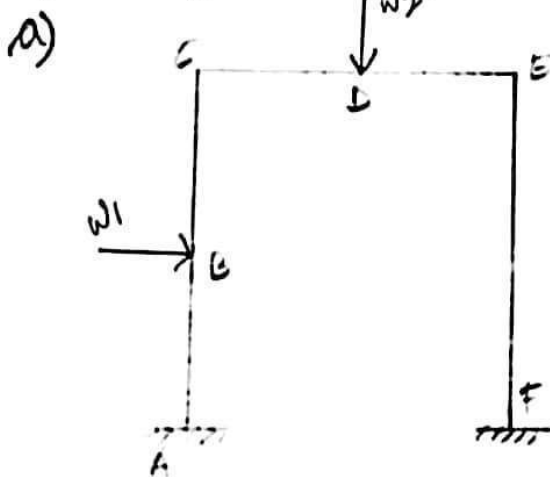
Lesson 44

- No. of Independent mechanisms that can be developed
 $K = N - D_s$.

N = no. of possible location of plastic hinge.

D_s = degree of static indeterminacy.

for eg: Compute the no. of Independent mechanisms for the given cases.



$$N = 6 \text{ (A, B, C, D, E, F)}.$$

$$D_s = 6 - 3 = 3.$$

$$K = N - D_s = 6 - 3 = 3 \quad \left\{ \begin{array}{l} 2 \text{ beam mech.} \\ 1 \text{ sway mech.} \end{array} \right.$$

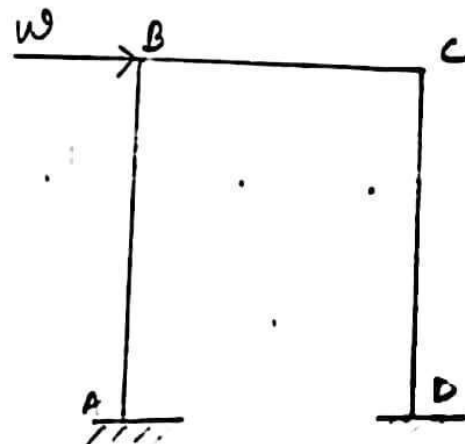
b)

$$N = 4 \text{ (A, B, C, D)}.$$

$$D_s = 6 - 3 = 6 - 3 = 3.$$

$$K = N - D_s = 4 - 3 = 1$$

(Sway Mechanism)



Q Compute the collapse load for given case.

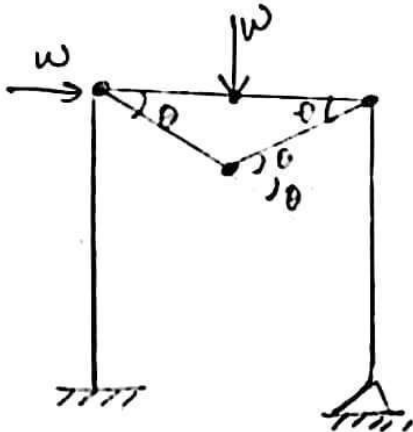
$$N = 4$$

$$D_s = 5 - 3 = 2$$

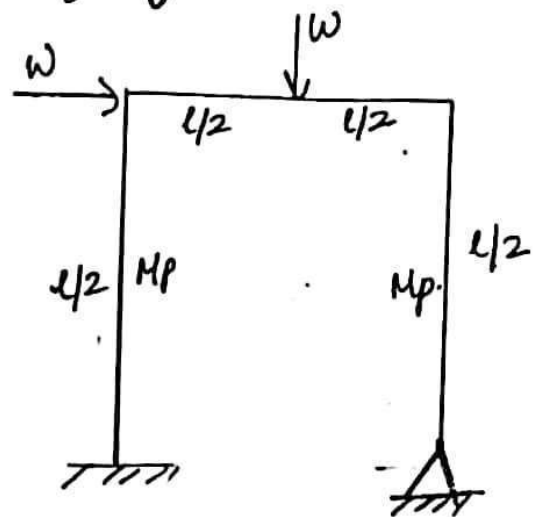
$$n = D_s + 1 = 3$$

$$K = 4 - 2 = 2$$

Case 1) Beam Mechanism



$$W_u = \frac{8M_p}{l}$$

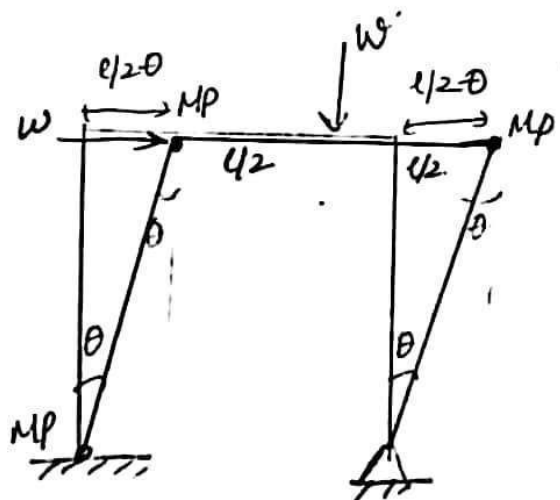


Case 2) Sway Mechanism

$$u_e = u_i$$

$$W \cdot \frac{l}{2} \theta + W \cdot 0 = M_p \theta + M_p \theta + M_p \theta$$

$$W_u = \frac{6M_p}{l}$$

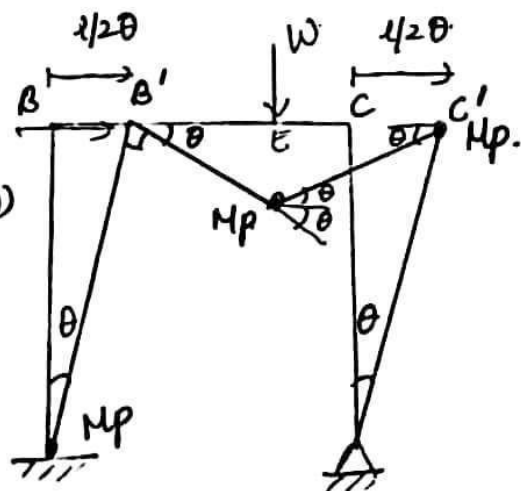


Case 3) Combined Mechanism

$$u_e = u_i$$

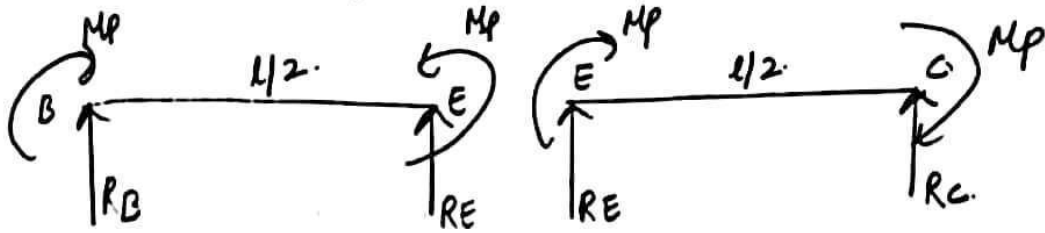
$$W \cdot \frac{l}{2} \theta = M_p \theta + M_p (\theta + \theta) + M_p (\theta + \theta)$$

$$W_u = \frac{5M_p}{l}$$



Hence, true collapse load $w_u = \frac{5M_p}{l}$

2) Also compute the moment at "B" at collapse



$$R_E = -\left(\frac{5M_p}{l} - \frac{4M_p}{l}\right) = -\frac{M_p}{l}$$

$$R_B = +\frac{4M_p}{l} \quad (R_E + R_B = 0)$$

$$\sum M_B = 0$$

$$R_E \cdot \frac{l}{2} + M_p - M_B = 0$$

$$M_B = -\frac{M_p}{l} \cdot \frac{l}{2} + M_p = \frac{M_p}{2}$$

$$\boxed{M_B = \frac{wl}{10}}$$

$$\sum M_C = 0$$

$$R_E \cdot \frac{l}{2} + M_p + M_p = 0$$

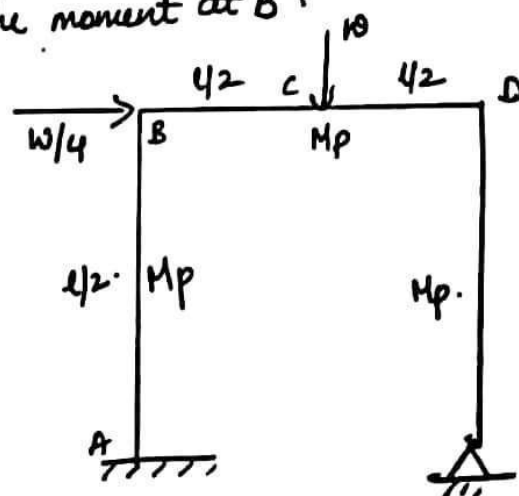
$$R_E = -\frac{4M_p}{l}$$

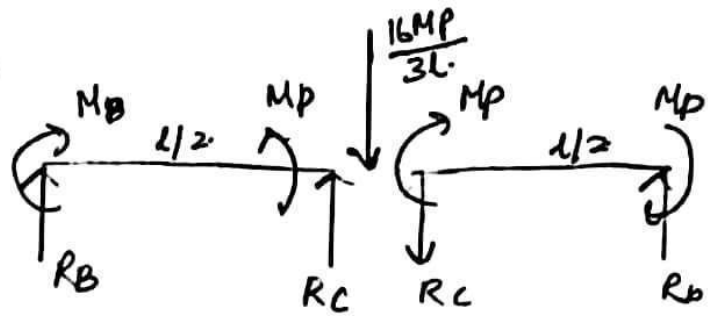
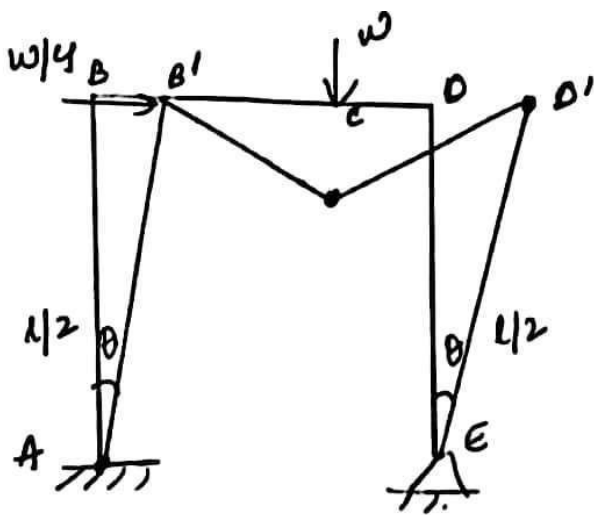
$$\sum F_y = 0$$

$$R_E + R_C = 0 \Rightarrow R_C = -R_E$$

$$R_C = \frac{4M_p}{l}$$

Q For the portal frame, show below collapse load based on on combined mechanism is worked out as $\frac{16M_p}{l}$. At collapse condition, compute the moment at B.





$$R_C = \frac{16Mp}{3l} - \frac{4Mp}{l}$$

$$\sum M_D = 0$$

$$R_C \times \frac{l}{2} = 2Mp$$

$$R_C = \frac{4Mp}{3l}$$

$$R_C = \frac{4Mp}{l}$$

$$R_B = 4Mp/3l$$

$$R_D = \frac{4Mp}{l}$$

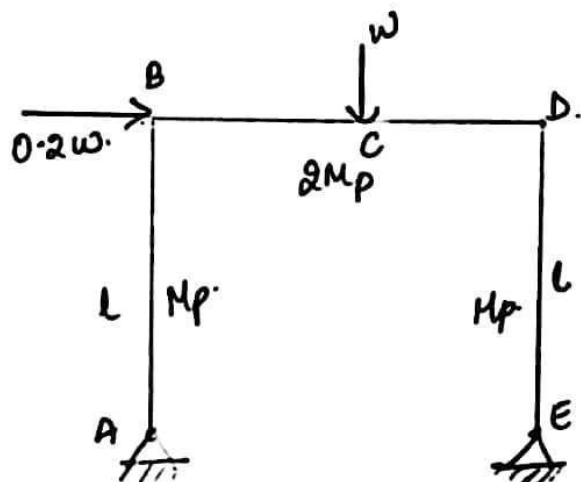
$$\sum M_B = 0$$

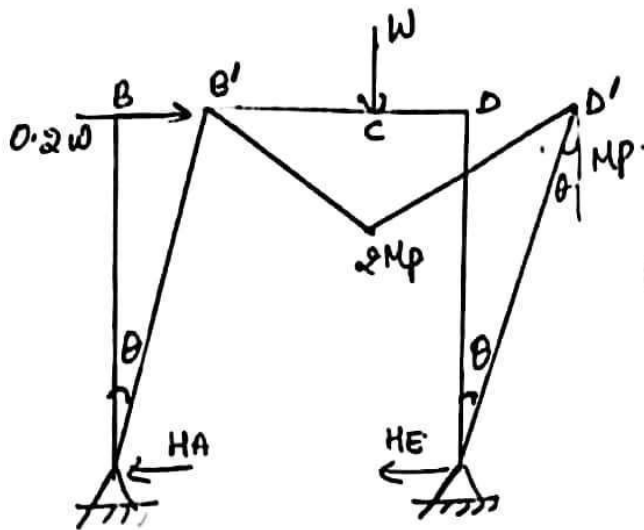
$$\frac{4Mp}{3l} \times \frac{l}{2} - Mp + M_B = 0$$

$$M_B = Mp - \frac{2Mp}{3} = \frac{Mp}{3}$$

$$M_B = \frac{3Wl}{16 \times 3} = \frac{Wl}{16}$$

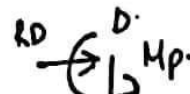
Q Collapse moment for the frame. given is worked out as $M_p = \frac{Wl}{5}$. What is the horizontal reaction at "A" at collapse condition





$$\sum F_x = 0$$

$$H_A + H_E = 0.2W \quad \text{--- (i)}$$



$$R_D - H_E = 0$$

$$R_D = H_E \quad \text{--- (ii)}$$

$$H_E \times L - M_P = 0$$

$$H_E = \frac{M_P}{L} = \frac{wL \times \frac{1}{5}}{L}$$

$$H_E = \frac{w}{5}$$

from eq (i) $H_A = 0.2W - H_E = 0.2W - 0.2W$

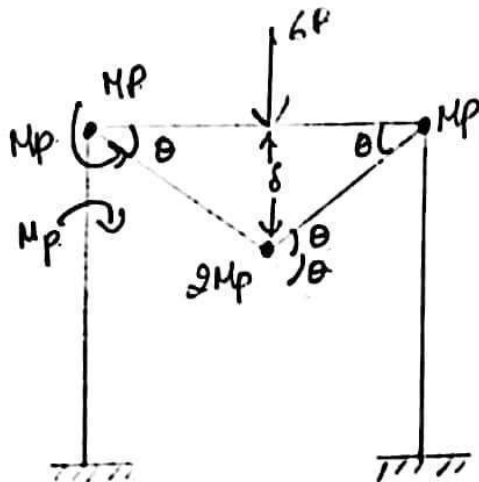
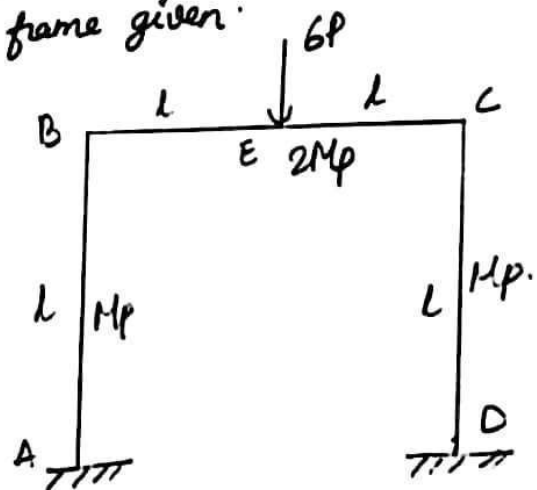
$$H_A = 0$$

Q At the instance of collapse, what is the horizontal thrust at joint A in the frame given.

$$N = 5$$

$$D.O.F = 6 - 3 = 3$$

$$K = 5 - 3 = 2$$



As both structure & loading is symmetric sway is not possible, i.e. only beam mechanism would be observed.

$$u_e = u_i$$

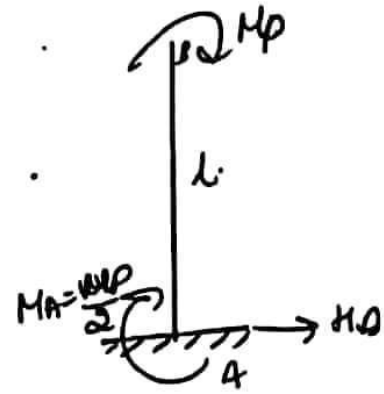
$$6P \times l \theta = M_p \theta + 2M_p(\theta + \theta) + M_p \theta$$

$$M_p = Pl - (i).$$

$$\sum M_B = 0.$$

$$H_A \times l - M_p - \frac{M_p}{2} = 0$$

$$H_A = \frac{3}{2} \frac{M_p}{l} = \frac{3}{2} \cdot \frac{Pl}{l} = \frac{3}{2} P.$$



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